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AVAILABILITY OF PERIODICALLY TESTED SYSTEMS

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There is at the present time a need in accurate models to assess the availability of periodically tested stand-by systems. This paper shows how to improve the well known "saw-tooth curve" model in order to take into account various reliability parameters. A model is developed to assess the pointwise and the mean availabilities of periodically tested stand-by systems. Exact and approximation formulae are given. In addition, the model developed herein leads to optimize the test interval in order to minimize the mean unavaibility. A safety diesel is given as an example.

INTRODUCTION

Reliability engineers working in the nuclear safety field have to deal with several kinds of problems. As you know, "Fault-tree analysis" can be used successfully for static problems, "Markov processes" fit very well for dynamic problems but some trouble arises when the problem is neither quite static nor dynamic, as for instance assessments of the availability of periodically tested stand-by systems. In that case, the well known "saw-tooth curve" model is generally used but it is a very limited model allowing to take into account only few parameters.

This paper intends to show how the "saw-tooth curve" model can be strongly improved to give pointwise and mean availabilities and their limits when the number of test intervals gets large. It also shows how to optimize the test interval and how to find very good approximation formulae easy to hand-calculate.

To help understanding of the model,a typical periodically tested stand-by system is considered as an example:a safety diesel in a nuclear power plant.

RELEVANT RELIABILITY PARAMETERS

Before describing the model it is necessary to determine the reliability parameters to be taken into account. In this view,considering a safety diesel is very useful.

During normal operation of the plant,the diesel is on stand-by position. So,if a failure occurs,it is impossible to know that it has occured and therefore to reveal the state of the diesel,it is necessary to try to start it. This is achieved according to a specified test policy. Here are appearing the 2 first parameters to be taken into account:

τ = test interval.

π = test duration.

The probability of diesel failure when it is on stand-by position is obviously depending on the time elapsed. This can then be taken into account by a stand-by failure rate λ.

When the diesel is started to be tested,it can fail because the sudden change in its state. This is not depending on the time,so a probability γ to fail upon demand has to be used.

During the test, the diesel is in a running state, then a running failure rate λ' is needed to assess the probability of diesel failure when running.

If a failure is detected after a test has been performed, repair begins at once and a repair rate μ is necessary to evaluate the probability of the diesel repairing.

To summarize, 6 parameters (τ, π, λ, γ, λ', μ) have to be used to modelise the diesel behaviour from a reliability point of view. This is true for every periodically tested stand-by system.

POINTWISE AVAILABILITY

At the time $t_1 = 0$, it is intended to perform later on tests at times $t_2, t_3, \ldots, t_i$.

Just before the time t_i scheduled to perform the test n°i, the system can be in either of the 3 following states:

- under repair : probability a_i
- failed on stand-by : probability b_i
- available : probability c_i

When the system is under repair, the test is cancelled. In the other cases the test can be really performed.

Just after the time scheduled for ending the test n°i, the system can be in either of the 4 following states:

- under repair : probability e_i
- failed on stand-by : probability g_i
- detected failure : probability d_i
- available : probability f_i

Repair and stand-by failure states only occur when the test has been previously cancelled.

During the test performance, two different situations have to be considered:

- 1st situation : the diesel is not available for its safety function.
- 2nd situation : the diesel is available for its safety function.

Very different from a reliability point of view, these 2 situations are however very close, mathematically speaking.

The test interval can be divided into 2 parts: the part scheduled to perform the test which lasts π and the part when the diesel is on stand-by position which lasts $\phi = \tau - \pi$.

Let us consider a time δ belonging to the part scheduled to perform the test. For the first situation, the diesel can be available at this time δ only if:

- the scheduled test has been cancelled (a_i)
- the repair has been ended at $\delta' < \delta$
- the system has not failed again between δ' and δ

So we obtain for $\delta \in [0, \pi]$:

$$A_i(\delta) = a_i \frac{\mu}{\mu - \lambda} \left[e^{-\lambda\delta} - e^{-\mu\delta} \right] \qquad (1)$$

Let us consider now a time Δ after the test has been achieved. Three cases have to be considered to find the pointwise availability:

- system under repair at the end of the test, repair finished at $\Delta' < \Delta \leq \phi$, no system failure since Δ':

$$(e_i + d_i) \frac{\mu}{\mu-\lambda} \left[e^{-\lambda\Delta} - e^{-\mu\Delta} \right]$$

- system in a non-detected failure state at the time scheduled to end the test. In this case the system remains unavailable.

