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**USEFULNESS OF THE MONTE CARLO METHOD
IN RELIABILITY CALCULATIONS**

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CEA-CONF-4303

SLIDE 1

The invitation for this paper was sent to Dr LANORE and she has also written the abstract. Unfortunately, in September she was in an accident and she has not been at work since then. This paper is written mainly using her abstract, reports and notes. However, as I think that some of you might be unfamiliar with the Monte Carlo work in Saclay, I have decided first to give some background information and after this short introduction I will go to the subject given in the abstract.

SLIDE 2

The work which will be discussed has been carried on at the LEP, Laboratory for Radiation Shielding Studies in the Nuclear Research Center of Saclay, France. In the organisation the LEP is a part of the SERMA, Service for Reactor Studies and Applied Mathematics.

SLIDE 3

In the next slide we have three examples of the Monte Carlo programs developed at the LEP. The work on the Monte Carlo method was started in the beginning of the sixties by writing several preliminary programs for radiation shielding.

This work led about 1969 to a large three-dimensional neutron and gamma transport code TRIPOLI. In this code the energy variable is treated in the multigroup approximation, but the number of energy groups is practically not limited. The program can solve deep penetration problems by utilizing variance reduction techniques based on the exponential transform and on the biasing of the angular scattering laws. The present version of the program can be used both in the stationary source problems and in the criticality problems.

The Program MERCURE-4 integrates neutron or gamma-ray line of sight point attenuation kernels by the Monte Carlo method in three-dimensional configurations and in the multigroup approximation. The program has an interesting adaptive importance sampling algorithm so that the effort of a user not familiar with Monte Carlo is reduced to minimum.

The program JERK simulates the point defect behaviour in a lattice by the Monte Carlo method. The simulation proceeds from one event to another, such as creation, annihilation, or transformation of a defect, thus avoiding the treatment of each elementary jump. Variation reduction techniques have been studied also in this program.

SLIDE 4

The work on the reliability was started at the LEP some years ago in collaboration with the Department of Nuclear Safety. Two different types of Monte Carlo applications have been studied : first, the uncertainty analysis, and secondly, the simulation of the behaviour of complex systems in time.

SLIDE 5

In the reliability calculations the basic description of the system to be analysed is quite often given by a fault tree. As an example, we have in the next slide a simplified spray system with the following components : a tank, in series with the tank a valve, a pump and two motor valves in parallel. In a failure of this spray system either the tank has leaked, or somebody has left the valve closed, or the pump doesn't work or neither of the motor valves opens.

SLIDE 6

In the next slide we have the corresponding fault tree. This is an upside-down tree ; the component failures denoted by the circles, by the leaves, are at the bottom of the tree. Through the logical gates AND and OR these elementary events lead to the system failure, the top event of the tree.

In general, the fault tree is a logical diagram, consisting of the boolean gates and of the leaves, the component failures. The tree represents the logical dependence of the system failure on the component failures. This representation permits the calculation of the system failure probability as a function of the component failure probabilities.

SLIDE 7

We go now back to the Monte Carlo applications. In practice the reliability parameters of the system components are known with a confidence interval which might sometimes be very wide. In the uncertainty analysis the problem is to evaluate the uncertainty or dispersion in the system failure probability due to the uncertainties in the reliability data of the components. In this case, the reliability parameters of the components are considered as random variables with known distributions. The system failure probability is a function of these random variables. Hence, the problem is to construct the distribution of a function of several random variables with known distributions. A Monte Carlo program PATREC-MC has been written to solve the problem for systems given in the fault tree representation. In the beginning of each Monte Carlo trial the individual component parameters are random sampled according to their distributions and then the deterministic program PATREC is utilized to calculate the corresponding value of the system failure probability. Using the trial values thus generated the program finally constructs the distribution of the system failure probability and evaluates its most important statistical characteristics.

In the present version of PATREC-MC we have five optional probability laws for the system components. These laws contain at most three parameters.

