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NUCLEAR CRITICAL OPALESCENCE, A PRECURSOR TO PION CONDENSATION

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ABSTRACT

It is shown that pion condensation in nuclei, a long range phenomenon, has a precursor in the disordered phase, the local ordering of spins which becomes of infinite range at the critical point. A new physical effect arising from this short range order is predicted, namely the enhancement of the static nuclear pion field near the critical momentum. This phenomenon is strongly reminiscent of the critical opalescence observed in the scattering of neutrons by antiferromagnetic substances.

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Since Migdal's first suggestion of the possible existence of a transition in nuclei to a condensed pion phase [1] much effort has been spent on the finding of signatures of the condensed phase. On the experimental side it has not been possible to disprove that the condensed phase exists in actual nuclei, nor to prove that it does and the question remains open. Most theoretical works however seem to agree on the conclusion that the critical density is higher than the normal nuclear density ρ_0 . In the laboratory an increase in density can be achieved only in a transient way, which restricts the possibility of experimentations and complicates the interpretation.

The condensed phase is characterized by a non-vanishing value of an order parameter which is the expectation value of the pion field $\langle\varphi\rangle$. As it is difficult to establish whether the nuclei are in the ordered phase or not, it is interesting to investigate instead of the order parameter itself, its fluctuations which are present even in the disordered phase. They are very large in the vicinity of the transition point where they manifest themselves by critical phenomena, for instance critical opalescence. Even if not critical, the usual nuclear densities might be sufficiently close to the transition density for such phenomena to appear, signing the proximity of the phase transition. This is the problem we want to explore in this article.

The coupling between the nucleons and the pion field is of the form $\underline{\sigma}.\underline{\nabla}\varphi$ which tends to orientate the nucleon spins in the direction of $\underline{\nabla}\varphi$. An order parameter $\langle\varphi\rangle \neq 0$ with a non vanishing wave vector thus represents a long range ordering in the spins so that the transition to the pion condensed phase resembles a magnetic transition. Actually it is closer to an antiferromagnetic one since the spins are not all aligned in the same direction.

To prove the existence of an ordering in the nucleonic spins it is in principle possible to perform the equivalent of a Bragg magnetic scattering. A measurement with a spin sensitive probe will display strong peaks in the cross section when the momentum transfered to the nuclear system coincides with the characteristic momentum of the condensate, realizing the equivalent of the magnetic scattering of neutrons. In order to minimize the rescattering problems of the probe with the medium, a weakly interacting probe is preferable. Pion photoproduction has been suggested by one of us (M. E.). However it is not clear, even in the case of a condensed state that this Bragg peak could be displayed. The reason is that the condensed pion momentum may have a preferred direction in the nucleus. If this lack of isotropy is realized the momenta of the condensed pions should be parallel for all nuclei in the target in order to have the equivalent of a monocrystalline target, i. e. the experiment requires oriented nuclei.

The detection of critical phenomena instead is not submitted to the same restriction. Near the Curie point of a ferromagnet, neutrons undergo a strong scattering in the forward direction even with a polycrystalline target [2,3]; this is the magnetic critical opalescence which is due to the strong magnetization fluctuations and reflects the short range ordering in the medium [4]. As the Curie point is approached, the range of the ordering increases, becoming infinite at the Curie point. In contradistinction with the Bragg scattering, the critical scattering does not require a single domain since the short range ordering does not have a preferred direction.

In pion condensation the role of the inverse temperature is played by the nuclear density which can hardly be varied. A parallel situation would occur in magnetism if one were constrained to a unique temperature. If one admits the hypothesis that nuclei are subcritical, this temperature would lie above the Curie point so that the ordered phase would be out of reach. Nevertheless the existence of a strong forward scattering would be by itself the signature of the proximity of a ferromagnetic phase transition.

The aim of this article is to show that similar critical phenomena are relevant to the pion condensate problem and that according to the current descriptions based on linear response theory the phase transition is sufficently close for the critical effects to be detectable. The maximum effects will not occur in the forward direction i.e. at zero momentum as in the ferromagnetic case, but at a finite momentum related to the critical one q_c, as for antiferromagnets.

The local ordering of the spins is described by the spin-spin correlation function which is the quantity studied in the magnetic systems since it is the only measurable one. However, the nuclear case presents the remarkable feature that the pion field responsible for the spin ordering is experimentally accessible. We will concentrate therefore on the properties of the field and especially of its propagator which is a measure of the fluctuations of the order parameter.