- system available at the end of the test, no failure between o and Δ:

$$f_i \, e^{-\lambda\Delta} = \left[1 - (e_i + g_i + d_i)\right] e^{-\lambda\Delta}$$

Gathering these results give the following formula for the system availability during stand-by position:

$$A_i(\Delta) = e^{-\lambda\Delta} \left[1 - g_i + \frac{\lambda}{\mu-\lambda} (e_i + d_i)\right] - \frac{\mu}{\mu-\lambda} (e_i + d_i) \, e^{-\mu\Delta} \quad \text{.............(2)}$$

(This formula can be used for the two situations; the formula n°1 can only be used for the first situation but it is easy to establish a similar formula for the second case).

It is now necessary to determine the values of a_i, b_i, e_i, d_i and g_i to calculate the system pointwise availability.

a_{i+1} = P{system under repair at the time t_{i+1} scheduled to perform test n°i+1}

This event occurs if:

OR { - a repair has begun at the end of the test n°i
- the system was under repair at the end of the test n°i

AND - the repair is not achieved at the time scheduled to perform the test n°i+1.

So we obtain: $a_{i+1} = (e_i + d_i) \, e^{-\mu\phi}$ ....(3)

The other coefficients can be found in the same way. This achieved, it is possible to perform a "step by step" calculation of the system pointwise availability if initial conditions are given. During the first test interval, the system pointwise availability is given by a simple exponential law :

$A(\delta) = e^{-\lambda\delta}$, therefore the initial conditions are:

$$a_2 = 0 \; ; \; b_2 = 1 - e^{-\lambda\tau} \quad \text{..(4)}$$

It is possible to show that, when the number of test intervals gets large, the coefficients a_i, b_i... reach steady-state values a, b... . In practical cases these assymptotic values are reached after only a little number of test intervals (3 or 4).

Figures n°1 and 2 show for the 2 situations the diesel pointwise unavailability. As you can see we obtain curves rather different from common "saw-tooth curves".

MEAN AND STEADY-STATE AVAILABILITY. APPROXIMATIONS

It is very easy, using the previous formulae 1 and 2 to assess in each test interval the system mean availability because these formulae are not difficult to be analytically integrated. The stars on figures n°1 and 2 show, for each test interval, the diesel mean unavailability. With data used to draw these curves the assymptotic mean unavailability, that is to say the steady-state unavailability, is reached very quickly.

In practical cases, approximations are generally possible because we have:

$\lambda.\tau << 1 \;\; ; \;\; 1 - e^{-\mu\tau} \approx 1 \;\; ; \;\; \mu >> \lambda$

Then it is possible to find approximation formulae from the exact formulae:

$$\bar{I}_1 \approx \frac{\pi}{\tau} + \frac{\lambda\phi}{\mu\tau} + \frac{\gamma_t}{\mu\tau} + \frac{\lambda\phi^2}{2\tau} \quad \ldots\ldots (5)$$

$$\bar{I}_2 \approx \gamma\frac{\pi}{\tau} + \frac{\lambda\phi}{\mu\tau} + \frac{\gamma_t}{\mu\tau} + \frac{\lambda\phi^2}{2\tau} + (1-\gamma)\,\frac{\lambda'\pi^2}{2\tau} + \frac{\lambda\pi\phi}{\tau} \quad \ldots\ldots (6)$$

In these formulae, γ_t is the total probability for the system to fail because of the test: $\gamma_t = \gamma + (1-\gamma)\lambda'\pi$. These formulae are of very practical use because they are easy to hand-calculate. $\bar{I}_1$ corresponds to the first situation and $\bar{I}_2$ to the second one.

It is now possible to compare the exact results with those given by the approximation formulae:

TABLE 1 - Comparison between exact and approximation results of steady state unavailability.

	exact values	approximation values
$\bar{I}_1$	$1.147\ 10^{-2}$	$1.15\ 10^{-2}$
$\bar{I}_2$	$8.73\ 10^{-3}$	$8.8\ 10^{-3}$

As you can see these results match very well. $\bar{I}_1$ is higher than $\bar{I}_2$, this is due to the fact in the first situation the diesel is unavailable during the test is performed, therefore the test duration has a strong influence on the system unavailability. This influence does not exist for the second situation.

TEST INTERVAL OPTIMIZATION

If the diesel could not fail on stand-by position ($\lambda = 0$), it would be of no interest to perform any test. On the other hand, if the diesel could not fail because of tests, it would be interesting to test it frequently. In the first case, the test interval would be equal to infinity (no test at all), in the second case it would be equal to zero (continuous test). Between these two extremes, the feeling is that an optimum test interval might exist.

