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CONSTRAINTS FROM JET CALCULUS ON QUARK RECOMBINATION

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Within the QCD jet calculus formalism, we deduce an equation describing recombination of quarks and antiquarks into mesons within a quark or gluon jet. This equation relates the recombination function $R(x_1,x_2,x)$ used in current literature to the fragmentation function for producing that same meson out of the parton initiating the jet. We submit currently used recombination functions to our consistency test, taking as input mainly the u-quark fragmentation "data" into π^+ mesons, but also s-quark fragmentation into K^- mesons. The constraint is well satisfied at large Q^2 for large moments. Our results depend on one parameter, Q_o^2, the constraint equation being satisfied for small values of this parameter.

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I. INTRODUCTION

Over the past few years, the success of parton-model calculations in current-initiated reactions has led to general acceptance of quarks and gluons as useful calculational descriptions of hadronic structure functions. Clearly the next step is to use these same quark-content functions to predict purely hadronic reactions. Use in high transverse momentum (p_t) reactions for prediction of, e.g., jet cross sections, is well advanced; however, the application of these ideas to other regions of multiparticle phase space is still in its infancy.

A very intuitive approach applying parton-model concepts to low p_t inclusive reactions was suggested by Das and Hwa[1] to explain the empirical observation of Ochs[2] that inclusive pion production at large x and small p_t in proton-proton collisions reflected the proton structure function. Various versions and applications of this recombination model, as it has become called, have appeared[3-8]; most of these resemble the original Das-Hwa paper in essential concept although there are differences in implementation. The basic idea is: Forward non-diffractive meson production can be calculated by assuming that most of the momentum in the meson came from a valence quark in the beam particle. This quark recombined with a sea quark of low momentum, also from the beam, to create a meson.

To compute the x distribution of the mesons from this process two hitherto unstudied functions were introduced: $F(x_1,x_2)$, describing the probability that the incident beam has a quark of the correct flavor at x_1 and an antiquark of the correct flavor at x_2, and a recombination function $R(x_1,x_2,x)$ which tells how the two constituents join to create a meson of momentum $x_1 + x_2 = x$. Originally, both functions were created from various physical arguments; a fair number of the improvements of subsequent papers are directed towards refining these arguments.

One should be careful about specifying parton model functions entering in such a calculation, however, since the functions are not completely arbitrary. The Q^2 dependence of hadron structure functions, and presumably of other quantities related to hadronic wave functions, is known. According to quantum chromo-dynamics, which predicts the various couplings among quarks(q), antiquarks($\bar{q}$), and gluons(g), a very energetic quark generates a cloud of partons (q, $\bar{q}$, and g) that accompany it. The development of this cloud (with the Q^2 of the probing current) is described by the Altarelli-Parisi[9] evolution equations. The application of the physics in these equations to the description of jet production from quarks was formalized by Konishi, Ukawa, and Veneziano[10,11] (KUV). With their jet calculus, it is possible to calculate quickly various properties of a jet including the distribution of multiple constituents in longitudinal momentum fraction.

We adopt the basic ideas of jet calculus to get an expression for the probability of finding a quark at x_1 and an antiquark at x_2 and then require that they recombine to make the final state meson with momentum fraction x. The resulting function must be a major contribution to the fragmentation function for the quark that produced the jet, that is, the probability that a given quark will materialize into a specific hadron with momentum fraction x. Such quark fragmentation functions have been phenomenologically determined by Field and Feynman from electron-positron jet production and hadron lepto-production data.[12]

Hence, as a matter of principle, the recombination function R is not completely independent of other functions used in parton model phenomenology. In fact, it is severely constrained by the fragmentation function D. This relationship will be derived in Section II, and although the general solution for R in terms of D is difficult, with present popular forms for R the moments of D can be written in terms of the moments of R. We believe that the relation between R and D is non-trivial because, as pointed out by Owens,[13] D should satisfy an evolution equation of the Altarelli-Parisi type. Results of our calculations are presented in Section III where a detailed comparison with the fragmentation function of the u-quark into positive pions is made. Comparison with fragmentation of s-quarks into K^- mesons also is satisfactory. Section IV contains our conclusions and discussion of implications of our results.

II. DERIVATION OF THE RELATION BETWEEN FRAGMENTATION AND RECOMBINATION FUNCTIONS

We briefly sketch the logic behind our relation: R measures the probability that two quarks will stick together to form a meson and D measures the probability that a meson will be found in a particular quark jet. If we can compute from first principles the probability that the quark jet in question contains the two partons necessary to form the meson, we can multiply this probability by R to obtain D. (This fragmentation function will be denoted $D_{M,i}$.)

The jet calculus[10,11] leads to the probability for finding two partons a_1, a_2 with momentum between x_1 and $x_1 + dx_1$, x_2 and $x_2 + dx_2$, respectively, when they originated from parton i,

$$\frac{1}{\sigma_{i\to jet}} \frac{d\sigma(i\to a_1a_2 + X)}{dx_1\, dx_2} \equiv D_{a_1a_2,\, i}(x_1,x_2,Q^2) =$$

$$\sum_{b_1,b_2,j} \int_0^Y dy \int_0^1 dx\; dz\,dw_1dw_2\; D_{a_1b_1}(w_1,y)\; D_{a_2b_2}(w_2,y) \quad X$$

$$\hat{P}_{j\to b_1b_2}(z)\; D_{ji}(x,Y-y)\; \delta(x_1-xzw_1)\; \delta(x_2-x(1-z)w_2) \quad . \qquad (2.1)$$