As in the report WASH 1400, we have supposed that the parameters have log-normal distributions. Experiments on the variance reduction in the estimation of certain statistical characteristics have been performed.

SLIDE 8

In the next slide we have an example of the distribution constructed by PATREC-MC. On the horizontal axis we have the system failure probability in a logarithmic scale. The problem is the simple spray system shown earlier in the fault tree example. The median value of the distribution is about 10^{-4} , the mean value is about by a factor of ten larger. The tail of the distribution is due to a large uncertainty given to the failure parameters of the tank.

As a conclusion of the uncertainty analysis, I would say that the Department of Nuclear Safety has more and more calculated not the value of the system failure probability but its distribution. The program PATREC-MC has been a powerful tool in these calculations.

SLIDE 9

The simulation of the behaviour of complex systems is the second Monte Carlo application to the reliability calculations. Here again we have the system in the fault tree representation. The probability laws of the component failures are supposed to be given. The problem is to solve the reliability or availability of the system.

The program MONARC 2 has been written to solve the problem by the Monte Carlo simulation. I should immediately say that MONARC 2 is not the first Monte Carlo program in reliability ; there are American programs beginning from the program SAFTE of Garrick and others about ten years ago. There are also European programs written for example in Ispra, Italy, and in Riso, Denmark. However, Dr Lanore has included in MONARC 2 perhaps a larger variety of different features than is the case in most earlier programs.

The MONARC-2 solution is based on the exact simulation of the system. For each component the time to the next failure (or repair) is sampled at random according to the given probability laws. The simulation proceeds from one event to another, evaluating always the state of the system and accumulating the number of the system and subsystem failures as well as the durations of these failures.

Finally, the accumulated data is used to estimate the reliability of the system and its subsystems.

There are at least two types of method competing with the Monte Carlo simulation in fault tree calculations :

- first of all we have the exact solution. The French program PATREC which is used in PATREC-MC is perhaps the best example of programs based on the exact solution.

- secondly we have the approximate solutions, most often based on the method of the minimal cut sets. The minimal cut sets are the simplest failure modes of the system, the minimal combinations

of component failures which can cause a system failure.

These two competing methods have developed quite a lot during the last years thus decreasing the need of Monte Carlo simulations. However, both in the exact and in the minimal cut set calculations the computing time depends strongly on the complexity of the system. Hence, even when these other methods could be used, the Monte Carlo method becomes interesting, if the system is complex, in other words, if the fault tree contains a large number of gates, leaves and dependencies. A dependency occurs when the failure of one leaf has an effect on the failure probability of another leaf. The simplest dependency is a repetition; the same component failure appears twice or more in the fault tree. In the Monte Carlo simulation the computing time depends mainly on the number of elementary events needed to obtain enough accuracy in the results and not so much on the complexity of the system. In addition, the Monte Carlo simulation supplies useful information about the most important subsystems allowing later simplification of the fault tree.

SLIDE 10

As an example, we have in the following slide the fault tree of a residual heat removal system. This tree contains more than 200 gates and leaves; all the triangles in the tree are subtrees containing several leaves connected by a gate OR. Furthermore, the tree contains 24 repetitions; you can find that for example the triangle K appears five times in the tree. The problem was studied using three different programs:

- first the program PATREC based on the exact solution,
- secondly the program PREP KITT based on the minimal cut sets,
- and finally, the Monte Carlo program MONARC 2

The Monte Carlo method was the only one able to handle the whole system without simplifications.

A system can be complex also because of its operating scheme. The Monte Carlo method can treat exactly a large variety of particular laws for failures and repairs. For example, some components might have failures which are detected only during inspection. Further the system may have scheduled shutdown for revision and refueling or unscheduled shutdowns when a certain subsystem fails or when the

subsystem is not repaired in a given time.

The normal fault tree representation considers only two states of components, namely up and down. Therefore, it cannot treat in details the important problem of stand-by components which have three possible states. The reliability of systems with stand-by components can be calculated for example by using so-called Markov diagrams, but this solution becomes too complicated for larger systems. In a Monte Carlo simulation program the possibility of stand-by components can be easily included.

