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PROTON AND NEUTRON DENSITIES FROM ELASTIC ELECTRON SCATTERING

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I. Introduction

The best determination of nuclear sizes and shapes comes from elastic electron scattering experiments. The pioneering work done at Stanford with high energy electrons showed that the nucleus does not have sharp boundaries, but does have some structure in its interior region. The work done at Stanford demonstrated that high energy electrons provide a probe of unmatched accuracy to study nuclear structure. In contrast to any other probe, electron scattering measures unambiguously the charge and current densities of nuclei. By varying the momentum transfer q, one gets a Fourier (or Hankel) transform of the spatial distribution of nucleons. If one had measurements from $q = 0$ up to $q = \infty$, he would only have to invert the transform to determine charge and magnetization densities of protons and neutrons. But the limitations due to accelerators and experimental techniques have for a long time prevented the momentum transfer to be sufficiently high to explore the fine details of nuclear distributions. This was a major problem, since the progress achieved by self-consistent field theories is so impressive that modern calculations differ essentially by fluctuations in the central region of the nucleus by about 10 %. It was precisely in this central region where the uncertainties due to the finite q_{max} were the largest. The recent developments of new accelerators and new experimental techniques have provided higher beam intensities, better energy resolution ($\Delta E/E \sim 10^{-4}$), higher momentum transfer ($q \sim 3.5$ fm^{-1}) and made it possible to determine experimental data with significantly increased accuracy, especially in the domain of systematic errors. Careful investigations to cross check the model dependence of the results have proved that for ^{58}Ni and ^{208}Pb the combination of very precise muonic X-ray energies and (e,e) data up to 3.5 fm^{-1} do yield a reliable estimate of the charge density to the order of 1 %.

The information on single particle densities is much more difficult to obtain by elastic electron scattering. Measurement of the charge difference between neighbouring nuclei provides very valuable information, but it is difficult to isolate the effect of core polarisation in an unambiguous way. For selected cases it has been shown that magnetic scattering can measure with extreme accuracy the shape of the valence orbits for both protons and neutrons.

Review papers and recent references on this subject can be found in the proceedings of the Amsterdam conference held in September 1978 [1]. I will focus only on very recent experiments (on medium and heavy nuclei) of some significance for our understanding of the structure of the nucleus. The very interesting work done at 180° has already been reviewed in this conference by L. Lapikas and results on the few body problem will be discussed later by R. Arnold and Y.E. Kim.

II. ^{12}C radius

^{12}C is commonly used as a reference nucleus for determining the normalization of relative measurements. Up to now the experimental situation was not clear. This is reflected in the spread of the measured values of the ^{12}C r.m.s. radius, which are statistically incompatible with quoted errors [2]. This is partly due to disagreement between various experiments but it also reflects different conceptions of fitting the data. The inclution of data points in the diffraction minimum where dispersive effects may be important or the imposition of constraint for the fall-off of the density at the tail region are issues treated differently in the various analysis that have appeared in the literature.

To clarify the situation new measurements have been performed at Mainz and the National Bureau of Standards (NBS). The analysis of the NBS experiment has just been completed and will be published soon [3]. A very careful determination of the various experimental parameters has significantly reduced the systematical errors. Cross-sections have been measured in the momentum transfer from 0.1 to 1.fm^{-1} with an uncertainty of 0.5 % for counting statistics and 0.3 % for systematics. These results have been combined with previous experiments to determine the charge density of ^{12}C. The r.m.s. radius of this density is r = 2.472 fm ± 0.006 fm (stat.) ± 0.010 (syst.) ± 0.011 (abs.norm.) which gives a total uncertainty of ± 0.016 fm. This result is in good agreement with the value found by I. Sick [4], r = 2.468 ± 0.018 fm, from a "model-independent" analysis of IKO and Stanford data. The new data from Mainz seem to agree well [5] although a previous result is not compatible [6]. Measurements of muonic X-rays at SIN by the Fribourg group have determined the 2p-1s transitions to 6 eV. A preliminary analysis gives a radius r = 2.467 ± 0.020 fm (statistical error only) [7].

The determination of the ^{12}C cross-section seems to be definitive in the region of momentum transfer up to 1.fm^{-1}. New measurements at Mainz aim at measuring more accurately the region of the steep slope in the vicinity of the diffraction minimum but this should not affect the ^{12}C radius.