The cross section for parton i to produce a jet is $\sigma_{i\to jet}$, with $D_{a_1a_2,i}(x_1,x_2,Q^2)$ the joint probability distribution for parton i with 4-momentum squared Q^2 to produce in its jet partons a_1 and a_2 at longitudinal momenta x_1 and x_2. The functions D_{ab} play the role of parton "propagators" in the variable $Y \sim \ln(\ln Q^2)$ of the Altarelli-Parisi equations, the vertices P are the fundamental QCD vertices (quark → quark + gluon, etc.) as used in those equations. The partons a_1 and a_2 can then be dressed into hadrons of interest by use of phenomenological functions; the resulting di-hadron distributions possess reasonable size and properties.[14] The δ-functions impose momentum conservation upon the momentum fraction variables x, z, w_1, and w_2. The KUV variable[11] is

$$Y = (2\pi b)^{-1} \ln[1 + \alpha_o b \ln (Q^2/\Lambda^2)] \quad , \qquad (2.2)$$

where $12\pi b = 11N_c - 2N_f$ for N_c colors and N_f flavors and α_o and Λ^2 are constants determining the strength and scale of the QCD coupling.

If we then require that partons a_1 and a_2 recombine to make a meson, we obtain an expression for the fragmentation function $D_{M,i}$ for quark i making meson M,

$$D_{M,i}(x,Y) = \sum_{a_i,b_i,j} \int_{Y_o(Q_o^2)}^{Y} dy \int_0^1 d\xi \, dz \, dw_1 \, dw_2 \, dx_1 \, dx_2 \quad X$$

$$R^M_{a_1a_2}(x_1,x_2,x) \; D_{a_1b_1}(w_1,y-Y_o) \; D_{a_2b_2}(w_2,y-Y_o) \; P_{j\to b_1b_2}(z) \quad X$$

$$D_{ji}(\xi,Y-y) \; \delta(x_1-\xi z w_1) \; \delta(x_2-\xi(1-z)w_2) \quad . \qquad (2.3)$$

The integral over y in (2.3) has a lower limit Y_o determined by $Q^2 = Q_o^2$ in Eq. (2.2), consistent with the requirement that some minimum energy is needed to make meson M. A pictorial representation of Eq. (2.3) is given in Fig. (1).

We now need a form for R^M. In this paper we concentrate our attention on the form used by previous workers in the field, although it is a very special one and perhaps should be generalized or altered as we discuss later. The recombination function of Das and Hwa,[1] and the more general form of Van Hove,[7] can be written as

$$R(x_1,x_2,x) = x\int_o^1 d\eta\ \tilde{P}_{a_1a_2\to M}(\eta)\ \delta(x_1-\eta x)\ \delta(x_2-[1-\eta]x) \quad . \tag{2.4}$$

When this is substituted into Eq. (2.3) and the delta functions removed, we find

$$D_{M,i}(x,Y) = \sum_j \int_{Y_o}^{Y} dy \int_x^1 \frac{dt}{t}\ D_{ji}\ (\frac{x}{t},Y-y)\ f_j(t,y) \quad , \tag{2.5}$$

where

$$f_j(t,y) = t\int_0^1 d\eta\ \tilde{P}_{a_1a_2\to M}(\eta) \quad \int_{\eta t}^{1-(1-\eta)t} \frac{dz}{z(1-z)}\ D_{a_1b_1}(\frac{\eta t}{z},y-Y_o) \quad X$$

$$D_{a_2b_2}(\frac{(1-\eta)t}{(1-z)},y-Y_o)\ P_{j\to b_1b_2}\ (z) \quad . \tag{2.6}$$

We find it most convenient to work with moments of these functions rather than with the functions themselves, since for the simple R under study this removes all integrations over the x variables. If we define the moments as

$$D^n_{M,i}\ (Y) = \int_0^1 x^n\ D_{M,i}(x,Y)\ dx \quad , \tag{2.7}$$

then (2.5) becomes

$$D^n_{M,i}(Y) = \sum_j \int_{Y_o}^{Y} dy\ D^n_{ji}(Y-y)\ f^n_j(y) \quad , \tag{2.8}$$

with

$$f_j^n(y) = \int_0^1 \int_0^1 d\xi_1 \, d\xi_2 \, \theta(1-\xi_1-\xi_2) \, (\xi_1+\xi_2)^n \, \times$$

$$\tilde{P}_{a_1a_2 M} \left(\frac{\xi_1}{\xi_1+\xi_2}\right) \psi_{a_1a_2j} \, (\xi_1,\xi_2,y) \quad , \tag{2.9a}$$

and

$$\psi_{a_1a_2j} \, (\xi_1,\xi_2,y) = \sum_{b_1b_2} \int_{\xi_1}^{1-\xi_2} \frac{dz}{z(1-z)} \, D_{a_1b_1} \left(\frac{\xi_1}{z}, y-Y_o\right) D_{a_2b_2} \left(\frac{\xi_2}{1-z}, y-Y_o\right)$$

$$\times \quad P_{j\to b_1b_2}(z) \quad . \tag{2.9b}$$

The special case proposed by Das and Hwa and used by most other workers in this field has

$$\tilde{P}_{a_1a_2\to M} \left(\frac{\xi_1}{\xi_1+\xi_2}\right) = \frac{\xi_1\xi_2}{(\xi_1+\xi_2)^2} \, C_{a_1a_2\to M} \quad . \tag{2.10}$$

This has the virtue, for our purposes, that the moments f^n reduce to double moments of ψ,

$$f_j^n \, (y) = \sum_{\substack{m \\ a_1a_2}} \binom{n-2}{m} \int_0^1 d\xi_1 \int_0^1 d\xi_2 \, \theta(1-\xi_1-\xi_2)$$

$$\times \quad \xi_1^{n-1-m} \, \xi_2^{m+1} \, \psi_{a_1a_2j}(\xi_1,\xi_2,y) \, C_{a_1a_2\to M} \quad , \tag{2.11}$$

which can be calculated explicitly,

$$f_j^n(y) = \sum_{\substack{m \\ a_1 a_2}} \binom{n-2}{m} C_{a_1 a_2 \to M} D_{a_1 b_1}^{n-1-m}(y-Y_o) D_{a_2 b_2}^{m+1}(y-Y_o) P_{j \to b_1 b_2}^{n-1-m,m+1} \quad . \tag{2.12}$$

Thus the moments of the fragmentation functions can be calculated from Eq. (2.8) using only the known moments of basic D_{ij} parton "propagators" and the known vertices, P, plus knowledge of which quarks combine to make which mesons.