SLIDE 11

In the next slide we have the example of an electric power supply. This schematic picture is not a fault tree, it is just a simplified functionally equivalent diagram showing that the power supply consists of two parallel sources, the nets N_1 and N_2 , and of the two stand-by diesels D_1 and D_2 . The probability laws of failure and repair are supposed to be known for the two nets and the two diesels. In addition, we have supposed that with a given probability a diesel doesn't start when needed. The availability of this relatively simple system has been calculated using the Monte Carlo program MONARC 2 and the results were very close to the results obtained by a Markov diagram solution. In this case the Markov diagram solution leads to a set of fourteen first-order differential equations.

The most difficult case from the point of view of the Monte Carlo simulation is a very reliable system which has few system failures for a lot of elementary events. This case corresponds to the case of the deep penetration in the neutron transport theory. The natural solution of the difficulties is to seek for a suitable importance sampling scheme, to increase the probability of interesting events and to correct the bias by weighting. Dr LANORE has successfully used the following simple importance sampling scheme in some particular cases :

SLIDE 12

For each component the probability of failure is multiplied by the factor k . Hence, the new probability p^x is equal to k times p , and the weight of the component i :

$$W_i = \frac{p_i}{p_i^x} = \frac{1}{k}, \quad \text{if the component is down. On the other hand,}$$

$$W_i' = \frac{1-p_i}{1-p_i^x} = \frac{1-p_i}{1-kp_i}, \quad \text{if the component is up.}$$

For the whole system and the history j the weight is :

$$W_j = \delta_j \prod_{i \in C_j} W_i \prod_{i \in C_j'} W_i'$$

(W_j is equal to δ_j times the product of the weight of all the down components multiplied by the product of the weights of all the up components). δ_j is equal to 1, if the system failure occurs and 0 otherwise.

The experiments for example in the case of the electric power supply show that the importance sampling can considerably decrease the computing time. However, there are limitations in the use of the simple technique introduced and further development is needed for the treatment of more complex situations.

I have still some words on the uncertainties. As said before, in practice the reliability parameters of components are known with a confidence interval sometimes very wide. To take this fact into account in a MONARC 2 calculation we can again suppose that the component parameters are log-normally distributed. In the beginning of each MONARC 2 history these parameters can be random sampled according to their distributions. This calculation doesn't give the distribution of the system reliability, it only gives the mean value of the distribution. Yet the result tells us something about the distribution which is not always the case in the calculations with so-called best estimate values.

SLIDE 13

In writing this paper I have supposed that the audience consists mainly of persons who have used the Monte Carlo method in neutron transport calculations. With the last slide I only want to say that the jump to the reliability applications is not so long. When somebody knows a Monte Carlo transport code, he has no difficulties in understanding the logic of a Monte Carlo reliability program. In addition, he quite often knows all the tricks related with the variation reduction better than persons working in the field of reliability.

