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SURFACE PROPERTIES OF SEMI-INFINITE FERMI SYSTEMS

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Abstract

A functional relation between the kinetic energy density and the total density is used to analyze the surface properties of semi-infinite Fermi systems. We find an explicit expression for the surface thickness in which the role of the infinite matter compressibility, binding energy and non-locality effects is clearly shown. The method, which holds both for nuclear and electronic systems (liquid metals), yields a very simple relation between the surface thickness and the surface energy.

## 1. Introduction

One of the most exciting feature of finite quantum systems is their surface. In nuclei, the surface energy is well known from empirical mass fits to be of the order of  $20 A^{2/3}$  Mev [1], where  $A$  is the mass number of the nucleus. Another characteristic quantity which can be defined without much ambiguity is the surface thickness of the one body density  $\rho(\vec{r})$ . The slope of  $\rho$  at the half density point [2] or the slope of the radial moments [3] provide natural definitions of the surface thickness. From elastic electron scattering and muonic atom X-ray data [4] we know that (apart from shell fluctuations of the order of 10 %) the surface thickness  $t$  remains practically constant throughout the periodic table. Typical values deduced from these analyses [3] are  $t = 2.33$  fm for  $^{40}\text{Ca}$ ,  $t = 2.15$  for  $^{48}\text{Ca}$ ,  $t = 2.33$  for  $^{90}\text{Zr}$  and  $t = 2.29$  for  $^{208}\text{Pb}$ , with uncertainties of the order of 1 %. These empirical values for the surface energy and surface thickness provide a stringent test for Hartree-Fock and other structure theories. These surface properties have been studied in the past by many authors both in the framework of semiclassical approaches [5-9] and in the Hartree-Fock approximation [6,10,13] .

The main purpose of this work is to study what are the theoretical ingredients that characterize the principle

features of the surface. We show, in the case of semi-infinite systems, that there exists a simple relationship between the surface thickness, non locality of the force and bulk properties of the infinite matter like binding energy per particle, incompressibility and Fermi momentum. The relative importance of the different ingredients depends on the structure of the density dependent terms of the interaction. We have also derived a relationship between the surface energy, surface thickness and the abovementioned bulk properties of the infinite systems. A simplified form of this relation was already known for an electron gas in liquid metals [14].

The method we propose is based on the existence of an energy density functional, i.e. we admit that the total energy of a many fermion system can be approximated solely in the form of a functional of the diagonal one-body density  $\rho(r)$ . This assumption implies that there exists an approximate functional relation between the kinetic energy density  $\tau(r)$  and  $\rho(r)$ . This point is analyzed in some detail. For a restricted class of variations with respect to the equilibrium state considered here, we conclude that such a relation does exist. Further this assumption also implies that the potential energy does not depend on the complete one-body density matrix  $\rho(r,r')$  but only on  $\rho(r)$ , its derivative and  $\tau(r)$ . This is already the case for the effective forces of the Skyrme or Moszkowski type [15]. However for the more realistic finite range forces, the exchange terms can be reduced to the desired form by a suitable expansion of the density matrix in terms of  $\rho(r)$  and its derivatives [16].

In section 2 the general formalism is presented, and the physical meaning of the ingredients entering in the main formulae for the surface thickness and energy are discussed. In addition, we study the validity of the functional relation between  $\tau(x)$  and  $\rho(x)$ . In section 3 the numerical results obtained with the present method are compared with exact Hartree-Fock calculations of semi-infinite nuclear systems.

## 2. The formalism

### 2.1 - Variational equations for energy density functionals

In the present paper we shall adopt the energy density formalism. In particular we suppose that the energy  $E$  of a many fermion system can be written in the form of a functional of the density  $\rho(\vec{x}) = \sum_i \psi_i^\dagger(\vec{x}) \psi_i(\vec{x})$  and of the kinetic energy density  $\tau(\vec{x}) = \sum_i \vec{\nabla} \psi_i(\vec{x}) \vec{\nabla} \psi_i^\dagger(\vec{x})$  :