The coefficients C are typically determined by recombination modelists as follows: Only the valence quarks in the produced meson are considered as it is believed that the sea develops later from gluons spawned by these valence quarks. Then, the constraint

$$\int_0^1 d\xi_1 \int_0^1 d\xi_2 \, R(\xi_1,\xi_2) = 1 \quad , \tag{2.13}$$

is either imposed or approximated. This forces all such pairs to make pseudoscalars rather than the other possible mesons which could be formed, e.g., vector or tensor mesons. Hence, the size is fixed. Typically, therefore,

$$C_{u\bar{d} \to \pi^+} = C_{\bar{d}u \to \pi^+} \leq 6 \quad . \tag{2.14}$$

Most applications choose 6 for this, to satisfy Eq. (2.13), although Van Hove[7] chooses 4.0.

III. RESULTS FOR QUARK FRAGMENTATION

Before comparing the fragmentation functions calculated from Eq. (2.8) with those extracted from the e^+e^- jet data, we must face the fact that (2.8) produces moments with different Q^2 dependence than that normally expected of fragmentation functions. Typically, one would compute the Q^2 behavior of the $D_{M,i}$ functions using one of the "quark propagators" $D^n_{ij}(y)$. Our formula is more complicated than this and can *a priori* produce quite different behavior.

However, both the jet calculus and the ordinary evolution equations for the fragmentation functions are expected to be valid for Q^2 very large. Furthermore, if Q^2 is large, and if both m and n in Eq. (2.12) are large, then the Q^2 evolution of the (p + n) th moment of Eq. (2.8) is the same as that derived from the transposed Altarelli-Parisi equations given by Owens[13] for the evolution of the fragmentation $D_{m,i}$'s. We can see this by going to a basis in which the D_{ij} functions are diagonal and of the form $e^{y\lambda_m}$, with λ_m an eigenvalue of the Altarelli-Parisi set. In Eq. (2.8),

$$D^{n+p}(Y) \sim \int_{Y_o}^{Y} dy\, e^{(Y-y)\,(\lambda_{n+p})}\, e^{(y-Y_o)\lambda_p}\, e^{(y-Y_o)\lambda_n}$$

$$\sim e^{(\lambda_p+\lambda_n)\,(Y-Y_o)} - e^{\lambda_{n+p}(Y-Y_o)} \quad . \tag{3.1}$$

Examination of the leading eigenvalues of the Altarelli-Parisi equations shows that if both n and p are large, then $|\lambda_p+\lambda_n| > |\lambda_{n+p}|$

so the Y dependence is given by $\sim e^{Y\lambda_{n+p}}$, exactly as in Owens' equations. (The eigenvalues are negative. Also, Y_o and Y are the values of y at opposite ends of the diagram, Fig. 1.)

We therefore will concentrate our comparison at large Q^2 and large moment, with the idea that the fragmentation functions must match here if the recombination model is to satisfy the constraint. The parameters Λ^2 and α_o in Eq. (2.2) must be specified; $\Lambda^2 = 0.25$, a relatively standard value, and α_o is taken as 10.0. It is desirable that our basic results be insensitive to α_o for large α_o; the dependence on α_o roughly indicated by $Y-Y_o$ in Eq. (3.1) assures this. Numerical calculation confirms this insensitivity as very little (few percent) change in the double moments of Eqs. (2.8) - (2.11) occurred when α_o was varied up to 10^6.

The only quantity which might be termed a free parameter is Q_o^2, which determines the lower limit of the y integration in Eqs. (2.3) - (2.8). It seems reasonable to assume that Q_o^2 might be small, on the basis of transverse momentum for produced pions relative to the jet axis. In fig. 2a and 2b we show two large moments, n = 20 and n = 27, of the fragmentation function $D_{\pi^+,u}$, as a function of Q^2. The dashed curve labeled OFF is calculated from the QCD evolution equation of Owens with the Field-Feynman empirical fit for $D^n_{\pi^+,u}$ as the initial "value". The solid curves are calculated from Eqs. (2.8), (2.9a), and (2.9b) with the somewhat special, popular choice for R, Eqs. (2.4) and (2.10). The Q_o^2 values, as indicated on the curves, are 0.3, 0.5, and 1.5; clearly, the smaller Q_o^2 values reproduce the shape of the O-FF fragmentation function best in the range Q^2 plotted.

In the calculation of these solid curves, the coefficient $C_{u\bar{d}\to\pi^+}$ in Eq. (2.12) is set equal to unity. Comparing the solid and dashed curves, therefore, we note that the actual value of C is quite sensitive to the value of Q_o^2 chosen.

