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CONTROLING MAGNETIC FIELD PROFILES

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CONTROLLING MAGNETIC FIELD PROFILES

by

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ABSTRACT

A method for designing solenoid magnets with controlled field profiles is discussed. The method, originated by D. B. Montgomery, minimizes both the field errors and the power consumption. An NOS time-sharing computer program for the CDC-6600, entitled MAGCOR, has been constructed to provide an interactive magnet design capability. Results obtained during the design of magnets for a radial line electron accelerator are presented.

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## CONTROLLING MAGNETIC FIELD PROFILES

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Solenoid and cusp magnets have many applications in plasma physics and particle beam accelerator design. These include providing fields for beam propagation in drift regions, external fields for magnetic insulation, and cusp fields for beam rotation. A common problem which occurs in these applications is field non-uniformity caused by gaps in the winding which are required for accelerating electrodes or vacuum pumping ports. The usual cure is to add correction coils at each end of the gap or to increase the winding density, both of which tend to increase the field in the gap. Although this works well for gaps which are much smaller than the coil radius, it is less suitable for larger gaps. Here the amount of extra flux required is so large that a single pair of correction coils are unable to improve the field in the center of the gap without perturbing the initially uniform field in regions adjacent to the gap. Correction of these secondary perturbations by additional coils carrying negative current works well for many cases, but a systematic method for synthesizing the proper correction coil current distribution would be desirable.

Montgomery<sup>1</sup> has described an appropriate method in his book on solenoid magnet design. It is based on a least squares treatment which minimizes the sum of the field errors at specified points. The correction current distribution resulting from the least squares fit will in many cases be impractically large or irregular. The method overcomes this difficulty by including a provision for minimizing the power consumed ( $I^2R$ ) as well as the field errors. A choice of weighting coefficients enables the user to emphasize either power or field error control. Because the method employs a matrix relating the correction coil currents to any specified field distribution, it can be used to produce almost any desired field profile, including, for example, magnetic cusps.

A computer program entitled MAGCOR has been constructed for the Sandia NOS computer to provide an interactive magnet design capability. The first part of the program computes the exact analytic radial and axial magnetic fields for a distribution of point current loops in the absence of conducting or magnetic boundaries. The magnetic stream function can also be computed to provide field lines for output plots. The method evaluates the standard formulae<sup>2</sup> and uses a complete elliptic integral subroutine. The second half of MAGCOR employs a correction routine based on Montgomery's method to improve the field uniformity. More general field control can be added easily.

The remainder of this report describes the field correction method in greater detail and discusses a series of examples drawn from solenoid magnets designed for a multistage electron accelerator.<sup>3</sup>

## II. Field Correction Methods

The radial magnetic field at a point  $r_m, z_m$  produced by a filamentary current loop located at the origin of the cylindrical coordinate system (see Fig. 1) is given by<sup>2</sup> (mks units)

$$B_r = \frac{\mu_0 I}{2\pi} \frac{z_m}{r_m \left[ (a + r_m)^2 + z_m^2 \right]^{1/2}} \left[ -K + \frac{(a^2 + r_m^2 + z_m^2)}{(a - r_m)^2 + z_m^2} E \right], \quad (1)$$

where  $I$  is the current carried by the loop and  $K$  and  $E$  are complete elliptic integrals of the first and second kind. The argument of the elliptic integrals is  $k$ , where  $k^2 = 4a r_m / [(a + r_m)^2 + z_m^2]$ . Similar expressions are available for  $B_z$  the axial field component, and the magnetic stream function  $\psi$ , namely

$$B_z = \frac{\mu_0 I}{2\pi} \frac{1}{\left[ (a + r_m)^2 + z_m^2 \right]^{1/2}} \left[ K + \frac{(a^2 - r_m^2 - z_m^2)}{(a - r_m)^2 + z_m^2} E \right], \quad (2)$$

$$\psi_\theta = \frac{\mu_0 I}{\pi k} (a r_m)^{1/2} \left[ \left( 1 - \frac{k}{2} \right)^2 K - E \right]. \quad (3)$$