Below the condensation threshold, the static (zero frequency) nuclear pion field can be obtained from the methods developped by Delorme et al [5] which, by consideration of a form factor $v(q^2)$ in the p-wave pion-nucleon interaction, can be brought in parallel with usual investigations of pion condensation in nuclear matter (see e.g. the work of Bäckman and Weise [6]). For an assembly of nucleon sources located at $\underset{\sim}{x}_i$ in an infinite nuclear medium, one finds in momentum space :

$$\varphi(\omega=0,\underset{\sim}{q}) = i\frac{g_r}{2m_N}\sum_i\left(1+\xi\chi_e/3\right)\frac{\underset{\sim}{\sigma}_i\cdot\underset{\sim}{q}\; v(q^2)\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{x}_i}}{\left[m_\pi^2+q^2\left(1+\chi_e(q)v^2(q^2)\right)\right]} \qquad (1)$$

where we have suppressed the isospin variables and omitted the s-wave π-N interaction. The factor $(1+\xi\chi_e/3)$ which modifies the π-N coupling constant g_r is irrelevant for the ensuing discussion which concerns essentially the renormalization of the propagator. The quantity $(1+\chi_e)$ plays the role of the axial dielectric constant of the medium, χ_e being its (effective) axial static polarizability.

The polarizability χ_e originates from two sources : the first one is the virtual nucleonic excitation, predominantly the Δ_{33} resonance. We denote this part which arises from high excitations as the diamagnetic part in analogy with the notations currently used in solid state physics. The corresponding polarizability α is related to the off-shell p-wave scattering volume c_0 and the nuclear density ρ. In symmetric nuclear matter one has to first order in the density : $\alpha(x) = -4\pi\rho(x)\, c_0(\omega=0)$. The value of c_0 at $\omega = 0$ varies from 0.12 to 0.19 m_π^{-3} according to the extrapolation model. With $c_0 = 0.15\ m_\pi^{-3}$ the diamagnetic polarizability is $\alpha = -0.9$ at the nuclear matter density $\rho_0 = 0.17\ \mathrm{fm}^{-3}$.

In addition, a pionic field may flip the nucleonic spin via nuclear excitations (predominantly of low energy), giving rise to the paramagnetic response which depends on the nuclear spectrum (notice that we adopt a philosophy different from that of our previous works [5] where the paramagnetic effects were assumed to be incorporated in the spin distribution and only the diamagnetic part was included in the denominator of eq. (1)). For a qualitative discussion we take the case of an infinite Fermi gas where the static paramagnetic polarizability β (q) is at low momenta $(q < 2p_F)$: $\beta(q) \approx \beta\,(1 - q^2/12p_F^2)$ with $\beta = -2m_N p_F/\pi^2$ (p_F = Fermi momentum). At $\rho = \rho_0$, $\beta = -2.7$ so that $|\beta| > |\alpha|$ as expected from the closeness of the nuclear excitations as compared to the nucleonic ones.

This is the first order approximation which neglects the effect of baryon-baryon correlations. Inclusion of the latter replaces $\chi(q) = \alpha + \beta(q)$ by an effective value $\chi_e(q) = \chi(q)/(1 - g'\chi(q))$ (Lorentz-Lorenz effect [7]).. The value of g' is 1/3 for a pure Lorentz-Lorenz effect while a recent estimate [8] gives $g' = 0.5$. The magnitude of χ_e is thus appreciably reduced.

The expression (1) shows that the pion propagator is renormalized with respect to its free space value by a factor R :

$$R = (m_\pi^2 + q^2)\Big/\left[m_\pi^2 + q^2\left(1 + \chi_e(q^2)v^2(q^2)\right)\right] \qquad (2)$$

As $\chi_e < 0$ and $v(q^2) \to 0$ for $q^2 \to \infty$, R increases from 1 at q = 0, reaching again the value 1 at $q = \infty$. In order to simplify the discussion we use the first order expansion $v^2(q^2) \approx 1 - q^2 r^2/3$. This approximation gives the explicit dependence of the critical parameters on the r. m. s. radius r (its accuracy is around 10 %). Similarly we expand $\chi_e(q^2) v^2(q^2) \approx \chi_e (1 - q^2 s^2/3)$ with $\chi_e \equiv \chi_e(0)$ and $s^2 = r^2 + \beta/4 k_F^2 \chi (1 - g'\chi)$. Above $\rho \approx 0.5 \rho_0$, s is a slowly varying function of the density which remains close its limiting value r for infinite density and depends thus very moderately on g'.