It is of interest to examine these same fragmentation functions for fixed Q^2 as a function of n or moment. In Fig. 3 we show values of $D^n_{M,i}$ at fixed $Q^2 = 10^6$ GeV^2 as a function of n for the fragmentation of the u-quark into π^+ and the s-quark into K^-. For this comparison we have made the non-zero $C_{a_1,a_2\to M}$ coefficients in Eq. (2.12) equal to unity. Calculations of curves as in Figs. 2 and 3 were performed for several other Q_o^2 values in addition to 0.3, 0.5, and 1.5 GeV^2. On the basis of Fig. 2, comparison of curve shapes with O-FF suggests $Q_o^2 = 0.3 - 0.5$ is preferred. In Fig. 3a, it appears that the curves labeled $Q_o^2 = 0.3$ and $Q_o^2 = 1.5$ are least like O-FF. From this moment dependence we clearly prefer $Q_o^2 \cong 0.5$. The two cases $u\to\pi^+$ and $s\to K^-$ in Figs. 3a and 3b from our jet-calculus recombination calculations are identical because of the assumed flavor independence of the propagators and vertices. The Owens-Feynman-Field fragmentation functions become very similar also at this large Q^2. (We use the "effective" fragmentation functions of Ref. 12, which include the consequences of vector meson creation and decay.) Notice that for large moments the two curves are nearly parallel; they differ by a factor of about 5.5 in the case of $u\to\pi^+$ and a factor of 4-5 in the case of $s\to K^-$. These obey the constraint $C \leq 6$ following from Eq. (2.13) and used by Das and Hwa; these factors are slightly higher than the value $C = 4$ adopted by Van Hove.[7]

Thus we see that in spite of the apparently ad hoc methods used by Das and Hwa to guess their recombination function, it essentially passes our test of being consistent with the measured fragmentation functions given the following caveats:

i) The two cases shown in Fig. 3 involve recombination of the main quark in the jet with a 'created' quark; this is the case which recombination modelists would term "valence-sea" recombination. This is the major case addressed in their works and there is some uneasiness on their part about extending the model to other cases. We will discuss "sea-sea" recombination briefly in the next section.

ii) We have demonstrated agreement in the region where the two approaches should have similar Q^2 dependence, namely very large Q^2 and large moment. The formula, Eq. (2.8), with the naive form for R in fact generates fragmentation functions which have rather different Q^2 dependence from that normally assumed, at low Q^2. This behavior needs to be explored also, and the actual Q^2 dependence of R needs to be deduced.

The true R function must correct the difficulty illustrated in Fig. 4. Moments of our calculated u quark fragmentation function into π^+ are shown versus n for two different Q^2; these are the solid lines labelled by $Q^2 = 100$ and 10^5 GeV^2. For comparison, the dashed lines are the moments at the same values of Q^2 given by the Owens[13] evolution equation with the Field-Feynman[12] initialization. The dependence on Q^2 at the largest n shown is similar but not identical. The dashed curve falls by a factor of four between $Q^2 = 10^2$ GeV^2 and $Q^2 = 10^5$ GeV^2, whereas the solid curve falls only a factor of three. However at small n, there is substantial disagreement; the solid lines rise in this same Q^2 interval while the dashed lines fall. Two possible modifications of R which might solve this problem are: (1) R may actually have an ntrinsic Q^2 dependence, or (2) R may have a different x_1 and x_2 dependence from that assumed and explored here. (This will produce a different combination of the y dependent moments than that given in Eq. (2.11)).

Examination of the moments in Figs. 2 and 3 suggests that the reconstruction of the fragmentation functions from the moments will not lead to the same shape in x as the Owens-Feynman-Field curves. For example, the dashed and solid curves in Fig. 3 diverge most for small n, so the inverted curves should tend to diverge most at small x. The curves for $xD_{\pi^+,u}(x)$ obtained by use of Yndurain's method[15] for inverting the moments are shown in Fig. 5. For the reader's benefit, a complete set of curves for (a) $Q^2 = 10^6$ GeV^2, (b) $Q^2 = 10^5$ GeV^2, ..., (f) $Q^2 = 10^1$ GeV^2 is shown. The dashed lines, consistent with the code of the other figures, resulted from inverting the moments calculated with Owens'[13] evolution equation using Field-Feynman[12] for initialization. The solid curves are the results of inverting our recombination theorv moments from Eqs. (2.8) - (2.12).

These curves are labeled by the values used for $C_{\bar{u}d \to \pi^+}$, either 4 or 6 being the popular values[1,7] in the recent literature. At the highest Q^2 of 10^6 or 10^5 and somewhat lower the solid curves calculated for $C_{\bar{\pi}d \to \pi^+} = 4$ are in surprisingly "reasonable" agreement with the dashed (Owens-Feynman-Field). In fact, the curves for $C_{\bar{u}d \to \pi^+} = 6$ are "good", considering the stringent nature of our constraint equation. The entire recombination model approach was constructed for x "reasonably" large; the best agreement between solid and dashed curves is at $x \gtrsim 0.5$. In the mid - Q^2 range, 10^2 - 10^4 GeV^2, agreement exists between the dashed curve and solid curve labeled "$C_{\bar{u}d \to \pi^+} = 6$". When the smallest $Q^2 = 10$ GeV^2 is reached, Fig. 5f, the wrong Q^2 dependence produced by the popular R function becomes clearly evident; $C_{\bar{u}d \to \pi^+}$ must be 9 to have the dashed and solid curves match at $x \gtrsim 0.8$.

V. DISCUSSION AND CONCLUSIONS

We have derived an equation relating the quark fragmentation function D to the recombination function R used in the recombination model. The basis for this derivation is the jet calculus, which allows us to find the probability distribution in momentum fraction variables for two partons in a parton jet. If these two partons are quark and antiquark, the recombination function folds them into a meson. The result is an expression for D, the probability that an initial parton i will yield a meson with momentum fraction x of the momentum of i.