The axial field at a point  $r_m, z_m$  produced by a total number of  $L$  loops is given by superposition as  $B_m = \sum_{l=1}^L b_{lm} I_l$ , where  $b_{lm}$  is seen from Eq. (2) to be a strictly geometric factor. The remainder of this section will use the notation  $B_m = B_z(r_m, z_m)$  for brevity. Suppose that a particular field profile  $\bar{B}_m$  is desired at  $M$  locations, given by  $r_m, z_m$ , for  $m = 1$  to  $M$ . The uncorrected field distribution at the  $M$  points will be defined as  $B_m$ . A correction field must be produced of magnitude  $\bar{B}_m - B_m = B_{cm}$  by an additional series of correction current loops of number  $N$ . These can in principle be located

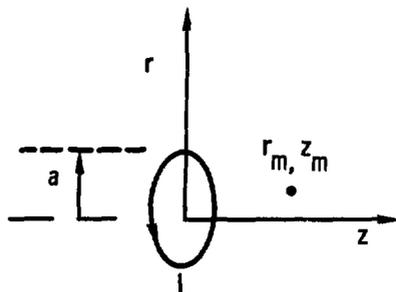


FIGURE 1

Filamentary current loop configuration.

anywhere, but the currents required will be minimized by locating them as close to the region to be corrected as possible. The correction field will be given as  $B_{cm}^* = \sum_{n=1}^N b_{nm} I_n$ , which in general will only equal  $B_{cm}$  if  $N = M$ .

The method of Montgomery provides a way for determining the distribution of currents to minimize the squared field errors at the  $M$  locations. It can also be used to control the power consumption. If we define the field errors at the  $M$  points as  $d_m = B_{cm} - \sum_{n=1}^N b_{nm} I_n$  and a power parameter  $P_n = I_n^2 p_n r_n$  associated with each correction loop, we can form the weighted sum

$$Q = \sum_{m=1}^M c_m d_m^2 + \sum_{n=1}^N p_n r_n I_n^2, \quad (4)$$

where  $r_n$  is the radius of the  $n$ th correction loop. The coefficients  $c_m$  and  $p_n$  can be adjusted with respect to each other to emphasize either field error or power control. Typically, it is sufficient to let all  $c_m$ 's equal unity and to let all  $p_n$ 's be equal to some value  $p_{no}$  which can then be varied with a reasonably small number of computer program runs.

The current distribution  $I_n$  is obtained by minimizing the weighted sum  $Q$  with respect to the currents  $I_n$ , i.e., by setting  $\frac{\partial Q}{\partial I_n} = 0$ . This provides a set of  $N$  equations which can be solved by standard matrix methods for  $I_n$ . The matrix equation is given by

$$A_{in} I_n = \sum_{m=1}^M c_m B_{cm} b_{nm}, \quad (5)$$

where the off-diagonal elements of the array  $A_{in}$  are given by

$$A_{in} = \sum_{m=1}^M c_m b_{im} b_{nm}, \quad (6)$$

and the diagonal elements are given by

$$A_{ii} = \sum_{m=1}^M c_m b_{im} b_{im} + p_i r_i \quad (7)$$

Because the method is based on a least squares fit, it is not necessary for  $N$ , the number of correction coils, to equal  $M$ , the number of field control locations.

As described in the introduction, the computer program MAGCOR begins with a specified set of solenoid windings and computes an uncorrected field distribution  $B_m$  from Eq. (2). If, for instance, a constant axial field  $B_0$  was desired over  $M$  points located within a winding gap, the proper correction field to be computed would simply be  $B_{cm} = B_0 - B_m$ . The second half of MAGCOR would use a set of  $N$  correction loops placed symmetrically near the ends of the gap to correct the field errors. Eq. (5) would normally be solved for the currents  $I_n$  with  $c_m = 1$  and  $p_n = p_{nc}$ .

### III. Example A: Correction of a Gap

A typical example of a system requiring field control is shown in Fig. 2. Two 26.7 cm long solenoids are separated by 5.1 cm. The solenoids are wound at a uniform pitch with 3.15 turns/cm; each turn carries 5000 amp. This particular case models an aperture for accelerating electrodes in a multistage accelerator.<sup>3</sup> The uncorrected axial field along the axis is plotted in Fig. 3 as a dashed line. MAGCOR was used to locate two pairs of correction coils to improve the field uniformity in the gap. The first pair was located directly adjacent to the gap and carried twelve turns each of positive current. The second pair was located 2.54 cm

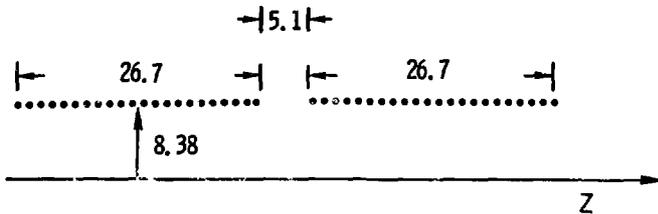


FIGURE 2

Simple uncorrected solenoid with a winding gap for accelerating electrodes. All dimensions are in centimeters.