USEFULNESS OF THE
MONTE CARLO METHOD
IN RELIABILITY
CALCULATIONS

CEN Saclay

SERMA/LEP

LEP = Laboratoire

d'Etudes de Protection

• TRIPOLI

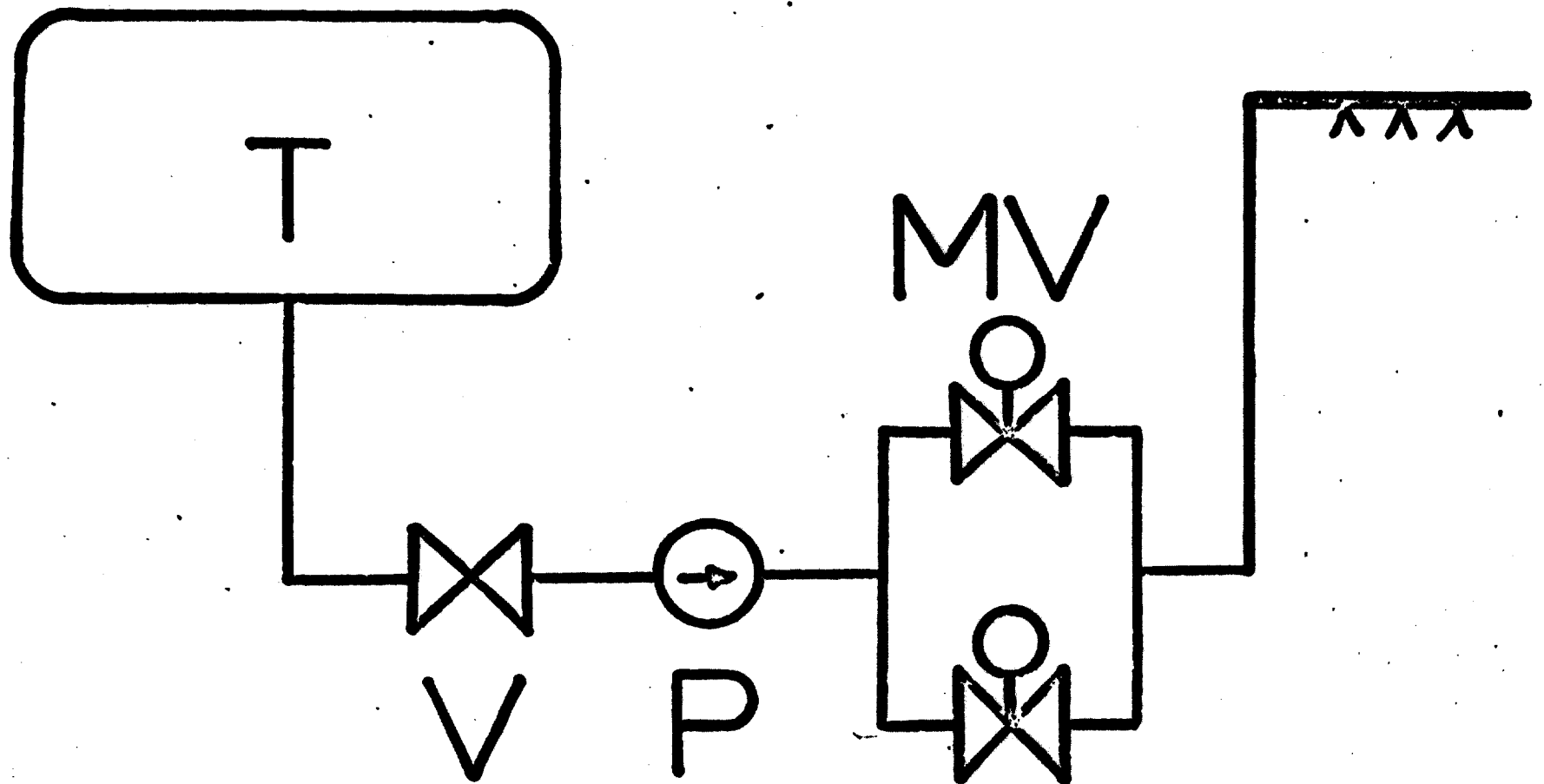
• MERCURE-4

• JERK

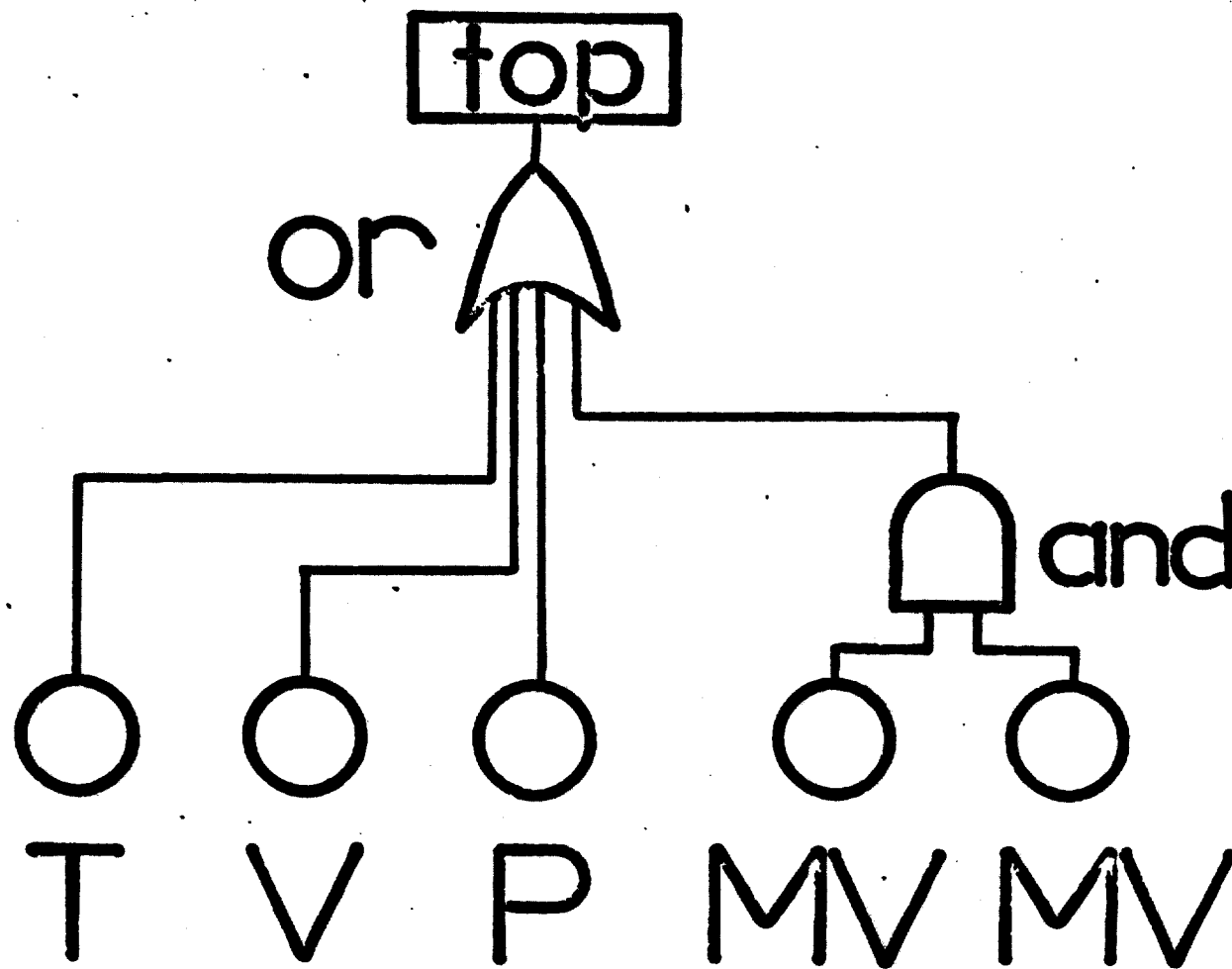