The renormalization factor R has then a single maximum at a value q_m of the momentum practically independent on the density : $q_m^2 \approx -m_\pi^2 + \sqrt{3}\, m_\pi / s$. The evolution of R when one varies the density (which fixes the parameters χ_e and s) can be traced from the properties of the poles of the propagator. A double pole is obtained when :

$$(1 + \chi_e)^2 + \frac{4}{3} m_\pi^2 s^2 \chi_e = 0 \tag{3}$$

This equation is equivalent to the two implicit equations in the density (the indices 1 and 2 correspond to the + and - signs) :

$$\chi_e^{1,2} = -\left(1 + \frac{2}{3} m_\pi^2 s^2\right) \pm \frac{2}{\sqrt{3}} m_\pi s \left(1 + \frac{1}{3} m_\pi^2 s^2\right)^{1/2} \tag{4}$$

Except in the region of low densities($\rho/\rho_0 < 0.5$) which are not of interest for our purpose, the variation of the right hand side with the density is very small and may be neglected in practice. As on the other hand the r. h. s. is nearly independent on the Lorentz-Lorenz parameter g', the two polarizabilities χ_e^1 and χ_e^2 are well determined, irrespective of its value. With $c_0 = 0.15\, m_\pi^{-3}$ and $r = 0.48\, m_\pi^{-1}$, we find numerically : $\chi_e^1 \approx -0.57$ and $\chi_e^2 \approx -1.75$.

These two characteristic values define three regions . For $|\chi_e| < |\chi_e^1|$ the two poles are negative and correspond thus to unphysical values of q^2. Between χ_e^1 and χ_e^2, they are complex conjugate. They become real and positive beyond the second characteristic point $\chi_e^2 \equiv \chi_e^c$ which thus corresponds to the condensation threshold +. The double pole q_c^2 is then the critical momentum of the condensation :

$$q_c^2 = -2 m_\pi^2/(1 + \chi_e^c) \approx -m_\pi^2 + \sqrt{3}\, m_\pi / s_c \approx q_m^2 \tag{5}$$

Numerically we get $q_c^2 = 2.6\, m_\pi^2$.

+A relation similar to equ. (3) has been derived by Ericson and Myhrer [9] from the condition for a pion-nucleus bound state at $\omega = 0$.

For what concerns the determination of the critical density ρ_c, the exact value of the Lorentz-Lorenz parameter is essential : in order for the polarizability to reach the effective value -1.75, the density has to be $\rho_c/\rho_0 \approx 1.3$ for $g' = 1/3$ but as large as $\rho_c/\rho_0 \approx 6$ for $g' = 0.5$. When g' attains values of the order of 0.5, the critical density becomes very sensitive to small changes in the critical polarizability χ_e^c induced by variations of the input data. Beyond the value $g' = |\chi_e^c|^{-1} = 0.57$ the condensation cannot occur at all even for large compressions but this does not prevent the occurence of critical phenomena. Hence the critical polarizability is well determined while the critical density is very uncertain or may even not exist.

In conclusion the maximum of the renormalization of the pion propagator becomes infinite at the critical density. Below the condensation threshold it can be expressed approximately as :

$$R_m = \chi_e^c / (\chi_e^c - \chi_e) \qquad (6)$$

Here χ_e^c is well known whereas χ_e for a given density depends sensitively on g'. For instance, at $\rho = \rho_0$ one finds $R_m = 5$ for $g' = 0.5$ but a much larger value is attained for $g' = 1/3$ where χ_e is close to χ_e^c. Away from q_m (for instance $q \lesssim m_\pi$), R has a smaller value which is not as sensitive to g'.

This enhancement of the pion field in this momentum range expresses the strong fluctuations of the order parameter. It appears in the subcritical region as a precursor of the Bragg peak corresponding to the condensed field. It is the critical opalescence phenomenon very similar to the one observed for antiferromagnets near the superstructure lines [10]. Our numerical estimations show that this phenomenon is pronounced at the ordinary density (at least in infinite nuclear matter) even if the critical density is far away or even cannot be reached at all. This may look suprising but our treatment suggests that the expansion parameter should not be $(\rho - \rho_c)$ but instead $(\chi_e - \chi_e^c)$.

Although experiments always involve a definite momentum (or a definite range of momenta), it is illustrative to discuss the behaviour of the field in x-space so as to see how the long range ordering of the spins is established. What we need is the Fourier transform of the expression (1). For simplicity we take one nucleon at $x_i = 0$ and consider form factor effects only in the propagator in consistency with the preceding discussion. Anyway our first order expansion of $v(q^2)$ does not allow a proper description of the high momentum components and thus of the close vicinity of the source.