The first approach to determine the optimum test interval is to look for which value of τ the derivative function of the steady state unavailability is equal to zero:

$\frac{d}{d\tau}\bar{I} = 0 \iff \tau_0 =$ optimum test interval.

For the first situation we find:

$$\tau'_0 = \left\{ \pi\left(\frac{2}{\lambda} + \pi\right) + \frac{2\gamma_t}{\mu\lambda} \right\}^{1/2} \quad \ldots\ldots (7)$$

In the same way, the 2nd situation gives:

$$\tau''_0 = \left\{ \pi\left[\frac{2\gamma}{\lambda} - \frac{2}{\mu} - \pi + \frac{\lambda'}{\lambda}\pi(1-\gamma)\right] + \frac{2\gamma_t}{\lambda\mu} \right\}^{1/2} \quad \ldots\ldots (8)$$

These formulae lead to the optimization of the steady-state unavailability.

Another approach consists of considering the real mean unavailability instead of the steady-state unavailability. The mean unavailability of the system is given by the following formula:

$$\bar{I}(o,k\tau) = 1 - \frac{1}{k}(\bar{A}_1 + \bar{A}_2 + \ldots + \bar{A}_k) \quad \ldots\ldots (9)$$

($\bar{A}_i$ is the mean availability in the test interval n°i).

This formula allows to assess the system mean unavailability for a given duration $T = k\tau$ like for instance one year. k is the number of test interval scheduled to be performed during T.

It is possible to consider the mean unavailability as a function of the test interval τ. This is shown on figure n°3 for the 2 situations we have previously defined.

Unavailability as a function of τ is a U-shaped curve which is quite flat in the neighbourhood of its minimum. The minimum of the curve corresponds to the optimum test interval. It is then easy to choose with the help of such a curve the test interval to be used to give the minimum mean unavailability for the system.

For the first situation the curve shows that the minimum is obtained for a test interval between 500 and 700 hours with a minimum mean unavailability of about $1.1\ 10^{-2}$.

In the same way the minimum is obtained, for the second situation, for a test interval between 250 and 350 hours with a minimum mean unavailability of about $6.9\ 10^{-3}$.

It is possible to find in each of the 2 situations an approximation formula to assess the minimum mean unavailability of the system:

$$\bar{I}'_m = \lambda(\tau'_o - \pi + \frac{1}{\mu}) \quad \ldots\ldots (10)$$

$$\bar{I}''_m = \lambda(\tau''_o + \frac{1}{\mu}) \quad \ldots\ldots (11)$$

Using formulae 7, 8, 10 and 11 we find:

$\tau'_o = 594$ h $\qquad I'_m = 1.14\ 10^{-2}$

$\tau''_o = 296$ h $\qquad I''_m = 7.\ 10^{-3}$

These results are perfectly correct with regard to those given by the curves.

So, using approximation formulae it is very easy to find the optimum test interval and the corresponding mean unavailability. As the curve is very flat in the neighbourhood of the optimum, it is possible to choose for the test interval an exact number of days, weeks or months.

Another interest of the U-shaped curve is that for a given value of the mean unavailability there are 2 corresponding values for τ. Then choosing the larger one (τ_n) leads to a minimum number of test to be performed. This is interesting from a cost point of view.

CONCLUSION

We have only tried in this paper to show how to improve the "saw-tooth curve" model. It was impossible to explain herein the whole theory in detail but this can be found in the reports we have realized on this subject (1), (2), (3), (4).

We hope to have shown the interest of this model and its ability to take into account many reliability parameters. Exact formulae are easy to use in computer code and we are developing the INDIGO code on these bases. Approximation formulae are easy to hand-calculate so, they are of very practical use.

Of course, it is possible, on these bases, to assess pointwise, mean and steady-state availabilities of more complex systems (series, parallel ...) but, may be, it could be a subject to propose to the third National Reliability Conference.

SYMBOLS USED

τ = test interval

π = test duration

λ = stand-by failure rate

λ' = running failure rate

μ = repair rate

γ = probability to fail upon demand

$A_1(\delta)$ or $A_i(\Delta)$ = pointwise availability

$\bar{I}_1$ or $\bar{I}_2$ = mean unavailability

$\bar{I}'_m$ or $\bar{I}''_m$ = minimum mean unavailability

$\bar{I}(o,T)$ = mean unavailability for a given period of time

τ'_o or τ''_o = optimum test interval.