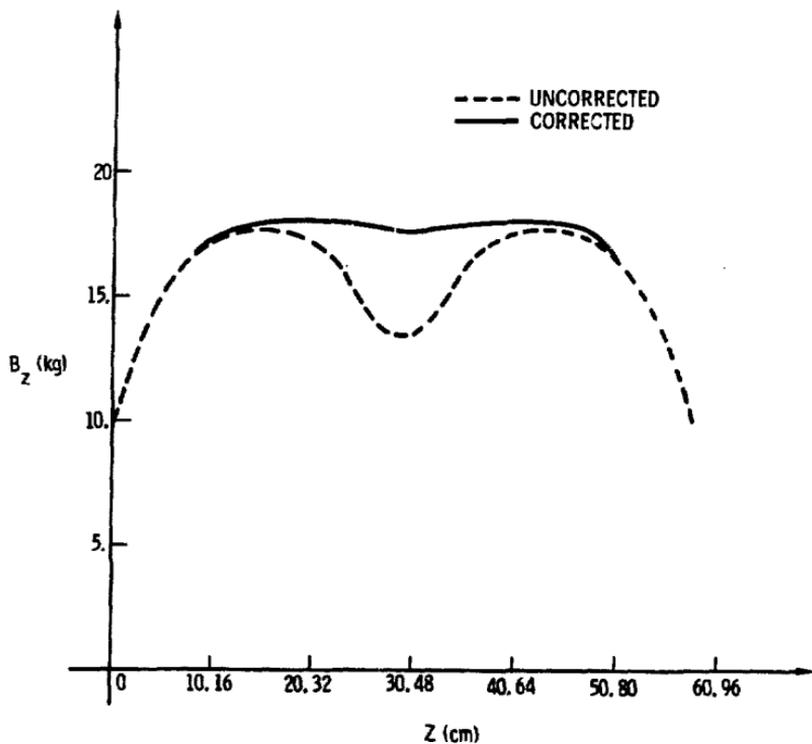


FIGURE 3

Comparison of corrected (solid line) and uncorrected (dashed line) magnetic fields near the axis.

from the ends of the gap, and each element carried seven turns wound in the opposite direction to provide a negative current. This reduced the peak-to-peak field variations in the gap from 25% to about 2.2%. The correction coils are actually defined in terms of amp-turns; half the turns at twice the current would be equivalent.

A solenoid quite similar to that shown in Fig. 2 was fabricated and corrected in the manner discussed above. The actual field profile along the axis was measured for comparison with the MAGCOR predictions.

The results of this comparison are shown in Fig. 4. (Note: the vertical scale in Fig. 4 is expanded for clarity.) The experimental configuration was somewhat more complicated than shown in Fig. 2, and used correction turns distributed in space according to the finite coil turn thickness. The experimental field variation in the gap was about 6%. The discrepancy between the computed and measured results was probably attributable to slight errors in manufacture and assembly. In addition, the measurement uncertainties are on the order of a few percent. Some improvement might be achieved by mounting the correction coils in axially adjustable frames, so that fine tuning of the final assembly would be possible. The accuracy of the correction was considered to be quite sufficient to permit the initial accelerator tests and to encourage the use of MAGCOR for more difficult problems.

#### IV. Example B: A Low Inductance Configuration

Although the configuration of Fig. 2 achieves a high degree of field uniformity, it does this at the expense of a relatively large volume containing magnetic energy. The drift tube radius in the actual accelerator is about 3 cm, so that a much smaller radius solenoid could be used. The field non-