The pion field $\varphi(\underline{x})$ is expressed in terms of the poles $q_\pm^2$ of the propagator :

$$q_{\pm}^2 = \left\{ (1+\chi_e) \pm \left[(1+\chi_e)^2 + \frac{4}{3} m_\pi^2 s^2 \chi_e \right]^{1/2} \right\} \Big/ \frac{2}{3} s^2 \chi_e \qquad (7)$$

One can distinguish two cases according to the value of the density (we exclude the condensation region $\rho > \rho_c$ where our model is not valid).

i) in the low density region $\rho < \rho_1$ (where ρ_1 corresponds to the first characteristic point $\chi_e^1 = -0.57$), the roots are both negative and the field has the form :

$$\varphi(\underset{\sim}{x}) \sim \left(|q_+|^2 - |q_-|^2\right)^{-1} \underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla} \left[\left(e^{-|q_-|x} - e^{-|q_+|x} \right) / x \right] \qquad (8)$$

At the lowest densities the Yukawa decay $e^{-|q_-|x}$ occurs with the effective mass m'_π in the medium : $|q_-| \approx m'_\pi = m_\pi/(1+\chi_e)^{1/2}$ (cf. ref. 5) and the other Yukawa function has a much shorter range which characterizes the nucleon form factor. When ρ tends to the value ρ_1 , both $|q_-|$ and $|q_+|$ converge to $|q_1| = \sqrt{2}\, m_\pi/(1+\chi_e^1)^{1/2}$ and $\varphi(\underset{\sim}{x})$ towards $(2|q_1|)^{-1} \underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla}\, e^{-|q_1|x}$.

ii) for an intermediate density $\rho_1 < \rho < \rho_c$, the roots are complex conjugate and the field becomes :

$$\varphi(\underset{\sim}{x}) \sim (2\eta\varepsilon)^{-1} \underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla} \left(\sin \varepsilon x \, e^{-\eta x} / x \right) \qquad (9)$$

where $\varepsilon = |\mathrm{Re}\, q_\pm|$ and $\eta = |\mathrm{Im}\, q_\pm|$. Between ρ_1 and ρ_c , ε increases from 0 to q_c and η decreases from $|q_1|$ to 0. The field is characterized by the occurrence of a sinusoïdal modulation which announces the ordered phase. When the critical density is approached, the Yukawa damping disappears and $\varphi(\underset{\sim}{x})$ tends to $\varphi_c(\underset{\sim}{x}) \sim (2\eta)^{-1} \underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla} \left(\sin q_c x / q_c x \right)$. As the field is the agent for spin orientation, the range of the spin ordering becomes infinite with that of the field (Coulomb-like x^{-1} behaviour) with the modulation $\sin q_c x$ characteristic of the condensation. In this theory based on the linear response, the critical exponent for the range of the spin correlation in the disordered phase is 1/2 since η goes to zero as $|\chi_e - \chi_e^c|^{1/2}$.

Experiments probing the nuclear pion field are thus of the utmost importance since they could reveal the proximity of a phase transition. The possible methods for its exploration have been briefly discussed in ref. (11). The most promising one seems to be the π^- annihilation into 2γ with a virtual π^+ of the nucleus proposed by T. Ericson and C. Wilkin [12] who have shown that it is a direct probe of the pion field. This process has been detected by the Louvain group [13] for π^- at rest. The ratio of 2γ versus 1γ emission has been measured in ^{9}Be and ^{12}C and a preliminary analysis indicates that it is enhanced by a factor about 3 as compared to the impulse approximation estimate, which if confirmed would reflect an enhancement of the pion field by a factor 1.7. It would be very exciting if this could be interpreted as a critical opalescence phenomenon. As a first approach, we can see whether this number makes sense compared with our infinite nuclear matter estimate.

In the present status of the analysis the momentum is not precisely known, a typical range being between 0.6 and 0.8 m_π. Our nuclear matter model gives an enhancement factor R (equ.(2)) of the order of 2.2. for $q = 0.7\ m_\pi$ and $g' = 1/3$ (for such low momenta, R is not very sensitive to g') , an encouraging value but far from conclusive.

First the evidence for the enhancement is only preliminary and based on unrefined impulse approximation estimate. Second our theoretical prediction refers to infinite nuclear matter while the experiments have been performed in small nuclei where surface effects can change drastically the expectations [5,14]. In addition the experiment does not strictly measure the static field because a certain unknown amount of energy could be transferred to the nucleus so that the polarizabilities are not the static ones. This can be of importance for the paramagnetic response which has a strong energy dependence. A useful information could be provided by a measurement of the photon energy so as to determine the energy transfer with the further advantage of the exploration of the momentum dependence of the renormalization.