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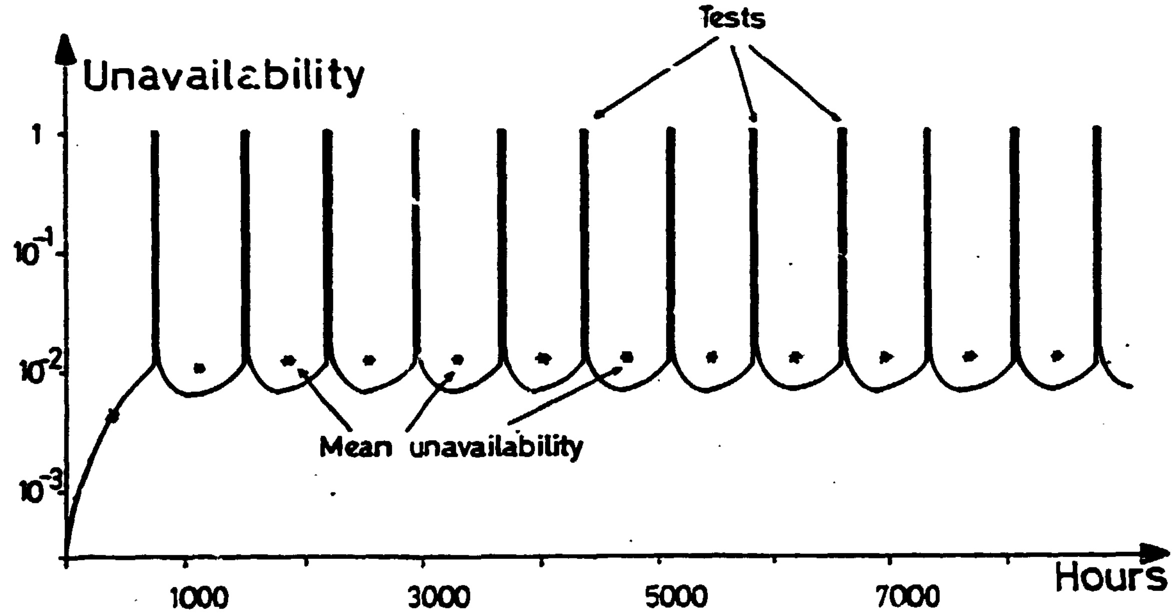

