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ANALYTIC PROPERTIES OF FORM FACTORS IN STRICTLY CONFINING MODELS

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Analytic properties of form factors in strictly confining models.

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An argument is presented, which shows that strict confinement implies the possible existence of an /unwanted/ branch point at $q^2=0$ in the form factors. In case of a bag extended to infinity in the relative time, the branch point is certainly there /provided that the form factor is non zero at $q^2=0$./

The non observation of free quarks has lead to the widely accepted notion of quark confinement. Recently, intensive work is going on to get confinement from basic theory /such as QCD might be/. On the other hand, models incorporating the notion of quark confinement /bag models/ have also been proposed[1,2] and elaborated in detail. In these models the Bethe-Salpeter /BS/ amplitude of a physical state is exactly zero outside the bag domain. It is clear that the experimantal situation does not really require this /e.g. an exponential fall off of the BS amplitude would be also acceptable/.

In order to stress that the BS amplitude is exactly zero outside the bag[3] we call these models strictly confining.

Assuming that the BS amplitude of the permanently bound states /$q\bar{q}$ mesons for simplicity/ is known, the form factors of the current matrix element $\langle A,P_A / J_\mu(o) / B,P_B \rangle$ are given by integrals of the structure:

$$F(q^2) = \int d^4x \; I(p_A, p_B, x)\, \Theta_A(p_A, x)\, \Theta_B(p_B, x)$$
$$= \int d^4x \, \exp(i\, q \cdot x/2)\, \psi_A^*(p_A, x)\, \psi_B(p_B, x) \qquad /1/$$
$$\times\, \Theta_A(p_A, x)\, \Theta_B(p_B, x),$$

where $q = p_A - P_B$ and x_μ is the relative coordinate of the quarks building up the meson. ψ_A and ψ_B are related to the BS amplitudes of A and B. The detailed structure / γ matrices, derivatives/ of the integrand is not important for our purposes. Our argument is based only on the assumption of strict confinement, which is manifestly shown in Eq./1/ by the presence of the factors $\Theta_A(p_A, x)$ and $\Theta_B(p_B, x)$. $\Theta_{A(B)}$ is a step function, which equals 1 inside and zero outside the bag of particle A/B/. Obviously $\Theta_B(p_B, x)$ depends only on the Lorentz invariant variables $p_B \cdot x/M$, x^2. The integration in Eq./1/ is carried out over the intersection of bags A and B.

Since Eq./1/ is Lorentz invariant, we may choose to work in the rest frame of A. In the following we always take

$p_A=(M_A,\vec{0})$, $p_B=(E,0,0,p)$. Then /for $M_A=M_B=M$/ $q^2=2M(M-E)$ and we have to carry out the analytic continuation of Eq./1/ in the variable E. It is clear that the integral representation Eq. /1/ is not suitable for the analytic continuation, since the arguments of Θ_B are $(Et-px^3)/M$ and $t^2-\vec{x}^2$, and the first one becomes imaginary for imaginary E /i.e.q^2/ thus Θ_B becomes undefined. A similar problem arises, if we rewrite Eq./1/ in terms of momentum space BS amplitudes:

$$F(q^2)=\int d^4k\ \tilde{\psi}_A^*(k+q/2)\,\tilde{\psi}_B(k)\ , \qquad /2/$$

where $\tilde{\psi}_{A(B)}(k)=(2\pi)^{-2}\int d^4k\,\exp(ik\cdot x)\,\psi_{A(B)}(x)$ and

the k integration is over the whole space. Obviously $\tilde{\psi}$ is an entire analytic function of k_μ and for some complex infinite arguments necessarily blows up exponentially. Since $\tilde{\psi}_B(k)$ in /2/ really depends on $p_B\cdot k/M$, k^2, for complex E the first argument becomes complex, and the integral /2/ becomes divergent. This does not mean that the analytic continuation of $F(q^2)$ is not possible, it means only that Eqs./1/ and /2/ are not suitable representations for analytic continuation. Since for nontrivial functions the integrals certainly can not be carried out explicitly, in order to carry out the analytic continuation a suitable integral representation should be devised. Unfortunately, we have been unable to find it, but we shall show that using only real $E\geq M$, it is possible to get information onthe analytic structure of $F(q^2)$.