III. The charge density of ^{40}Ca and ^{208}Pb

Because of its importance for nuclear theory, ^{40}Ca has been studied extensively by elastic electron scattering. The experiment by Bellicard et al. [8] at Stanford was the first to reach q_{max} = 3.4 fm^{-1}. It was the first time that an oscillatory modulation was needed in addition to a simple Fermi shape to fit the data at high q. However, even 3.4 fm^{-1} was not enough to determine the density in the center of the nucleus. A combined analysis of electron scattering and muonic X-rays data by Sick [9] has shown that there were two discrete sets of solutions for $\rho(r)$ which fit the experimental data equally well. The experiment had stopped at a momentum transfer too small, with insufficient counting statistics in a diffraction maximum. Thus, the sign of the following oscillation of the form factor was not determined.

Our group at Saclay has performed a new experiment at 500 MeV (Fig. 1). We have measured 18 differential cross-sections between 2.2 and 3.7 fm^{-1}, improving both statistical accuracy and the momentum transfer range. The results together with the data already used in ref.[8], plus the new muonic data of Wohlfahrt et al. [35] have been analyzed by Sick [10]. The form factor does change sign, which means that for all the nuclei measured up to now the form factor is always alternating between positive and negative. It confirms the assumptions of Lightbody and O'Connell [11], for the behaviour of the form factors at large q. The density at the origin, which is the integral of the form factor from q = 0 to q = ∞, has now an uncertainty lower than 1 % ; the dominant contributions to this uncertainty are coming from statistics and normalizations. However, theoretical corrections such as dispersive effects and meson exchange are not known to this accuracy. It is clear that 1 % is quite adequate to allow a useful comparison with any theory. An important point is that a 3 % renormalization of the data has an imperceptible effect on the charge density. This reflects the fact that the radius and the absolute normalization are pinned down by the very precise muonic X-ray data.

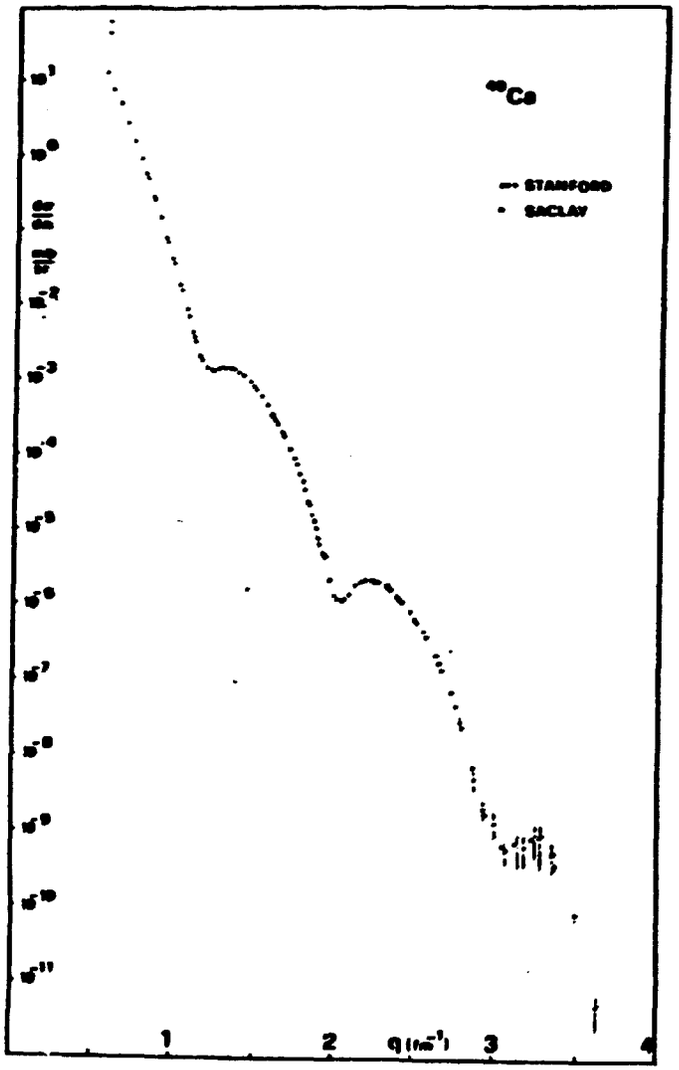

Fig. 1 - Elastic cross-section for electron scattering at 500 MeV from ^{40}Ca as a function of momentum transfer.