$$E = \int E(\rho, \tau) d\vec{x} = \frac{\hbar^2}{2m} \int \tau(\vec{x}) d\vec{x} + \int f(\rho) d\vec{x} + B \int |\nabla \rho|^2 d\vec{x} + \int g(\rho) \tau(\vec{x}) d\vec{x} \quad (1)$$

where  $f$  and  $g$  are functionals of  $\rho(x)$  and  $B$  is a constant. The underlying assumption in eq.(1) is theoretically suggested by the formalism of the density matrix expansion [16] which has been widely used to determine static and dynamic properties of various physical systems (nuclei, metals...) [17]. The microscopic Hartree-Fock (HF) equations can be derived starting

from eq.(1), by imposing the condition  $\frac{\delta E}{\delta \psi_1} = 0$ .

An alternative approach to HF consists of expressing the kinetic energy density  $\tau(x)$  as some function only of  $\rho(x)$ . Then the total energy of eq.(1) becomes a functional only of the density  $\rho(x)$  as

$$E = \int E(\rho) d\vec{x} \equiv \int E(\rho, \tau(\rho)) d\vec{x} \quad (2)$$

and can be minimized by imposing

$$\frac{\delta E}{\delta \rho} = 0 \quad (2')$$

which avoids the determination of the quantum states  $\psi_i$ .

The crucial point is of course the choice of the relation between  $\tau$  and  $\rho$ . It is well known that the Thomas-Fermi (TF) approximation  $\tau = \alpha \rho^{5/3}$ , where  $\alpha$  is a constant, is valid only for homogeneous systems. To study finite systems one has to generalize the TF approximation by including gradient-type corrections : [18-20]

$$\tau(x) = \alpha \rho^{5/3} + \beta \frac{|\nabla \rho|^2}{\rho} + \gamma \nabla^2 \rho \quad (3)$$

The form of eq.(3) is suggested by a semiclassical expansion of  $\tau(x)$  in terms of  $\rho(x)$  [20]. This expansion yields also explicit predictions for the coefficients  $\alpha = \frac{3}{5}(3\pi^2)^{2/3}$ ,  $\beta = \frac{1}{6}$ ,  $\gamma = \frac{1}{5}$ . However there is no fundamental reason why the semiclassical values of the parameters  $\alpha, \beta, \gamma$  should correspond to the best approximation of the true kinetic energy density. In section 3.2 we shall discuss in detail the problem of obtaining the best numerical values for  $\alpha, \beta$  and  $\gamma$ . For the remainder of

this section we shall outline some properties of the solutions of eqs. (2) and (2'). In particular we shall investigate the stationary condition 2' with respect to different density variations  $\delta\rho$ .

We firstly consider a monopole scaling deformation

$$\delta\rho = \epsilon \vec{\nabla}(\vec{r} \rho(r)) \quad (4)$$

Condition 2' with choice (4) yields the well known saturation condition - the generalized virial theorem. In the limit of very large systems this condition gives the value of  $k_F$  or equivalently the saturation density. In order to study the surface properties, choice (4) is not very convenient, and one should consider deformations involving explicitly the surface as, for example,

$$\delta\rho = \epsilon \vec{\nabla}(\vec{r} \frac{d\rho}{dr}) \quad (5)$$

with choice (5) there are no contributions coming from volume effects and one is explicitly considering surface effects.

Here we shall restrict the discussion to a particular form of functional (3) which is commonly used in nuclear physics

$$\int E(\rho\tau) d\vec{r} = \int \left[ \frac{\hbar^2}{2m} \tau(r) + C \rho^2 + F \rho^n + B |\nabla\rho|^2 + G \rho\tau \right] d\vec{r} \quad (6)$$

where the coefficients B,C,F,G are either phenomenologically adjusted [15,23] or calculated from a finite range two body interaction [16]. For nuclear systems n ranges from 2 to 3.