Let us note first that $F(q^2)$ depends on E in two ways. First, there is an explicit E dependence, second there is a dependence through $p=(E^2-M^2)^{\frac{1}{2}}$. This suggests that there might be branch points at $E=\pm M,\infty$. Obviously the E=M /i.e. $q^2=0$; $q^2=(M_1-M_2)^2$ in the general case/ is an unwanted singularity.

To prove that the E=M branch point is there, we proceed by indirect argumentation. Assuming that usual analyticity properties are valid, $F(q^2)$ does not have singularities for $\mathrm{Re}E>-M$ /i.e. $\mathrm{Re}q^2<4M^2$/. Therefore, starting from $E=E_1$ and continuing along the real axis to E=M and back again to $E=E_1$ should bring us back to the starting value of $F(q^2)$. This should be true even if we choose to go to the second Riemann sheet of $p=(E^2-M^2)^{\frac{1}{2}}$, i.e. if we change p from $p_1=(E_1^2-M^2)^{\frac{1}{2}}$ through p=0 to $p=-p_1$. This kind of continuation may be carried out using Eq. /1/ since the integral remains well defined /and finite/ throughout. However, it is easy to see that the integration region does depend on E /both directly and through $p=(E^2-M^2)^{\frac{1}{2}}$ /and we get different integration regions depending on whether we go to the second Riemenn sheet of p or not[4,5].

The situation is especially simple if the bag is unbounded in relative time t. As an example, take the bag to be $|\vec{x}|<R_s$ in the rest frame of the particle. Using cylindrical coordinates (z,ϱ,φ) the angle integration in Eq. /1/ is trivial. We choose to carry out the $\varrho=((x^1)^2+(x^2)^2)^{\frac{1}{2}}$ integration last. The integration region in the (t,z) plane is then shown on Fig.1.

It is ABCD for $E=E_1$ and the analytically continued region /along the path mentioned above/ is A'B'C'D'. The image of AB is A'B' so it is clear that we pick up an overall minus sign. This means that $F(q^2)$ does change if $I(p_A, p_B, x)$ has a part even under $p\to-p$. However, to get a nonvanishing form factor at $q^2=0$ /which should be the charge/ $I(p_A, p_B, x)$ has to have an even part. Therefore if the charge does not vanish, we do not get back to the same value of $F(q^2)$, which contradicts the assumption and proves the existence of the branch point at $q^2=0$. The proof may be extended for the case of time dependent boundary, i.e.: $R_s=R_s(t)$, and non-spherical space bag, too.

Another case is, when the bag is bounded in relative time, too. Here we can not give a general proof, however the same procedure of analytic continuation along the real path given above provides us with a method to decide whether the discontinuity is there or not there. Namely, if $I(p_A, p_B, x)$ is a known function, we can calculate the integrals at the starting and end points of the path numerically and see whether they are different or not. It is clear that such a numerical method may give a decisive answer only in case of nonvanishing discontinuity. To give an example consider a bag $|\vec{x}|<R_s$, $|t|<R_t$. It is rather interesting that the analytically continued integration region is sometimes <u>outside</u> the bag of particle A /which is taken to be at rest/. Using cylindrical coordinates as above, the intersections of the bags in the (t,z) plane are shown on Figs. 2/a/,2/b/. In case of Fig. 2/b/

the integration region changes sign, in case of Fig.2/a/ it changes in a more complicated way. To avoid a confusingly complicated drawing only the image of one part of the intersection is shown in Fig.2/a/. To check a specific example we have used the pion BS amplitude of Ref./2/ to establish by numerical integration that the pion electromagnetic form factor has a branch point at $q^2=0$ in that model.

Having established the possible existence of the branch point we stress that it originates solely from the assumption ot strict confinement. If this assumption is relaxed, the integration in Eq./1/ should be extended over the whole space-time, and our arguments concerning variation of the integration region as a function of E become meaningless [6]. It is quite obvious also, that the magnitude of the discontinuity across the new, unwanted cut is model dependent. In a particular model it may very well happen that the new branch point does not effectively change usual dispersion relations within the precision required by experimental data.

