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CHIRAL SYMMETRY IN PERTURBATIVE QCD

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Abstract :

We prove that the chiral symmetry of quantum chromodynamics with massless quarks is unbroken in perturbation theory. Dimensional regularization with $\gamma_5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$ is used. The ratio of the vector and axial vector renormalization constants is shown to be independent of the renormalization mass. The general results are explicitly verified to fourth order in g, the QCD coupling constant.

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In a recent paper, O. Nachtmann and W. Wetzel have claimed to[(1)] demonstrate that chiral SU(2) x SU(2) is broken by the lowest order QCD perturbation. This result is very surprising and, if true, would have many implications. At the very least it would cast doubt on the widely used method of dimensional regularization and the associated definition of γ_5 on which they base their calculation[(2)]. We will show that, unfortunately, their result is incorrect for rather uninteresting reasons ; namely : the renormalization has not been done properly.

Their claim is based on a comparison of the vacuum polarization diagrams for the vector and axial vector currents.

$$i\int d^4x\, e^{iq\cdot x}\langle T(j_\mu^a(x)\, j_\nu^b(0))\rangle = \delta^{ab}(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^V(q^2),$$

$$i\int d^4x\, e^{iq\cdot x}\langle T(j_\mu^{5a}(x)\, j_\nu^{5b}(0))\rangle = \delta^{ab}(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^A(q^2)$$

$$+\,\delta^{ab} q_\mu q_\nu\, \Pi_L^A(q^2)\ . \qquad (1)$$

Here a and b are isospin indices. Notice that only non-singlet currents are considered so there are no anomalies and the quark lines can be followed continuously through the graph from one current to the other. Were the graphs convergent, one could simply move the γ_5 from one vertex to the other ; in a massless theory, it would pass an even number of γ-matrices before encountering and cancelling the other γ_5. Thus, one could conclude that $\Pi^V = \Pi^A$ and $\Pi_L^A = 0$.
However, the graphs are not convergent and one conventionally proceeds by regulating the theory. Most methods of regulating break the chiral symmetry :

the conventional Pauli-Villars vector-current conserving regulation does [3] and so does the dimensional regulation used by Nachtmann and Wetzel. This is not necessarily a problem ; as emphasized by Bogoliubov and Shirkov [4] this may be compensated for by the appropriate addition of counter-terms to the Lagrangian. These counter-terms themselves will necessarily violate chiral symmetry. In particular, we must expect that the vector and axial vector vertex renormalization constants Z_V and Z_A will be unequal.

(One detail : Nachtmann and Wetzel consider, instead of the axial vector vertex $\gamma_\mu \gamma_5$, a three index antisymmetric tensor $\gamma_{\rho_1} \gamma_{\rho_2} \gamma_{\rho_3} \pm$ permutations. As long as the indices refer to ordinary space-time indices, which is the case here because the currents couple externally only, this is identical to the 't Hooft and Veltman definition $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in all dimensions [2].

More precisely they consider the Adler function [5]

$$D^{V,A}(q^2) = -12\pi^2 q^2 \frac{\partial}{\partial q^2} \Pi^{V,A}(q^2) \quad . \qquad (2)$$

These are calculated up to second-order in QCD in the Landau gauge. To this order, in the gauge, the result is finite without any renormalization and They obtain

$$D^V(q^2) = \frac{3}{2}\left(1 + \frac{g^2}{4\pi^2}\right) \; ,$$

$$D^A(q^2) = \frac{3}{2}\left(1 + \frac{11}{3}\,\frac{g^2}{4\pi^2}\right) \; . \qquad (3)$$

The difference between these two forms the basis for their claim that chiral symmetry is broken already in lowest order perturbation theory. Of course, they realize that (3) is not sufficient to establish the result ; to this order they are just multiples of one another. It is nececessary to examine the q^2 dependence and for this they use the renormalization group equation. However, it is no mere coincidence that if we define Z_V and Z_A by normalizing the vector and axial vector vertices at the same point M , then we find [6]

$$D^V(q^2) = \left(\frac{Z_A}{Z_V}\right)^2 D^A(q^2) \; ; \tag{4}$$

i.e.

$$Z = \frac{Z_A}{Z_V} = 1 - \frac{4}{3}\,\frac{g^2}{4\pi^2} + O(g^4) \; . \tag{5}$$

One would naturally think that the renormalized Adler functions $D_R^{V,A} = Z_{V,A}^2 D^{V,A}$ are the appropriate quantities to compare. However, Nachtmann and Wetzel claim that they satisfy different renormalization group equations and so evolve differently in q^2. The basis for this claim is the assertion that

$$\gamma = -M \frac{d}{dM} \ln Z \neq 0 \quad . \tag{6}$$

If (6) were correct, then surely D_R^V and D_R^A would have different q^2 dependence. They establish (6) by differentiating (5) and conclude that

$$\gamma = \frac{8}{3}\,\frac{g}{4\pi^2}\,\beta + \cdots$$

That this is a flaw has been noticed independently by I. Antoniadis [6]: because β is of order g^3 when $n \to 4$ it is essential to examine the singular part of the g^4 term in (5). Thus, one must really go to fourth order in QCD to establish their claim ; to order g^2, one can conclude nothing.