It is straightforward to evaluate the changes linear in  $\epsilon$  of the terms  $\int \rho^k d\vec{r}$  entering in eq. (6), with respect to variation (5), by parametrizing the density in terms of a Fermi function

$$\rho(x) = \rho_0 \frac{1}{1 + \exp\left(\frac{x-R}{a}\right)} \quad (7)$$

where  $a$  is the surface thickness. One then has

$$\delta \int \rho^k d\vec{r} = -\epsilon \rho_0^k \frac{k-1}{k+1} 4\pi R^3 \frac{1}{a} \left(1 + O\left(\frac{a}{R}\right)\right) \quad (8)$$

Using similar techniques one can evaluate the variations of the other terms of eq. (6) containing the gradient of the density ( $\int \frac{|\nabla \rho|^2}{\rho} d\vec{r}$ ,  $\int |\nabla \rho|^2 d\vec{r}$ ). The stationary condition  $2'$  then yields the following expression for the surface thickness of semi-infinite systems

$$a^2 = \frac{\frac{\hbar^2}{2m} \beta + \frac{2}{5} \rho_0 \left[ G(\beta - \gamma) + B \right]}{\frac{10}{11} \frac{n-1}{n+1} \bar{E} + \frac{4}{33} \frac{1}{(n+1)} K + \frac{3n-5}{22(n+1)} \frac{3}{10} \frac{\hbar^2}{m} k_F^2} \quad (9)$$

where we have introduced the binding energy in infinite matter

$$\bar{E} = \lim_{A \rightarrow \infty} \frac{E}{A} \quad (\bar{E} < 0) \text{ and the infinite matter incompressibility}$$

$$K = k_F^2 \frac{\partial^2 \bar{E}}{\partial k_F^2}. \quad \rho_0 \text{ is related to the Fermi momentum by } \rho_0 = \frac{2}{3\pi^2} k_F^3.$$

The surface thickness is often expressed in terms of the interval  $t$  in which the density falls from 90 % to 10 % of the central value  $\rho_0$ . For a Fermi distribution one has

$$t = 4 \ln 3 a \quad (10)$$



Using a similar procedure, by expanding the total energy (6) in terms of  $\frac{a}{R}$ , one gets, apart from some inessential corrections, the following formula for the surface energy :

$$\sigma = a \rho_0 \left( \frac{-10}{11} \frac{n-1}{n+1} \bar{E} + \frac{4}{33} \frac{1}{n+1} K + \frac{3n-5}{22(n+1)} \frac{3}{10} \frac{\hbar^2}{m} k_F^2 \right) \quad (11)$$

where  $\sigma$  is defined through :

$$E_{\text{tot}} = \bar{E} V \rho_0 + \sigma S$$

and the usual surface energy  $a_s$  is related to  $\sigma$  through :

$$a_s = \frac{4\pi}{\left(\frac{4}{3} \pi \rho_0\right)^{2/3}} \sigma \quad (12)$$

## 2.2 - Properties of the surface thickness

Eq. (9) represents one of the main results of the present work, since it permits us to evaluate, in a simple way, the surface thickness of a many-body system once the interaction force is known. It is interesting to discuss the role of the various terms entering in eq. (9). Firstly we note that the contribution of the Fermi energy  $\frac{3}{10} \frac{\hbar^2}{m} k_F^2$  is always rather small, so the surface thickness is determined by the binding energy and incompressibility. It is intuitive that a system with high incompressibility prefers a sharp surface which is indeed the case with eq. (9). Also the role of the energy can be easily understood if one notes that the chemical potential  $\lambda$  (for semi-infinite systems  $\lambda$  identifies with the binding energy  $\bar{E}$ ) determines the asymptotic form of the density :

$$\rho_r + \infty = \frac{1}{r^2} \exp \left( -6 \sqrt{-\frac{2m\lambda}{\hbar}} r \right) \quad (13)$$

Of course this asymptotic behaviour can also affect the surface region. From eq.(9) one also gets information on the role of exchange forces, since the effect of the non-local terms  $B$  and  $G$  is to make the surface more diffuse. This result can be explained if one remarks that an important effect of non local forces is to spread the single particle energy levels. Then the wave functions of such levels contribute in an "incoherent" way (the asymptotic behaviour of each level  $i$  is given by eq.(13) with  $\lambda$  replaced by  $\epsilon_i$ ) to the density in the surface region. Finally we remark that also in the extreme limit of suppressing the non local terms  $G$  and  $B$ , the surface thickness has a finite value, due to the presence of the Weizsäcker term  $\beta \frac{|\nabla\rho|^2}{\rho}$  in the kinetic energy density  $\tau(r)$ .