Our relation might be viewed as an integral equation which specifies R in terms of $D_{M,i}$. We could not solve this in general; instead we inserted popular Q^2 independent R functions and computed $D_{M,i}$. At large Q^2, for large moments, the computed $D_{M,i}$ function is much like the ones derived from other sources. This tends to support the belief of recombination modelists that their functions describe meson production at large x.

With the choice $Q_0^2 = 0.5$ GeV^2, the values $4 \leq \alpha \leq 6$, chosen by recombination model advocates to normalize R, give numerical agreement with $D_{M,i}$ at large x over a wide range of large Q^2. The Q^2 dependence of $D_{M,i}$ obtained from our equation with a Q^2 independent R is not, however, quite correct. Presumably the correct Q^2 and x_i dependence of R is specified by the "known" Q^2 dependence of $D_{M,i}$ and our relation between them. Applications of the recombination model to date have not attempted to include <u>any</u> Q^2 dependence in R nor to vary its x_1 dependence from the simple form studied here. We believe that investigation of this point is necessary before the recombination model can be regarded as having a solid foundation.

The results presented here have focused on valence-sea recombination in which the sea-quark ($\bar{u}$ or $\bar{d}$) is light, since that is where the recombination model has been most successful. We are currently studying two other situations which present additional complications, (a) valence-sea recombination in which the sea quark is heavy, and (b) sea-sea recombination with either light or heavy quarks. We shall briefly sketch the new physics involved to contrast with the case of light sea-quark recombination.

Field and Feynman[12] find a suppression of valence-sea fragmentation functions for cases such as $u \to K^+$, due presumably to problems of extracting the heavy mass of the $\bar{s}$ quark from the sea. Our calculation of $D_{M,i}$ from R does not at present allow for such suppression, since no quark mass effects are included. The same R would be used for $u \to K^+$ and for $\bar{s} \to K^+$, so the $D_{K^+,i}$ functions calculated from Eq. (2.3) would be the same. Manipulation of Q_o^2 can, of course, yield different values but it is not clear at present whether this is an appropriate solution to the problem.

Sea-sea recombination has presented some difficulties for advocates of the recombination model. Some applications have led to claims of a dramatically enhanced sea,[3] whereas others have found a not-so-enhanced sea.[7] Our calculation of the O-FF curve yields much smaller values for $D_{K^-,u}$ than $D_{\pi^-,u}$. On the other hand our recombined jet calculus result for $D_{K^-,u}$ is over an order of magnitude smaller yet. We suspect that this small value may be related to the necessity of sea enhancement in some recombination calculations.

Proper calculation of $D_{K^-,u}$ should take account of the mass effects discussed above. In addition, because the number of K's produced is small, contributions from other mechanisms which would normally be neglected must be included. One of these is recombination from gluons. Another, more important, effect is the recombination production of heavy mesons which decay to produce K's. For instance, there is a contribution to $u \rightarrow K^-$ from $u \rightarrow (s\bar{s}$ mesons, e.g., $\phi) \rightarrow K^+K^-$.

Recombination into heavy mesons and their subsequent decay is also expected to produce additional numbers of pseudoscalars, mainly at small x. This will certainly serve to increase agreement between the dashed and solid curves in Figs. 3 and 5. However, explicit calculation showed this contribution to have small effects in the region of agreement apparent between these curves in Fig. 5.

In conclusion, we have deduced an integral equation for the recombination function R in terms of the quark fragmentation function $D_{M,i}$. This equation suggests that the recombination function may have Q^2 dependence, contrary to popular assumptions in the literature. However, this intuitive, popular R function is shown by explicit calculation to have a wide range of reliability at large Q^2 for large moments of $D_{M,i}$.

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FIGURE LEGENDS

Fig. 1. Schematic representation in the jet calculus for the fragmentation function of parton i into meson M as given by Eq. (2.3) of the text. The circular blobs on the parton lines i, b_1, b_2 stand for the complete QCD evolution of a parton cloud out of which partons j, a_1, a_2, respectively, are selected. The partons a_1 and a_2 are recombined to make the di-parton state M through use of R.

Fig. 2. Moments (a) n = 20 and (b) n = 27 of the fragmentation function for a u quark into π^+ mesons. The dashed curves are calculated from Owens' evolution equations with the Field-Feynman fit to e^+e^- annihilation and hadron lepto-production data as the initializing condition. The solid curves show our results for the moments of the fragmentation function for u into π^+ calculated from Eqs. (2.8) - (2.12) of the text for Q_0^2 = 0.3, 0.5, and 1.5 GeV^2.

Fig. 3. Moments of fragmentation functions for (a) $u \to \pi^+$ and (b) $s \to K^-$ versus n, the moment number, for our calculation (solid line) compared with that (dashed line) of Owens-Feynman and Field. The dashed lines are calculated from Owens' evolution e⁻uations with the Field-Feynman fit of Ref. 12 for the initial (low Q^2) condition. In (a) we show our calculation for Q_o^2 = 0.3, 0.5, and 1.5 to illustrate

that the smaller Q_o^2 value produces too fast a fall off of D^n with moment number. For completeness, we note that the dashed curve rises at n = 2 to 0.037 for $u \rightarrow \pi^+$ and 0.022 for $s \rightarrow K^-$.

Fig. 4. Curves showing the moments of the (u-quark into π^+) fragmentation function versus n for $Q^2 = 10^2$ and 10^5 GeV^2. The dashed curves are calculated with the Owens evolution equation for D given the Field-Feynman initialization. The solid curves are given by Eqs. (2.8) - (2.12) with the recombination function of Das and Hwa, Ref. 1, C = 1, and $Q_o^2 = 0.5$ GeV^2.