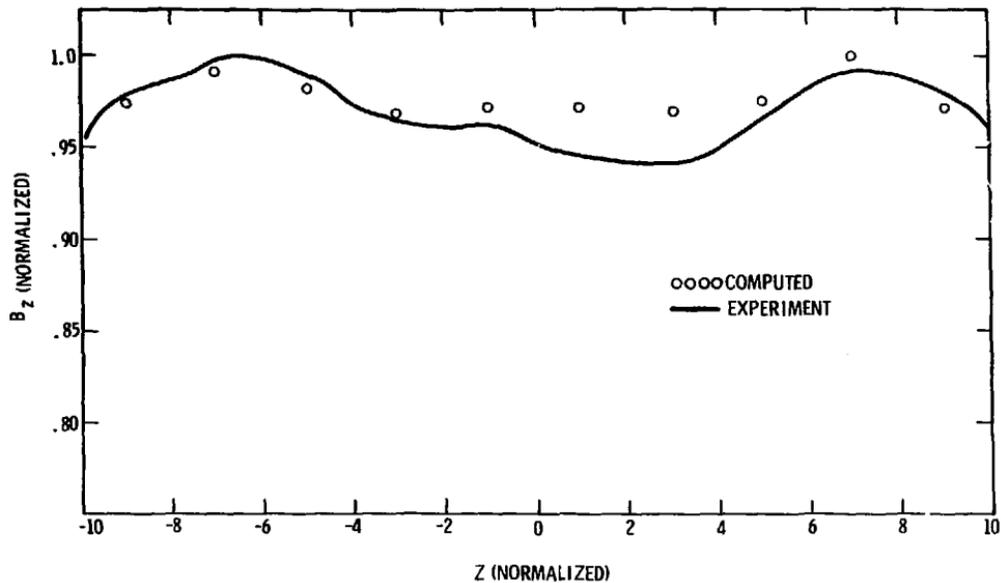


FIGURE 4

Comparison of experimentally measured field profile (open dots) with predicted field (solid line).

uniformities in the gap for the uncorrected configuration will be larger, so that some relaxation of the requirements on field homogeneity will be required.

A more efficient basic configuration is shown in Fig. 5. Here most of the drift tube length between the accelerator gaps is covered by a much smaller radius solenoid. If the small and large radius sections are wound with the same pitch, the field non-uniformities are too big to correct with a practical design. The length to diameter ratio in the center section is very large, so that the field in its center is nearly equal to that of an ideal infinite length solenoid; the opposite situation holds for the large radius trimming sections. It is thus necessary to reduce the winding density in the center section to minimize the field non-uniformities at the interfaces between the large and small radius coils before attempting the final correction with MAGCOR. Unrealistic correction currents would be required otherwise.

A design which achieves reasonable field uniformity with about 80% of the drift length wound at the small radius is shown in Fig. 6. The plane of symmetry at the far right is in the center of a single accelerating gap. The plane of symmetry between the electrodes is beyond the range of the left-hand portion of the figure. The small radius section is wound at 1.4 turns/cm, while the large radius section is again wound at 3.15 turns/cm. This ratio requires very little additional correction, consisting of a pair of single turn loops near the gap, plus the removal of two turns of the small radius coil near each gap edge (four turns per complete solenoid stage). The resulting axial field profile for a 5000 amp current is shown in Fig. 7.

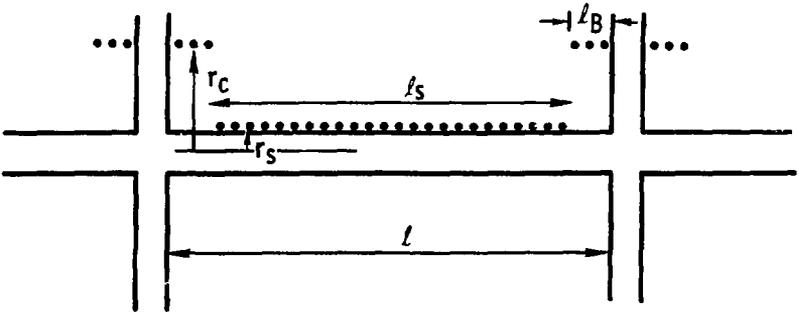


FIGURE 5

Basic low inductance solenoid configuration.