We would now like to outline a proof, in the dimensional regularization scheme, that in massless QCD the constants Z_V and Z_A can be chosen so that the vector and axial vector vertex function for SU(2) x SU(2) are "equal" to all orders ; i.e.

$$\Gamma_\mu^5 = \Gamma_\mu \gamma_5 \quad . \tag{7}$$

(The isotopic indices will be suppressed , but remember this is all for flavor non-singlet currents). This is also stated to be true in ref.1. Having established (7), one then shows in the same fashion that Z is independent of M and so

$$\gamma \equiv 0 \tag{8}$$

to all orders in perturbation theory.

Because the Adler functions are superficially convergent, Eq.(7) implies immediately that $D_R^V(q^2) = D_R^A(q^2)$ and Eq.(8) lays to rest any doubt one may have about their q^2 dependence being different. Finally, to have something very explicit, we examine the coefficient of the $\ln q^2$ term in order g^4 . If Eq.(4) holds beyond second order, the coefficient of this term must be the same in D^V and D^A while, according to Nachtamnn and Wetzel, it would be in the ratio of the coefficient of the g^2 term. We shall see, of course, that the coefficient is the same.

The proof of Eq.(7) proceeds by induction : assume it to be true for order $(m-2)$ in g . It is obviously true for order g^0 . Then use the equations

$$\Gamma_\mu(p',p) = Z_V \gamma_\mu + \int S \Gamma_\mu S K , \tag{9a}$$

and

$$\Gamma_\mu^5(p',p) = Z_A \gamma_\mu \gamma_5 + \int S \Gamma_\mu^5 S K . \tag{9b}$$

This is written symbollically following Bjorken and Drell[7], only note that the integral is in n-dimensions, S is the renormalized quark propagator and K is the renormalized two particle irreducible quark-quark scattering kernal. This equation forms the basis for the induction proof because K starts off in order g^2 and so $\Gamma^{(m)}$ is related to $\Gamma^{(m-2)}$, $\Gamma^{(m-4)}$, ... by this relation. The integrals in (9) are finite as $n \to 4$ after one subtraction by power counting because all subgraph renormalizations have been done. Thus, $\Gamma_\mu(p',p) - \Gamma_\mu(p_0',p_0)$ and $\Gamma^5_\mu(p',p) - \Gamma^5_\mu(p_0',p_0)$ for arbitrary fixed p_0 and p_0' are given by integrals convergent as $n \to 4$. But then the γ-algebra can be done as if in 4-dimensions and, since there must necessarily be an even number of γ-matrices occuring on either side of the vertex in any Feynman graph, in the massless theory, we may pass the γ_5 to the left within the integral and obtain

$$\Gamma^{5(m)}_\mu(p',p) = \left(\Gamma^{(m)}_\mu(p',p) - \Gamma^{(m)}_\mu(p_0',p_0)\right)\gamma_5 + \Gamma^{5(m)}_\mu(p_0',p_0).$$

The divergent part of the integral, as $n \to 4$, must be proportional to γ_μ or $\gamma_\mu\gamma_5$ by power counting so one can fix the normalization condition

$$\Gamma^{5(m)}_\mu(p_0',p_0) = \Gamma^{(m)}_\mu(p_0',p_0)\gamma_5$$

by simply choosing Z_A/Z_V so this is true. Then Eq.(7) follows. Incidentally, we have not required that the substraction point here be in any way related to the QCD substraction point M, but it clearly will be economical to do so. Indeed, the points of normalization for Γ_μ and Γ^5_μ could be different from each other ; in that case one would obtain $\Gamma^5_\mu = \tilde{Z}(g)\Gamma_\mu\gamma_5$ where $\tilde{Z}(g)$ is a non-trivial power series in g.

Now suppose that the normalization point for the vertices is chosen to be the QCD point, say $p^2 = p'^2 = (p-p')^2 = M^2$, and consider two different possible renormalization points M and M'. All of the changes in the internal QCD renormalization cancel out and Eq.(9) implies that

$$\frac{Z_V(M')}{Z_V(M)}\Gamma_\mu(p',p) - \Gamma'_\mu(p',p) = \left(\frac{Z_A(M')}{Z_A(M)}\Gamma^5_\mu(p',p) - \Gamma^{5\,\prime}_\mu(p',p)\right)\gamma_5,$$

because each side is the given by convergent integrals and so the γ-algebra may be done using the 4-dimensional rules. But then from (7) we have

$$Z(M') = \frac{Z_A(M')}{Z_V(M')} = \frac{Z_A(M)}{Z_V(M)} = Z(M), \qquad (10)$$

where Z is independent of M and $\gamma=0$, as stated in Eq.(8).

Finally, let us examine the contribution to $D^{V,A}$ up to fourth order in g using dimensional renormalization[2] in Landau gauge. In this method one simply subtracts off the poles at n = 4 which occur in the Green's functions rather than fixing their values at some point. As we have seen, in zeroth and second order the results are independent of q^2 and for us the important question is the coefficient of $\ln q^2/\mu^2$ in fourth order. The constant term in fourth order can be changed by finite renormalization, but the coefficient of $\ln q^2/\mu^2$ cannot because the second order term is constant. (μ^2 is the scale parameter introduced in the dimensional renormalization method).