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References

1. A.Chodos, R.L.Jaffe, K. Johnson, C.B.Thorn and V.F. Weisskopf, Phys.Rev. D9 3471 /1974/

2. G.Preparata, N. Craigie, Nucl.Phys. B102 476 /1976/
   G.Preparata, Geometrodynamics for quarks and hadrons: the mesonic states; CERN TH 2271 /1977/, unpublished.

3. We use the word bag in a somewhat unusal context to refer to the space-time region, where the BS amplitude is non-vanishing.

4. As a matter of fact the integration region calculated from the θ_A, θ_B functions for $p=-(E_1^2-M^2)^{\frac{1}{2}}$ directly does differ from the analytically continued region.

5. To get more confidence in our argumentation it is instructive to calculate the integral /1/ for simple cases. We have calculated explicitly the I=1 and $I=\exp(iq\cdot x/2)$ cases.

6. An infinite /and probably unrealistic/ bag, which does not change under Lorentz transformation is given by the boundaries: $t^2-\vec{x}^2=R^2$. Obviously in this case we do not expect a branch point at $q^2=0$.

Figure captions.

Fig.1. The intersection of the bags and its continuation for the case of bags unbounded in relative time.

Fig.2. The intersection of the bags and its continuation for the case of bags bounded in relative time, with $R_s=R_t$.

a. The case $(R_s^2-\rho^2)^{\frac{1}{2}} > R_s p/(E+M)$. Only the image of ABCD is shown.

b. The case $(R_s^2-\rho^2)^{\frac{1}{2}} < R_s p/(E+M)$.

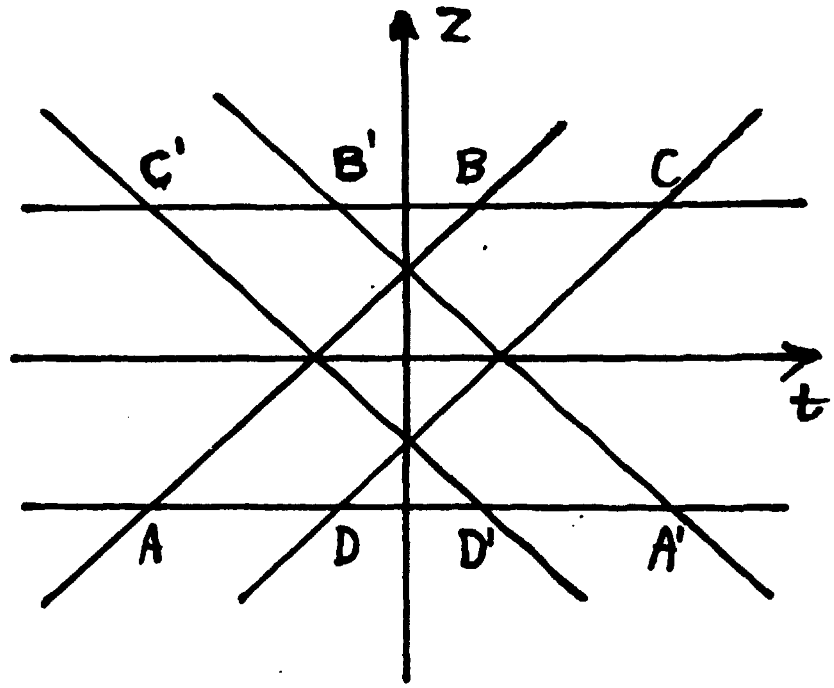

Fig. 1.