Up to now, we have not discussed the dependence of our results on the value of the power  $n$  entering in the energy functional (6). For nuclear systems reasonable values of  $n$  range between 2 and 3, which means that the contribution of the energy and of the incompressibility to the denominator of eq.(9), are equally important. A different result is obtained for liquid metals ( $n=1$ ) [17,21] where the contribution to the surface thickness comes essentially from the incompressibility and one gets the following simplified formula for the surface energy :

$$\sigma = \frac{1}{18} a K \rho_0 = 2 \frac{a}{x} \quad (14)$$

where we have defined the compressibility  $\chi$  as

$$\chi = \frac{1}{V} \frac{1}{\frac{\partial^2 E}{\partial V^2}} = \frac{9}{\rho_0 K} \quad (15)$$

Expressions similar to eq. (14) have been already found for liquid metals [14] and the fact that the product  $\chi\sigma$  is almost constant (of the order of  $1 \text{ \AA}^2$ ) for a whole class of different systems (metals and liquids) is experimentally known [22]. In our derivation of eq. (14) we have explicitly shown that the appearance of the incompressibility is due to a low power dependence of the energy functional on  $n$ . In the opposite limit ( $n$  large) the relevant quantity for the surface properties (eq. (9)) would be the binding energy.

### 2.3 - Validity of the functional relation between $\tau$ and $\rho$

In the derivation of eq. (9) for the surface thickness we have used eq. (3) for the functional relation between the kinetic energy density  $\tau$  and the density  $\rho$  in a variational calculation. This assumption is rather severe since eq. (3) is an approximation for the kinetic energy of the ground state of the system and there is no a priori reason why it should still be valid when we move away from the ground state, as happens in variational calculations.

The problem is then to know if the identity

$$\tau(\rho) = \tau^0(\rho) \quad (16)$$

where  $\tau^0(\rho)$  is given by eq. (3), is verified with sufficient accuracy for a whole class of states around the ground state. The general response to this question is negative as there are several instances in which eq. (16) fails. This happens systematically if we consider states  $\Psi$  containing current terms, as for example,

$$\Psi = e^{i \int \mathbf{c} \cdot \mathbf{f}(\mathbf{r})} \Psi_0 \quad (17)$$

Transformation (17) does not change the density  $\rho$ , but affects the kinetic energy :

$$\tau = \tau_0 + \epsilon^2 |\vec{\nabla} f|^2 \rho \quad \text{with } \rho = \rho_0 \quad (18)$$

and eq. (16) is clearly violated.

Also in the framework of static deformations (which do not involve current terms) there are important cases in which eq. (16) cannot be used. If one considers the following scaling quadrupole deformation :

$$\Psi(x, y, z) = \Psi_0(e^\lambda x, e^\lambda y, e^{-2\lambda} z) \quad (19)$$

then the total kinetic energy is changed by a volume effect. However if we utilize eq. (3) and consider the variations induced by eq. (19) on the density, we get changes coming only from surface effects, thereby proving that eq. (3) cannot be used in variational calculations involving scaling quadrupole transformations. Consequently one is restricted to the use of eq. (3) only to monopole deformations around a spherical ground state. We have explored in some detail the validity of eq. (16) in this case, by considering two different types of monopole

deformations, which have been already discussed in sect. 2.1, namely eq.(4) and eq.(5). Variation (4) is associated to the following scaling unitary transformation :

$$\psi = e^{i \frac{\epsilon}{2} \sum_i (\vec{r}_i \cdot \vec{p}_i + \vec{p}_i \cdot \vec{r}_i)} \psi_0 \quad (20)$$

It is straightforward to rigorously verify eq.(16) in the space of states defined by eq.(20). Qualitatively different results are obtained for variation (5). This deformation explicitly affects the surface and is associated to the monopole transformation :

$$\psi = e^{i \frac{\epsilon}{2} \sum_i \left( \frac{d\rho^0(r_i)}{dr_i} \frac{1}{\rho^0(r_i)} \vec{r}_i \cdot \vec{p}_i + h.c \right)} \psi_0 \quad (21)$$