Fig. 5. Fragmentation functions times momentum fraction x vs x for $u \rightarrow \pi^+$. The dashed curves labelled O-FF are calculated for the various Q^2 = (a)10^6, (b)10^5, (c)10^4, (d)10^3, (e)10^2, and (f)10 GeV^2 from Owens' evolution equations with Field-Feynman initialization. The solid curves are calculated from Eqs. (2.8) - (2.12) for our jet calculus recombination model with normalization of the recombination function fixed by the values $C_{u\bar{d} \rightarrow \pi^+} = 4$ or 6 as given in the literature. For a discussion of (f), see text.

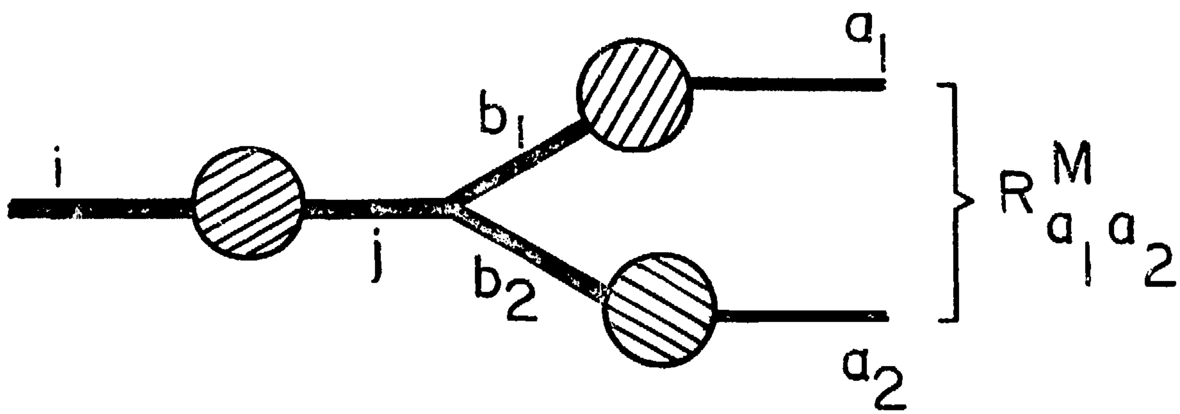


Fig. 1. Schematic representation in the jet calculus for the fragmentation function of parton i into meson M as given by Eq. (2.3) of the text.

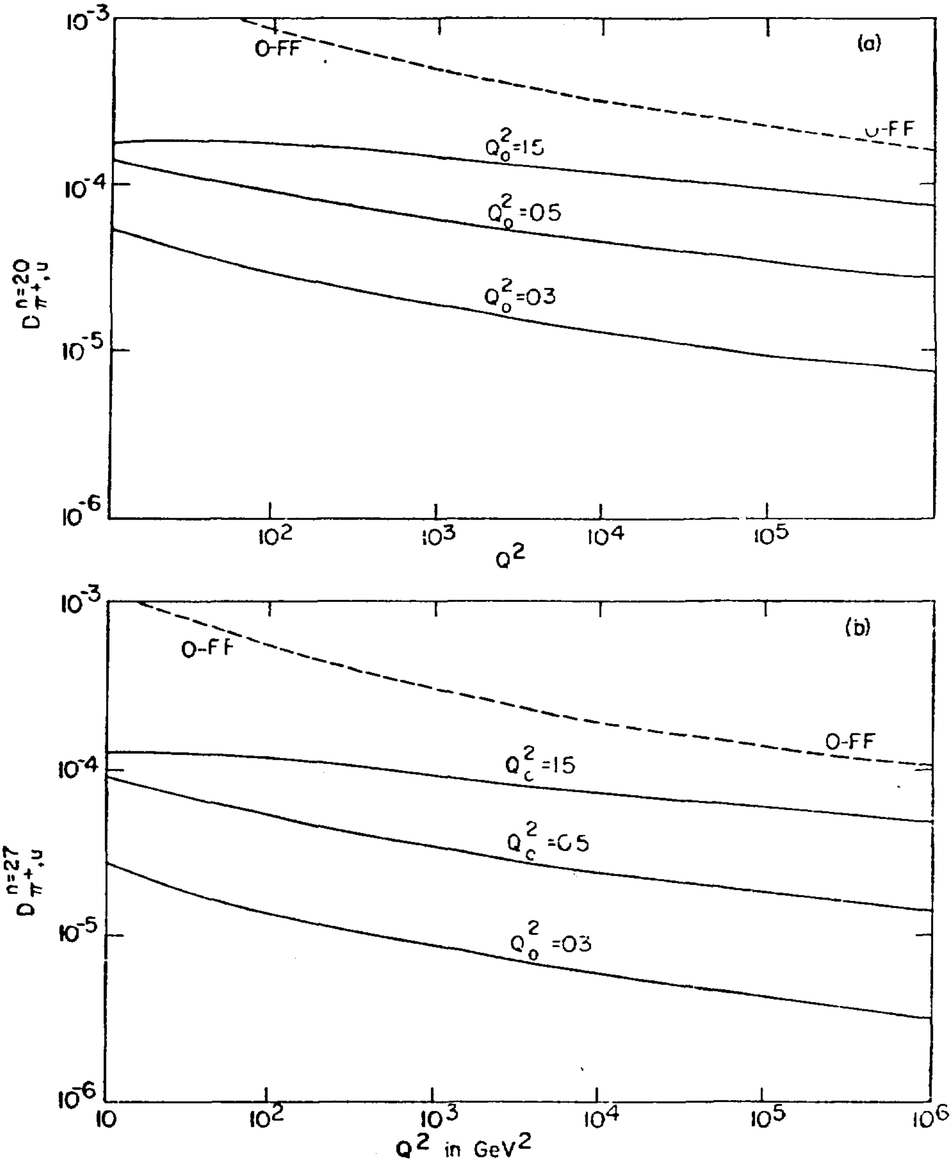


Fig. 2. Moments (a) n = 20 and (b) n = 27 of the fragmentation function for a u quark into π^+ mesons.