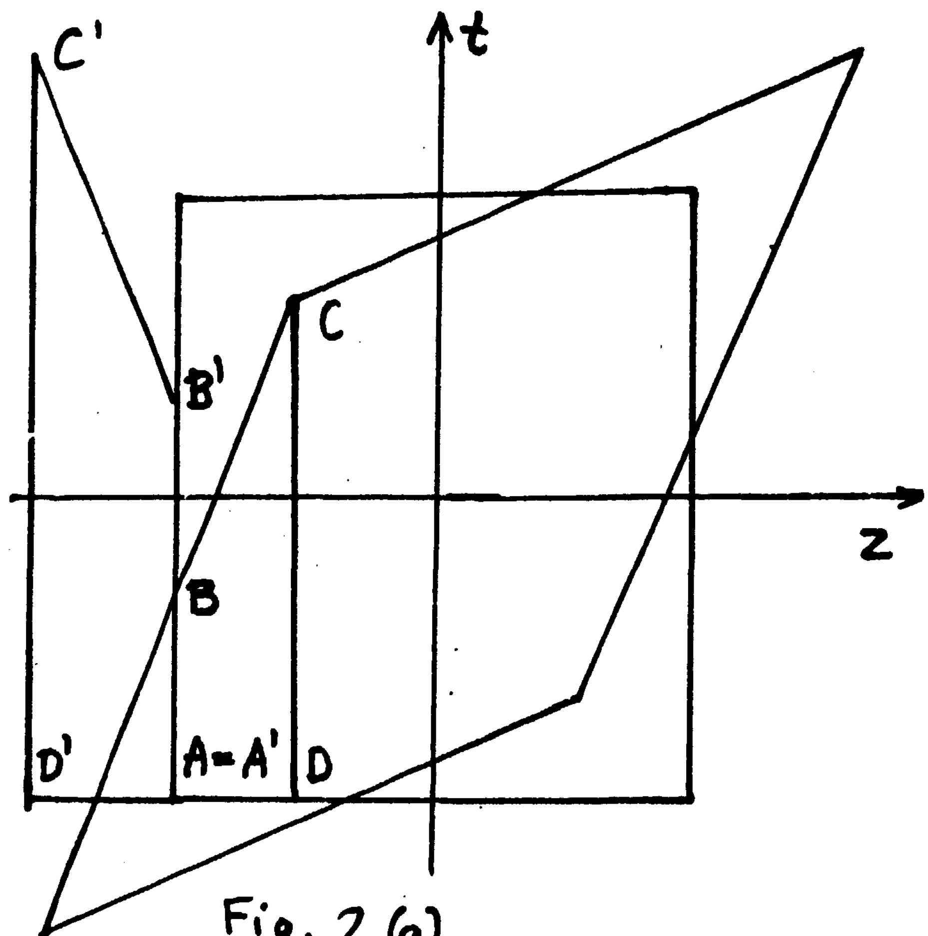

Fig. 2.(a)

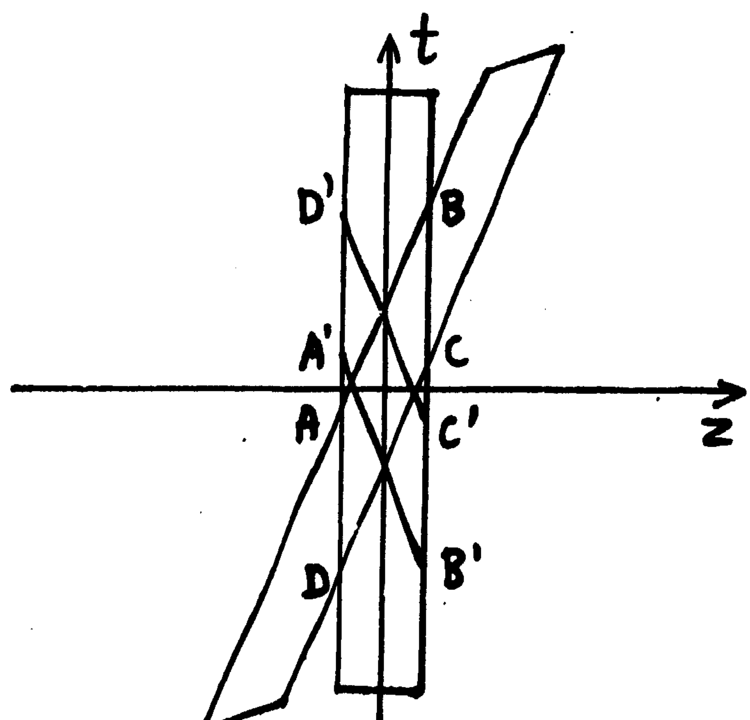

Fig. 2. (b)