Because of the derivative in Eq.(2) , $D^{V,A}$ are superficially convergent and there are no overlapping divergence problems. Graphs containing only quark self-energy insertions are renormalized to be finite independently of whether a vector or axial vector vertex appears, so the only graphs which can conceivably contribute differently to D^V and D^A are those containing a vertex insertion. To this order, these are one or two loop graphs and have been shown by 't Hooft and Veltman [2] to be the sum of harmless poles at n =4 plus a convergent part.

In general, the one loop vertex insertion contains a simple pole at n = 4 . In Landau gauge this is multiplied by a factor n - 4 leading to a constant which is different for vector and axial vector insertions. From the pole piece one obtains

$$\Gamma^{(2)}_{\mu}\Big|_{pole} = \frac{2}{4-n}\,\frac{g^2}{12\pi^2}\,\frac{(n-4)(n-1)}{n}\gamma_\mu = -\frac{1}{2}\,\frac{g^2}{4\pi^2}\gamma_\mu \equiv C_V\,\frac{g^2}{4\pi^2}\gamma_\mu, \tag{11a}$$

$$\Gamma_\mu^{5(2)} = \frac{2}{4-n}\ \frac{g^2}{12\pi^2}\cdot\frac{(n-4)(n-9)}{n}\,\gamma_\mu\gamma_5$$

$$= \frac{5}{6}\ \frac{g^2}{4\pi^2}\,\gamma_\mu\gamma_5 \equiv C_A\,\frac{g^2}{4\pi^2}\,\gamma_\mu\gamma_5 \ . \qquad (11b)$$

Because these are finite, in the dimensional renormalization scheme no subtraction is made and so the result (3) to second order, in which $D^V \neq D^A$ is obtained.

In fourth order, there are two classes of contributions to depending on where in the graph the q^2 -derivative in Eq.(2) acts ; those which contain one or two one loop insertionsand those containing a single two loop insertion . Because the zeroth and second order graphs are themselves constant, the first class can lead only to constant partswhich differ for V and A . The two loop insertions in general have a double pole and a simple pole at $n = 4$; these are converted in Landau gauge to a simple pole plus a constant. Both the constant and the residue of the pole depend on whether the current is vector or axial vector. The pole term is subtracted in renormalizing the vertex but the constant remains. However, this simply multiplies the zeroth order contribution and so is constant in $D^{V,A}$. The convergent portions will give rise to $\ln q^2/\mu^2$ but these will be the same then in D^V and D^A .

For completeness, the vertex renormalization to this order is just

$$Z_{V,A}^{d.r.} = 1 + \frac{1}{2}\left(\frac{g^2}{4\pi^2}\right)^2 \frac{1}{n-4}\left[C_{V,A}\left(3 z_3^{(1)} - 2 z_1^{(1)}\right) + z^{(2)}\right] \qquad (12)$$

in the dimensional renormalization scheme.

$z^{(2)}$ is the same for V and A and the combination $\left(3 z_3^{(1)} - 2 z_1^{(1)}\right)$ is clearly related to the QCD charge renormalization [8]

$$\frac{Z_3^{3/2}}{Z_1} = 1 + \frac{1}{2}\,\frac{g^2}{4\pi^2}\,\frac{1}{n-4}\left(3Z_3^{(1)} - 2Z_1^{(1)}\right) \tag{13}$$

From this one finds immediately that the coefficient of $\ln q^2/\mu^2$ is just $\left(\frac{g^2}{4\pi^2}\right)^2\left(\frac{3Z_3^{(1)}-2Z_1^{(1)}}{2}\right)$ in D^V and D^A, by simply using the renormalization group equation and the vector Ward identity.

Notice that in the dimensional renormalization scheme, the vector and axial vector vertices are not normalized so that Eq.(7) is true and so $\mu \frac{d}{d\mu} \ln Z^{d.r.} \neq 0$. The pole term in order g^4 has however the same form in both the dimensional renormalization scheme and in the scheme defined by (7) so that (12) can be used to verify (8) to this order.

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REFERENCES

[1] O. Nachtmann and W. Wetzel, Physics Letters 81B, 211 (1979)

[2] G. 't Hooft and M. Veltman, Nuclear Physics B44 , 189 (1972)

[3] See, for example, J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics, Mc Graw-Hill, N.Y. (1964)

[4] N.N. Bogoliubov and D.V. Shirkov, Introduction to the Theory of Quantum Fields, Interscience Publishers, Inc., N.Y. (1959)

[5] S.L. Adler, Phys. Rev. D10, 3714 (1974)

[6] The result was obtained independently by I. Antoniadis, Laboratoire de Physique Théorique de l'Ecole Normale Supérieure. I thank E.G. Floratos for telling me of this work.

[7] J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, Mc Graw Hill, N.Y. (1964)

[8] B.W. Lee and J. Zinn-Justin, Phys. Rev. D5, 3137 (1972)