MONTE CARLO APPLICATIONS TO RELIABILITY

- UNCERTAINTY ANALYSIS
- SIMULATION OF COMPLEX
SYSTEMS

FAULT TREE EXAMPLE



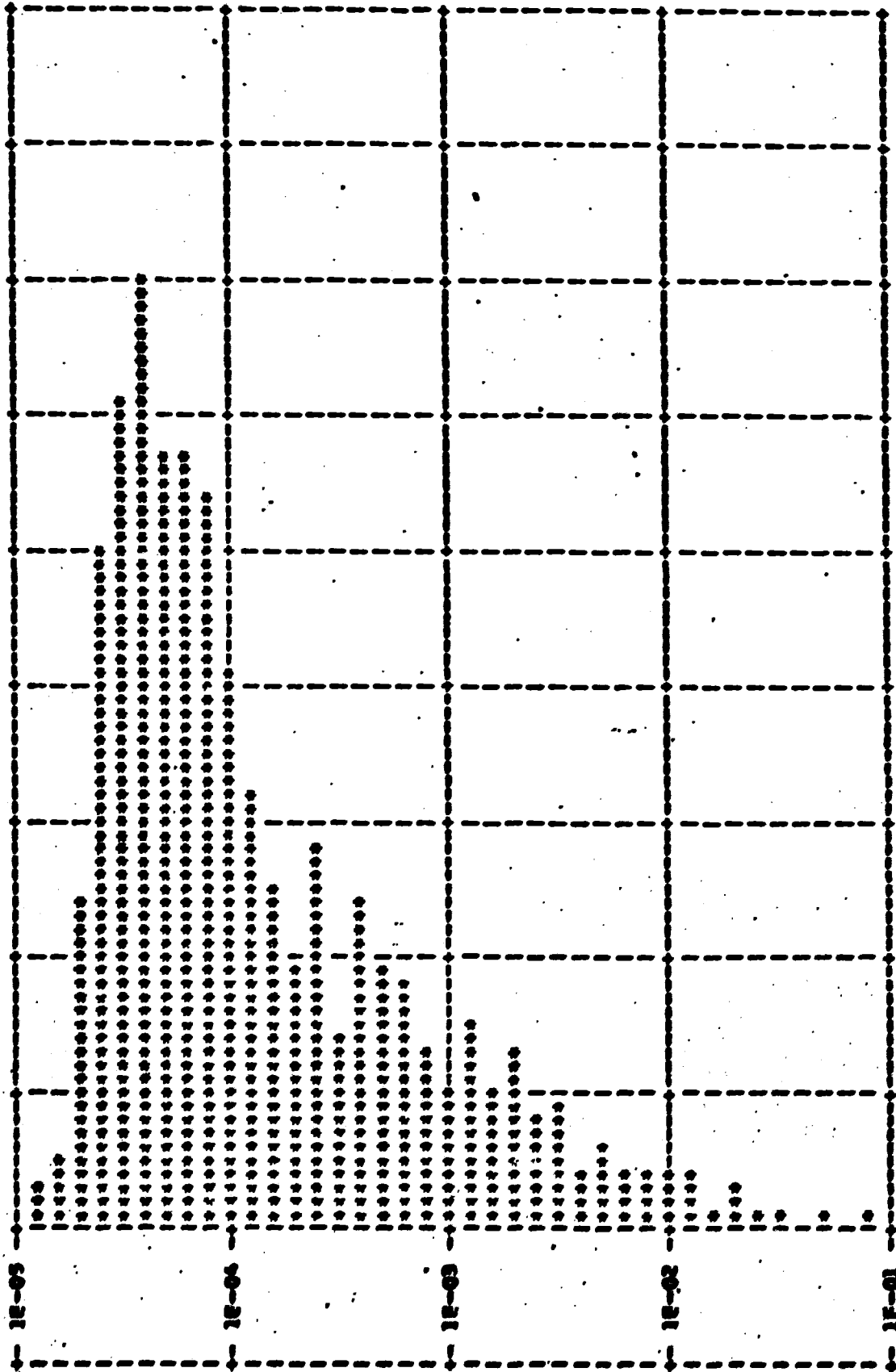
FAULT TREE EXAMPLE



UNCERTAINTY ANALYSIS

- PATREC-MC