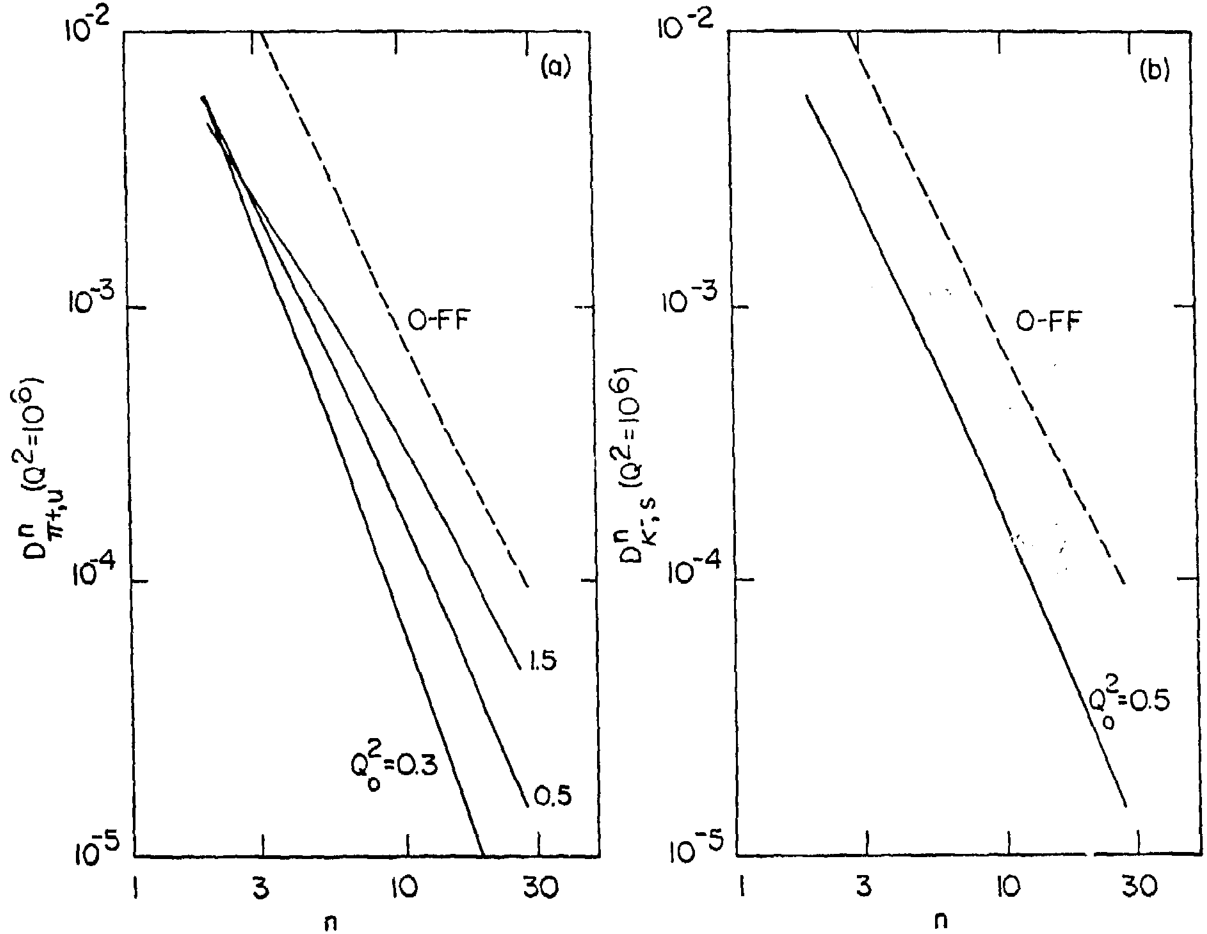
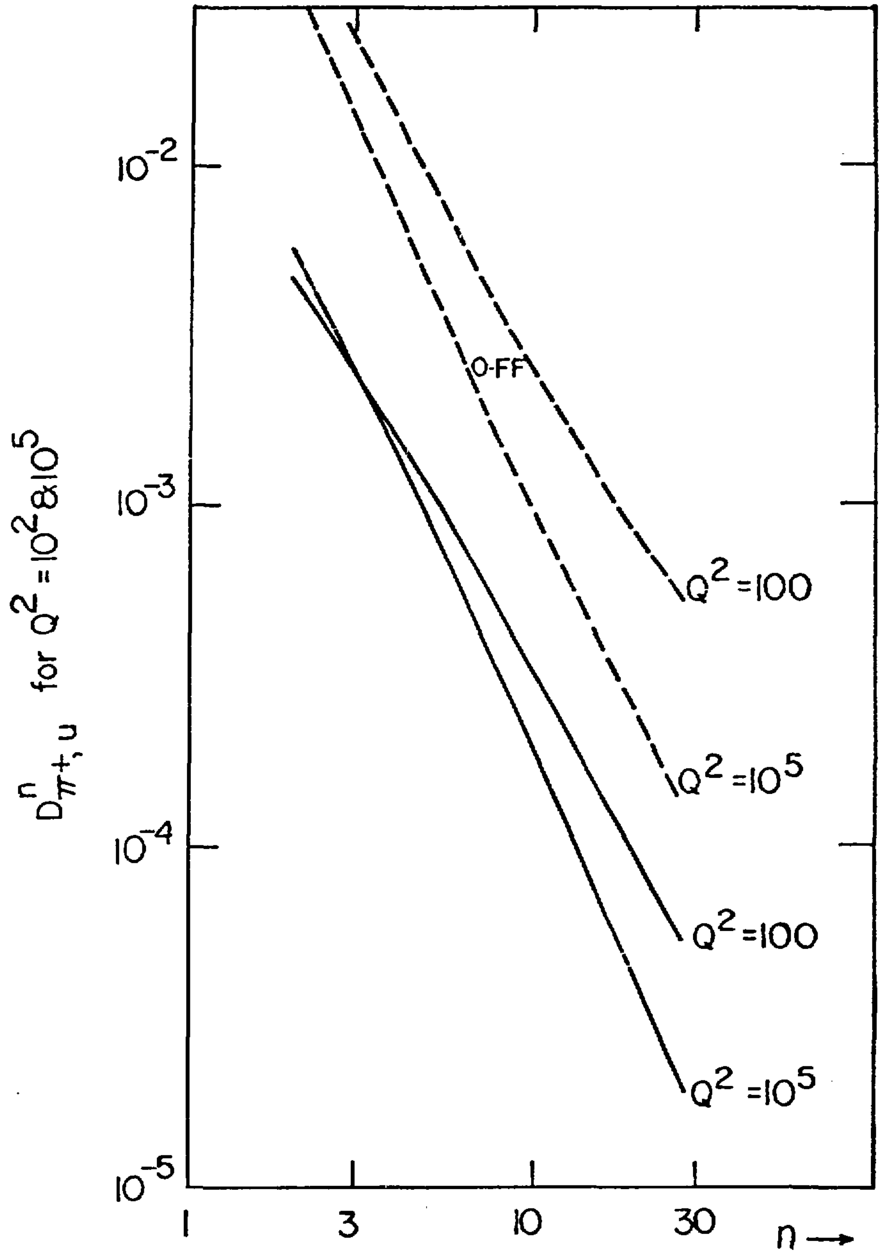