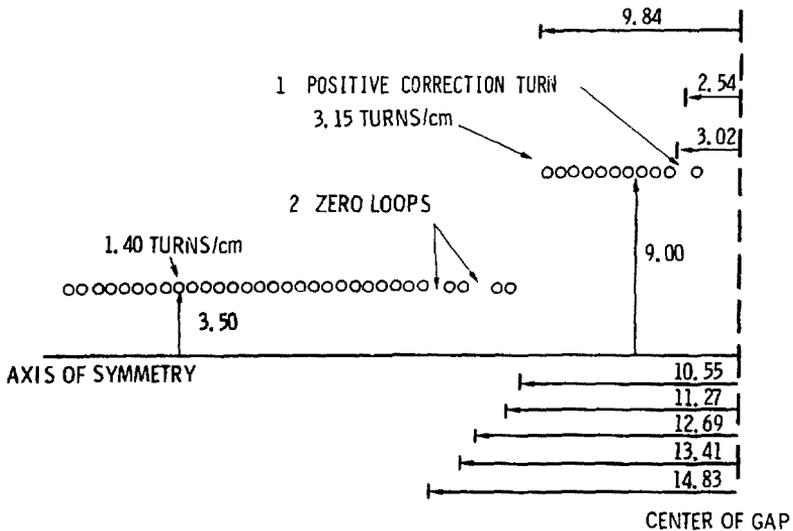


FIGURE 6

Detailed design for low inductance configuration. All dimensions are in centimeters.

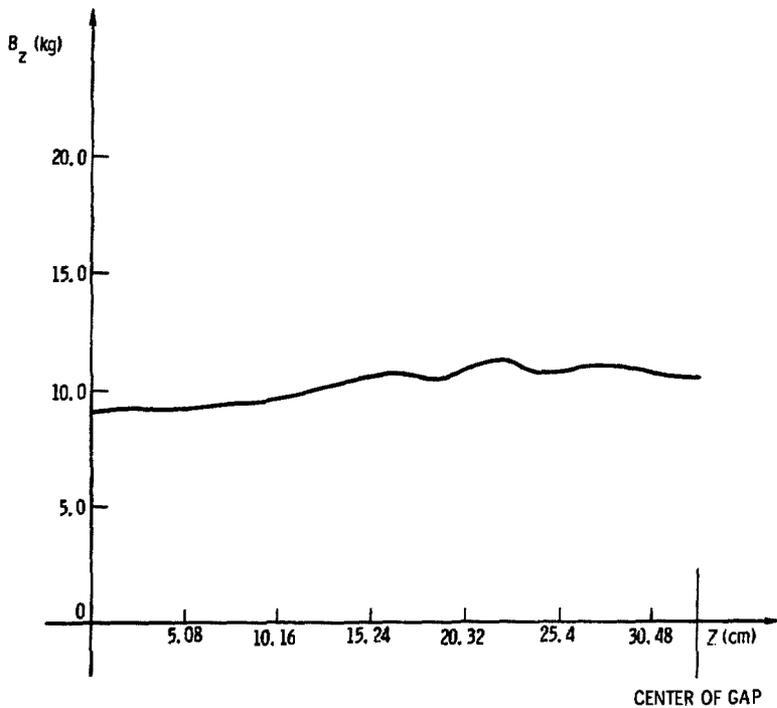


FIGURE 7  
Field profile for the design shown in Fig. 6.

The gradual fall-off at the left-hand boundary is a numerical artifact due to the omission of the other stages. The important parameter is the field variation near the gap, which is seen to have a peak-to-peak value of about 6%.

#### V. Example C: Correcting a Wide Gap

It is frequently necessary to correct for winding gaps which are more than a coil radius in extent. By far the most satisfactory method is to place the correction turns at a larger radius in the center of the gap as shown in Fig. 8. This places the correction flux directly where its most needed and reduces the number of turns required. This technique has been applied to a solenoid with a gap for a vacuum pumping port. Fig. 8 shows the actual distribution of correction windings which was designed with MAGCOR and fabricated. The magnetic field profiles near the axis before and after correction are shown in Fig. 9. The peak-to-peak variations were reduced from 67% to 9%. A more efficient design might have been possible if the winding density of the small radius coil had been increased near the gap. It would also be possible to use a more continuous winding distribution axially for the correction turns. The choice of a smaller number of discrete correction locations was dictated by constructional considerations.

#### VI. Summary

A technique for controlling the spatial profile of magnetic fields has been described and results drawn from design studies for a multi-stage electron accelerator have been discussed. Although all of these examples