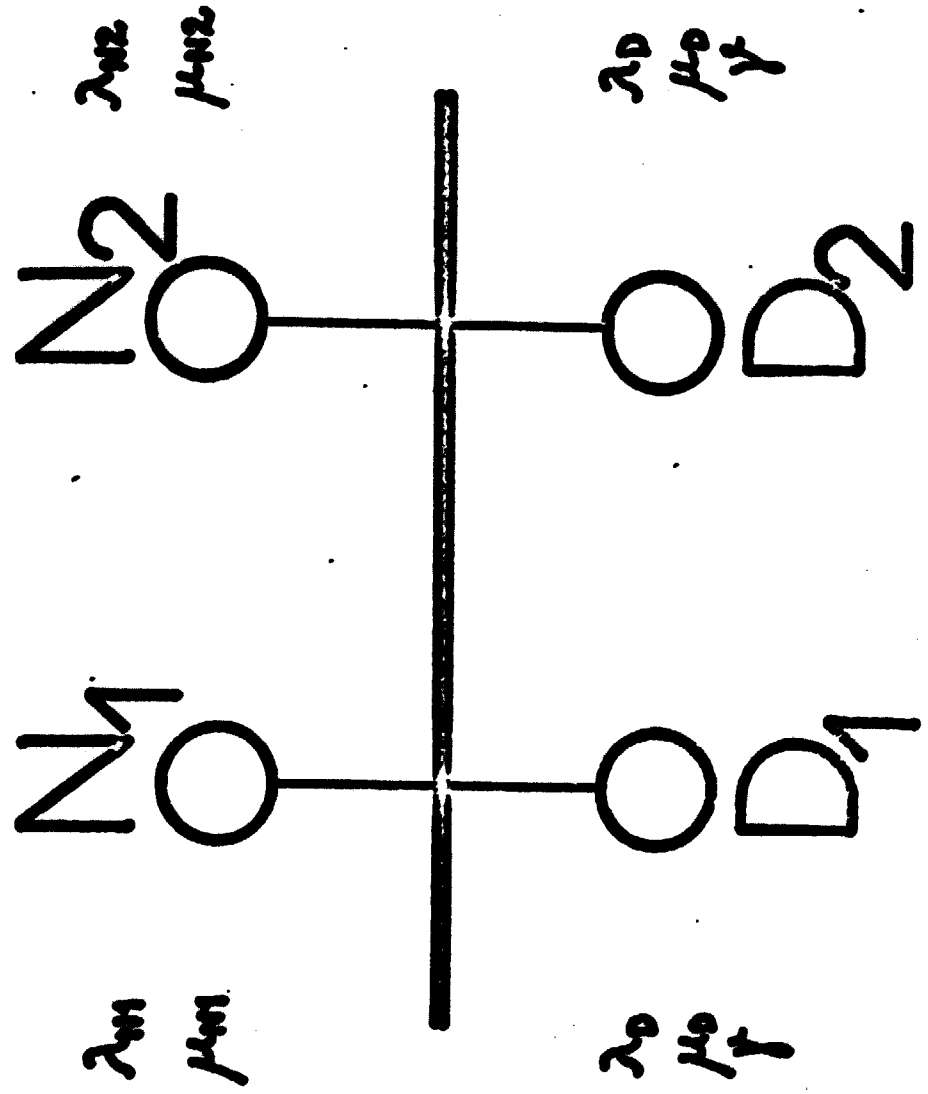
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SIMULATION OF COMPLEX SYSTEMS

- MONARC 2

POWER SUPPLY EXAMPLE



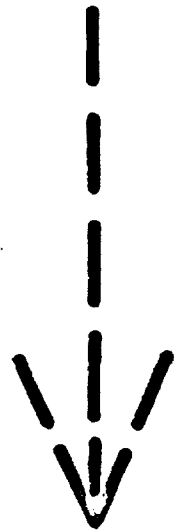
$$P_i^*(t) = k p_i(t)$$

$$W_i = \frac{P_i^*}{p_i^*} = 1/k \quad \text{down}$$

$$W_i = \frac{1 - P_i^*}{1 - p_i^*} = \frac{1 - P_i}{1 - k P_i} \quad \text{up}$$

$$W_j = \delta_j \prod_{i \in G_j} W_i \prod_{i \in G_j} W_i'$$

MONTE CARLO IN TRANSPORT THEORY



RELIABILITY