Fig. 3. Moments of fragmentation functions for (a) $u \to \pi^+$ and (b) $s \to K^-$ versus n, the moment number, for our calculation (solid line) compared with that (dashed line) of Owens-Feynman and Field.



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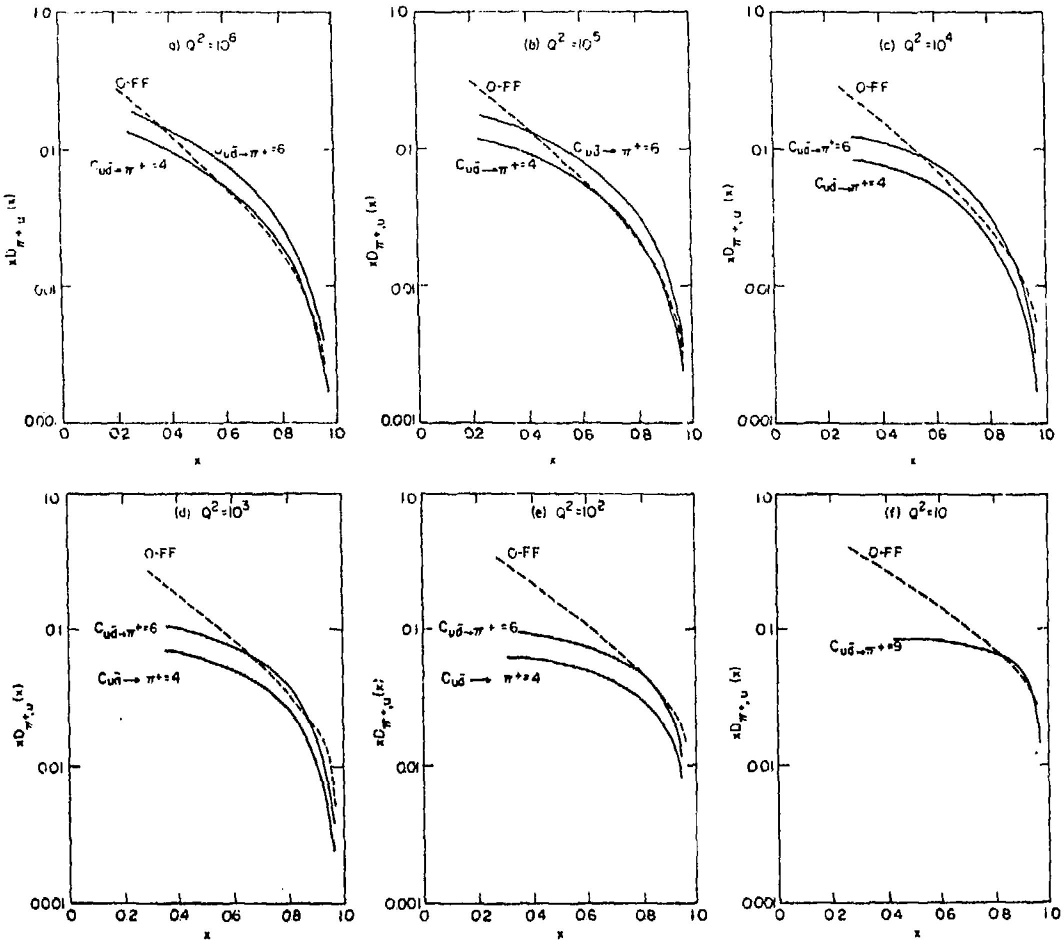

Fig. 5. Fragmentation functions times momentum fraction x vs x for $u\to\pi^+$.