Figure 1 Diesel unavailability - 1st situation

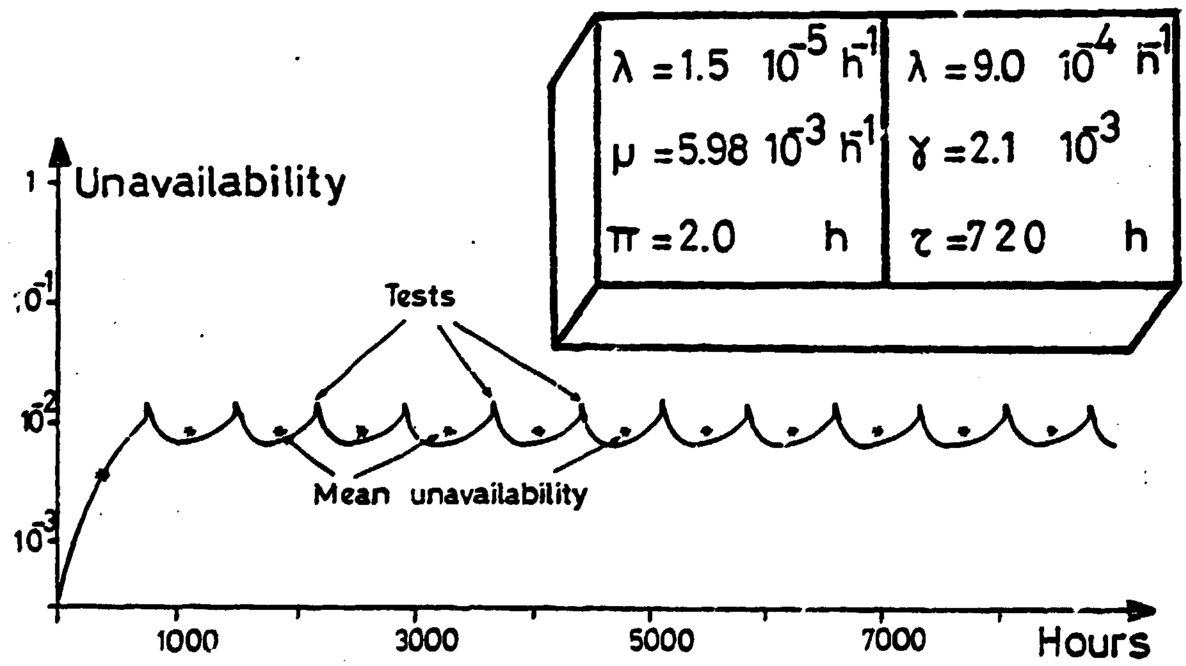

Figure 2 Diesel unavailability - 2nd situation

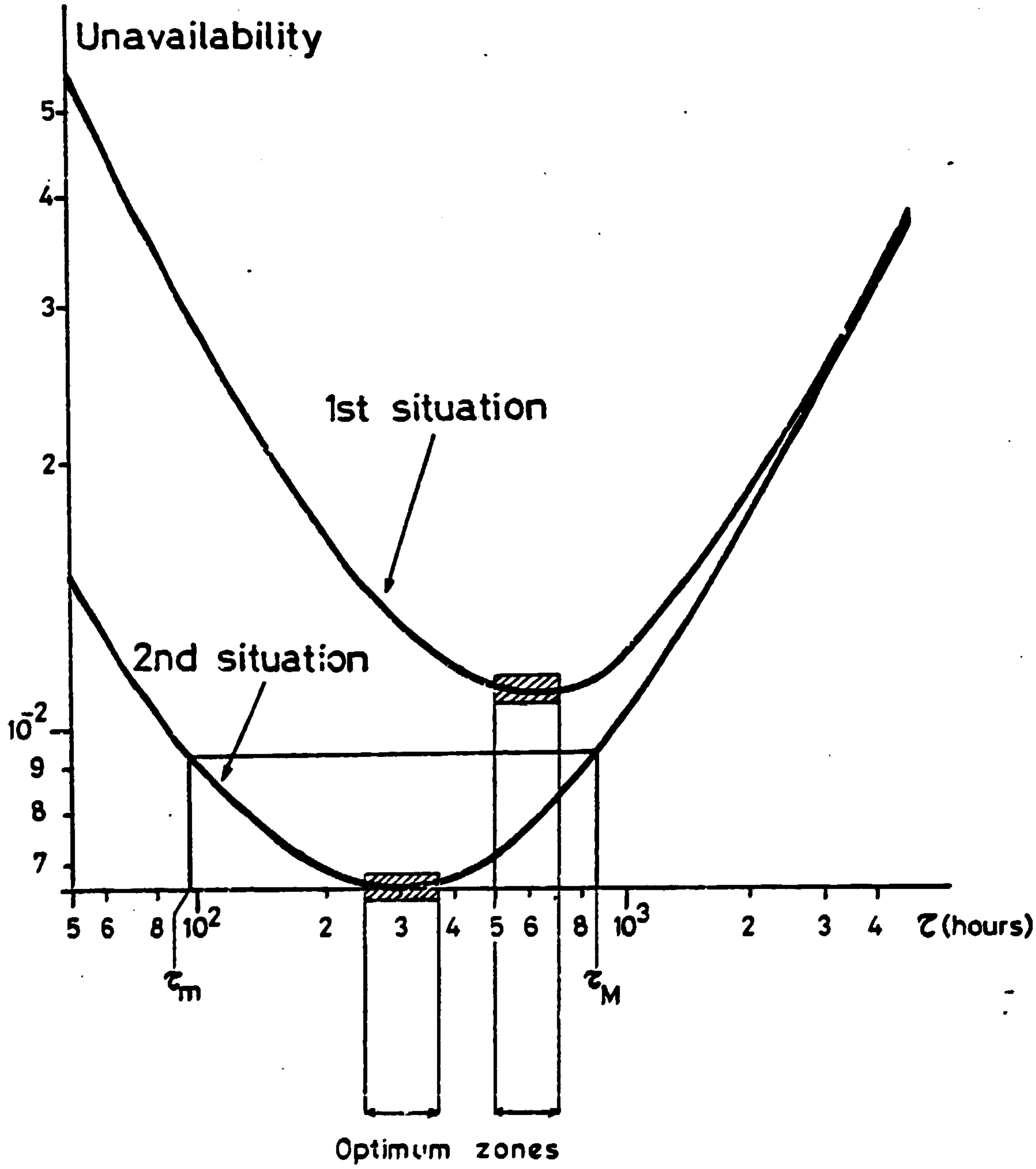

Figure 3 Optimum test interval research