We have explicitly evaluated the variations of the kinetic energy given by eq.(21). It turns out that eq.(16) is not exactly verified in this case because of some differences coming from the terms in  $\beta$  and  $\gamma$  of eq.(3). Anyway we have evaluated these effects and have shown that they are negligible in the evaluation of the surface thickness. This result is in agreement with the fact, discussed in sect.3.2, that the value of the parameters  $\alpha, \beta, \gamma$  is rather well defined independently of the shape of the density (different variants of the force correspond in fact to rather different values of the surface thickness).

In conclusion we can say that the kinetic energy density approximation (3) can be used with sufficient accuracy to perform variational calculations in spherical systems.

### 3. Results

#### 3.1 - Exact numerical Hartree-Fock calculations of large systems

In order to illustrate the validity of the methods described in previous sections we need some exact numerical solutions of the Hartree-Fock equations for semi-infinite systems using different energy density functionals. A number of these calculations have been performed in the past [10-13]. In order to perform a general analysis of the surface energy and thickness, we have developed a Hartree-Fock computer program adapted to the energy density functionals of the type (1). As a particular feature the code works in spherical coordinates, the whole set of bound states  $(n,l)$  being calculated in the selfconsistent HF mean field for  $N = Z$  systems with no Coulomb, or spin-orbit forces. The code has been used for systems with up to 7000 particles. For the most relevant quantities (see below) the asymptotic values are reached to a few percent at about 3000 particles. With less than 1000 particles, curvature and shell effects are still important. The surface energy is calculated as

$$a_S^{\text{HF}} = \lim_{A \rightarrow \infty} (E - \bar{E} A) / A^{2/3}$$

where  $E$  is the calculated total energy of the finite nucleus with  $A$  particles and  $\bar{E}$  is the binding energy of infinite nuclear matter. The surface thickness  $t^{\text{HF}}$  is evaluated as

the 10-90 % distance of the Hartree-Fock density.

We have performed complete calculations (i.e. reached the asymptotic values for  $a_B^{HF}$ ,  $t^{HF}$ , as well as for  $\alpha, \beta, \gamma$  - see below) for three functionals : i) Skyrme-III [23], ii) the modified force of Rivine, Treiner and Bohigas SKM [24] which contains a weaker density dependence, and iii) for the purely local force S-0 ( $B = G = 0$ ,  $C = -1000$ ,  $\gamma = 15000$ ,  $n=3$ ). We have failed in obtaining relevant results with the very non-local force Skyrme-V [23] because of instabilities due to large oscillations in the interior density. These three functionals which we considered cover a wide spectrum of bulk properties (see Tab. 1). Since in real double magic nuclei the force SKM gives results for the surface thickness which are very close to the experimental values, the prediction by this force in the limit  $A \rightarrow \infty$  should be considered as a realistic estimate of the surface thickness of the semi-infinite matter ( $t = 2.63$ ).

### 3.2 - Determination of the parameters $\alpha, \beta$ and $\gamma$ of the kinetic energy density approximation

To determine the surface thickness corresponding to a given energy density functional by using formula (9) one needs numerical values for the parameters  $\alpha, \beta$  and  $\gamma$  for the kinetic energy density approximation of eq. (3). In the past a number of different methods [18-20] have been proposed to determine theoretically the values of these parameters. Unfortunately these methods, based on semi-classical expansions

(in powers of  $\hbar$ ) of  $\tau(r)$  in terms of  $\rho(r)$ , are not valid beyond the classical turning point, i.e. in the external part of the surface region. Using the semi-classical strengths for the inhomogeneity terms ( $\beta = 1/36$ ,  $\gamma = 1/3$ ) yields only a poor parametrization of  $\tau(r)$ . Consequently, in the present work we have adopted a more pragmatic point of view which consists in keeping the form (3) and adjusting the parameters  $\alpha, \beta, \gamma$  so that the exact kinetic energy density is reproduced. In practice we have imposed the condition that the total kinetic energy be conserved

$$\int (\tau^{\text{HF}}(r) - \tau^{\text{appr.}}(r)) d\vec{r} = 0$$

and the following surface constraints

$$\int (\tau^{\text{HF}}(r) - \tau^{\text{appr.}}(r)) \frac{\partial \rho(r)}{\partial r} d\vec{r} = 0$$

$$\int (\tau^{\text{HF}}(r) - \tau^{\text{appr.}}(r)) \frac{\partial \rho(r)}{\partial r} \rho(r) d\vec{r} = 0$$