Finally we should add the important following remark : the experiment measures the ratio between the pion field $\varphi(\underset{\sim}{q})$ and the threshold photoproduction amplitude which according to soft pion theorems is nothing else than the matrix element of the axial current[15] $\underset{\sim}{A}\cdot\underset{\sim}{\varepsilon}$ ($\underset{\sim}{\varepsilon}$ = photon polarization vector) . In a previous work, it was shown that both are modified by polarization effects in the following way [5]:

$$\varphi(\underset{\sim}{q}) = i(g_r/2m_N)\left[\underset{\sim}{\sigma}(\underset{\sim}{q}) - (f_\pi/g_A)\underset{\sim}{P}(\underset{\sim}{q})\right]\cdot\underset{\sim}{q}/(m_\pi^2+q^2)$$

$$\underset{\sim}{A}(\underset{\sim}{q}) = -g_A\left[\underset{\sim}{\sigma}(\underset{\sim}{q}) - (f_\pi/g_A)\underset{\sim}{P}(\underset{\sim}{q})\right] \qquad (10)$$

In these expressions $\underset{\sim}{\sigma}(q)$ and $\underset{\sim}{P}(q)$ are the Fourier transforms of the spin and polarization densities respectively and g_A and f_π the axial and pion decay constants. Thus the experiment measures essentially the ratio between the longitudinal and transverse components of the quantity $\left[\underset{\sim}{\sigma}(\underset{\sim}{q}) - (f_\pi/g_A)\underset{\sim}{P}(\underset{\sim}{q})\right]$ so that it is a detector of renormalization only if both components renormalize differently, a property which depends on the nucleus and on the transition under investigation.

It is clear that much theoretical effort remains to be done in order to extend our nuclear matter treatment of the field fluctuations to finite systems. Some studies of the condensation problem in finite nuclei have been performed with the conclusion that they do not drastically differ from nuclear matter for what concerns the threshold [16,17]. It is thus natural to hope that the new physical effect that we predict at subcritical densities in infinite systems, namely an enhancement of the pion field near the critical momentum, will survive in actual nuclei.

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REFERENCES

1 . A. B. Migdal, Zh ETF 61 (1971) 2210 ; JETP 34 (1972) 1184.

2 . H. Palevsky and D. J. Hughes, Phys. Rev. 92 (1956) 202.

3 . M. Ericson, Thèse, Université de Paris (1958), Rapport CEA 1189.

4 . L. Van Hove, Phys. Rev. 95 (1954) 1374.

5 . J. Delorme, M. Ericson, A. Figureau and C. Thévenet, Ann. Phys. (NY) 102(1976)273.

6. S. O. Bäckman and W. Weise, to appear in Mesons in Nuclei (M. Rho and D. H. Wilkinson eds.).

7 . M. Ericson and T. E. O. Ericson, Ann. Phys. (NY) 36 (1966) 323.
G. Baym and G. E. Brown Nucl. Phys. A 247 (1975) 395.

8 . G. E. Brown, S. O. Bäckman, E. Oset and W. Weise, Nucl. Phys. A286 (1977) 191.

9 . T. E. O. Ericson and F. Myhrer, preprint TH 2434-CERN (1978)

10 . P. G. De Gennes, Thèse, Université de Paris (1959), Presses Universitaires de France ed.

11 . M. Ericson, to appear in Mesons in Nuclei (M. Rho and D. H. Wilkinson, eds.) preprint TH 2410-CERN (1977).

12 . T. E. O. Ericson and C. Wilkin, Phys. Lett. 57 B (1976) 1.

13 . J. Deutsch, private communication.
J. Deutsch, D. Favart, P. Lipnik, P. Macq, M. Lebrun and R. Prieels, contribution C17, 7th international conference on high-energy physics and nuclear structure, Zürich 1977.

14 . J. Delorme and A. Figureau, 4e session d'études, La Toussuire, C. 7. 1 (preprint LYCEN 7702).

15 . M. Ericson and A. Figureau, Nucl. Phys. B 3 (1967) 609 ; ibid. B 11 (1969) 621.

16 . S. A. Fayans, E. E. Saperstein and S. V. Tolokonnikov, J. Phys. G, 3(1977) L 51.

17. J. Meyer-ter-Vehn, preprint SIN (1978).