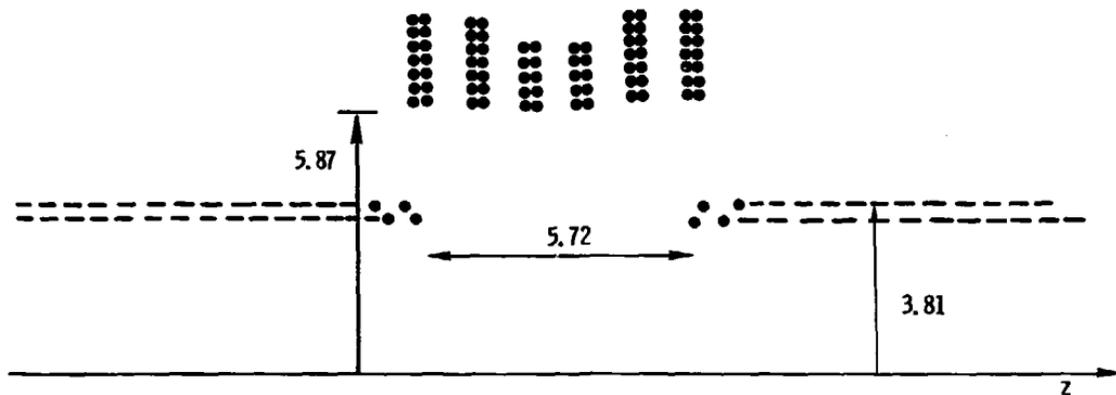


FIGURE 8

Solenoid with a wide gap for a vacuum port. Dashed lines indicate continuation of windings.

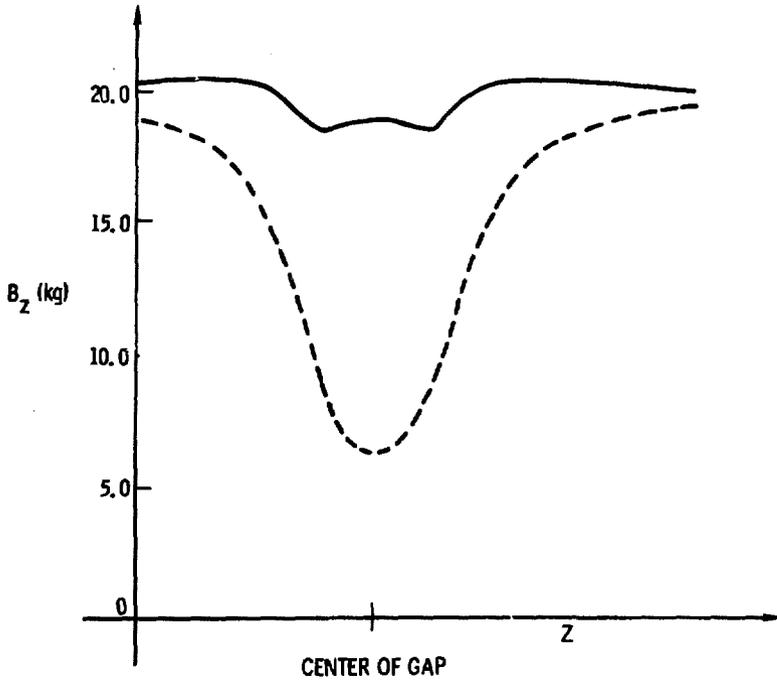


FIGURE 9

Corrected (solid line) and uncorrected (dashed line) field profiles for the design shown in Fig. 8.

have been variations on the basic theme of obtaining a homogeneous field profile, the technique has sufficient generality to allow the design of arbitrary field gradients.

Several common conclusions emerge from the design studies. The first of these is the fairly obvious one that the ratio of gap lengths to coil radii should be kept as small as possible to reduce the uncorrected field variations. Very large gaps can require more correction turns than exist in the main solenoids. It has also been found advantageous to always design a basic configuration in which all elements have at least comparable fields before attempting to design correction coils with MAGCOR. The low inductance configuration is a good example of this, where greatly differing winding densities were necessary in the large and small radius sections. Correction coils which can be displaced axially after assembly to fine tune the final configuration also seem to be useful.

MAGCOR presently provides an interactive design capability with fast turnaround using the NOS CDC-6600 computer. Typical run times for corrected designs are about ten seconds. For most applications it is sufficient to set all field error coefficients  $c_m$  to unity and all power coefficients to a constant value  $p_{no}$ . The best error control will be found for  $p_{no} = 0$ , but usually the correction currents will be unacceptably high. The value of  $p_{no}$  should be increased in succeeding runs until a reasonable compromise is achieved between currents and field errors. If this does not occur, either more correction coil locations or different positioning is required.

#### Acknowledgments

The many helpful suggestions of Kenneth R. Prestwich and R. Bruce Miller are gratefully acknowledged.

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3. R. Bruce Miller, private communication.

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