For the energy density functionals that we have considered, we found that the values of  $\alpha, \beta, \gamma$  depend smoothly on the number of particles, the asymptotic value being attained for  $A \geq 3000$  to within a few percent. The value of  $\alpha$  approaches, as expected, the standard nuclear matter value  $\alpha^{\text{m}} = \frac{3}{5} (3\pi^2)^{2/3}$ . The value of  $\beta$  ranges between 0.08 to 0.11 depending on the nuclear force. This constancy in the value of  $\beta$  contrasts with the dispersion of theoretical predictions ( $\beta = 1/4$  from the Weissäcker work [18],  $\beta = \frac{1}{5}$  in Willets work [19], and



$\beta = \frac{1}{36}$  from [20] ) and gives to us some confidence on the validity of this approach<sup>(+)</sup>. For the Laplacian term we obtain  $\gamma = 0.3$  for all functionals, which is about the semiclassical prediction [20].

As a typical example, in figure 1 are shown the exact Hartree-Fock (full line) and approximate (dashed line) kinetic energy densities, corresponding to a system of  $A = 4620$  particles calculated with the interaction SKM [24]. The plot refers to the surface region and shows the good accuracy of the approximation.

### 3.3 - Surface energy and surface thickness for simple energy density functionals

As an illustration of the accuracy of the formalism developed in the preceding sections we present a comparison of the estimates of the simple formulae of section 2, for the surface energy and surface thickness, with the results of exact Hartree-Fock calculations.

In table 1,  $t$  and  $a_s$  denote respectively the estimates for the surface thickness and surface energy, while  $t_{HF}$  and  $a_s^{HF}$  denote the corresponding Hartree-Fock values. Also indicated in table 1 are the relevant quantities entering in equations (9) : the incompressibility of nuclear

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<sup>(+)</sup> Krivine and Treiner have recently [25] proposed a parametrization of the type (3) in which  $\alpha$  and  $\beta$  are adjusted as a function of the number of particles to reproduce HF results of the standard double magic nuclei. Their values are in agreement with our results.

matter  $K$  and the coefficients of the non-locality terms in the energy density functional  $B, G$ . (The saturation properties  $\bar{E}$ ,  $k_F$  are practically the same for all the forces, and are not reported). As far as the role of the non-locality effects is concerned we remark that the contribution to the numerator of eq. (9) coming from the term  $G$ , which is related to the effective mass by  $\frac{\hbar^2}{2m^*} = \frac{\hbar^2}{2m} + G\rho$ , is always small.

From the results of table 1 we remark that formula (9) is practically exact in predicting the surface thickness. In contrast formula (11) for the surface energy is only accurate to  $\approx 20\%$ . The origin of this discrepancy lies in the fact that in the derivation of eq. (11), we have not allowed the parameter  $\alpha$  (see eq. 3) to depend on the nuclear radius  $R$  (we have used  $\alpha = \alpha^0$ ). The dependence of  $\alpha$  on the distance  $R$  affects the surface energy since it introduces a surface term in the expansion of the total energy performed to derive eq. (11). However this dependence does not affect the surface thickness, because it only introduces terms of lower order in  $\frac{1}{R}$  in the energetic balance of eqs. (2') and (5).

Formulas (9) and (11) are also useful in understanding the origin of differences of the results from one force to another. For example, we see that the smallness of the surface thickness and surface energy of force S-0 is essentially due to the absence of exchange term ( $B = 0$ ), while the differences between S-III and SKM are essentially due to the change in the incompressibility.

### Conclusions

The crucial point in the analysis of the surface properties carried out in the present work, was the approximation for the kinetic energy density as a function of the density  $\rho(r)$  given by eq.(3). The standard Thomas-Fermi approximation is in fact known to fail in the surface region, the role of inhomogeneous terms in the kinetic energy being essential to reproduce quantitatively correct results. A remarkable result of this work is that the coefficients  $\beta$  and  $\gamma$  of the terms  $\frac{|V\rho|^2}{\rho}$  and  $V^2\rho$  are shown to be rather well defined ( $\beta = 0.1$ ,  $\gamma = 0.3$ ) so that no major ambiguities seem to arise when using approximation (3) in the analysis of spherical systems.

The results presented in this paper offer a useful tool in the analysis of the surface properties of effective interactions. Formulas (9) and (11) for the surface thickness and energy can in fact be used either as a constraint for new phenomenological forces, or as a predictive estimate of the surface properties of more fundamental two-body interactions (via some approximations for the non local part of the density matrix [16]).

We have found useful to develop a general formalism in which the properties of very different systems (nuclei and metals) could be discussed in a common framework. This has permitted us to understand clearly the role of some physical quantities like the incompressibility, the binding energy and non locality effects in the surface region.

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- [25] H. Krivine and J. Treiner, Phys. Lett. B, in press.

Table Caption

Table 1 Surface thickness and surface energy calculated in the HF approximation and according to the formulas (9) and (11), for different energy density functionals. The infinite nuclear matter incompressibility  $K$  and the strengths of the finite range terms  $B$  and  $G$  are also indicated.

	S-0	S-III ref. [23]	SKM ref.[24]	SV ref.[23]
$t$ (fm)	1.54	2.23	2.63	2.66
$t^{HF}$ (fm)	1.54	2.20	2.64	-
$a_s$ (MeV)	15.7	24.4	22.1	27.5
$a_s^{HF}$ (MeV)	11.6	20.4	18.6	-
K (MeV)	336.	356.	216.	306.
B (MeV fm <sup>5</sup> )	0.	63.0	63.5	128.1
G (MeV fm <sup>5</sup> )	0.	44.4	34.7	215.5

- Table 1 -

Figure Caption

Figure 1      Exact Hartree-Fock (full line) and approximate (eq.3) (dashed line) kinetic energy densities corresponding to a system of  $A = 4880$  particles calculated with the functional SKM [24].



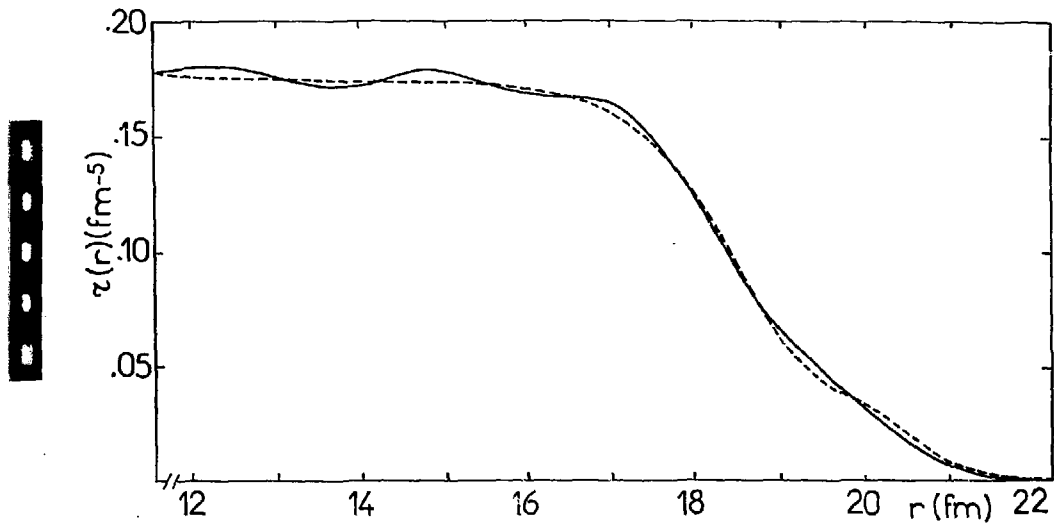


Fig-1