The experimental density of ^{40}Ca is shown in Fig. 2, together with its error band and two theoretical predictions. The one labeled DDHF is the result of a Hartree-Fock-Bogolyubov calculation by Decharge and Gogny [12] using the density dependent finite range force D1 described by Gogny at this conference. The overall agreement is good and is much closer to the experiment than in the case of ^{208}Pb [13], but again the theory predicts too much structure in the interior of the nucleus.

The first order correction to this type of mean field calculation is the inclusion of long range correlations calculated in the framework of the random phase approximation (RPA). Blaizot and Gogny [14] have developped the theory for the collective excitations of closed shell nuclei. They have shown that it is possible to

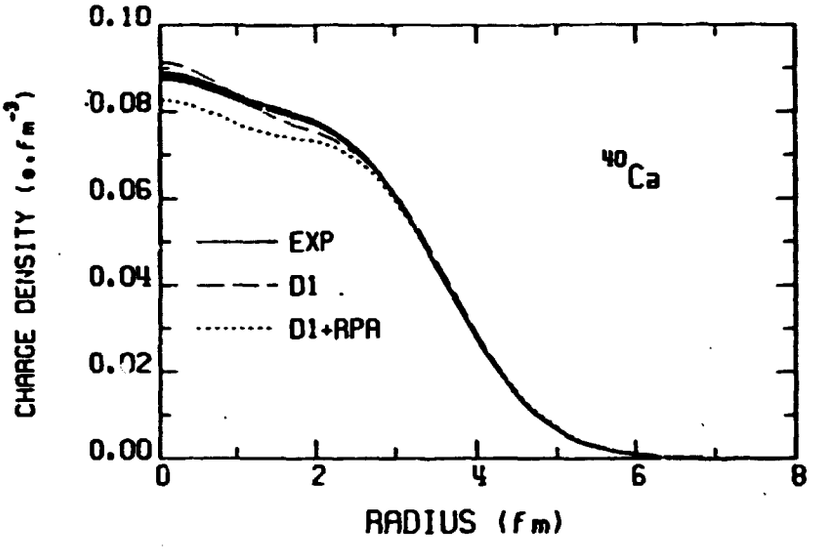

Fig. 2 – ^{40}Ca charge density. D1 is a theoretical mean field prediction from Gogny. D1 + RPA is including also ground state correlations.

perform a self-consistent description of these collective 1p-1h effects in a large basis that reproduce very well the position of the monopole giant resonance and the shape of the transition charge densities of ^{208}Pb. Moreover, they have shown that a different force for the description of the ground state and the collective excitations leads to a destruction of coherence of the calculated physical states. These RPA correlations have been included by Dechargé and Gogny [12] in the ground state charge distribution. The result is also shown on Fig. 2. Now the shape is almost perfect, but the density is too low. This can be corrected by a readjustment of the density dependence of the force that scales the density closer to the experimental value. It should be understood that in *all* DDHF calculations such an adjustment is done, but without RPA correlations. The inclusion of these corrections and a new determination of the density dependence of the effective force will probably give a much better agreement with experiment. But, already the inclusion of the RPA correlations have decreased by a factor 3 the chi-square of the mean field theory.

In the framework of the DDHFB theory, once the effective force has been determined, no parameter is free in the calculation. The pairing field is treated on the same footing as the mean field, and the RPA correlations are derived from the same force. It is then of particular interest to determine for the best cases throughout the periodic table what is the agreement between theory and experiment. One of the best cases is ^{208}Pb because it is the heaviest doubly magic nucleus. We have mentioned previously [13] that the best theories predict too much structure in the interior of the nucleus. Fig. 3 shows that the RPA correlations improve the agreement with the experiment very significantly. The chi-square of the mean field theory is decreased by a factor 2 when the RPA correlations are included. But it is not sufficient, and there is still something missing. However, for both ^{40}Ca and ^{208}Pb, there is a large improvement with the inclusion of the long range correlations.

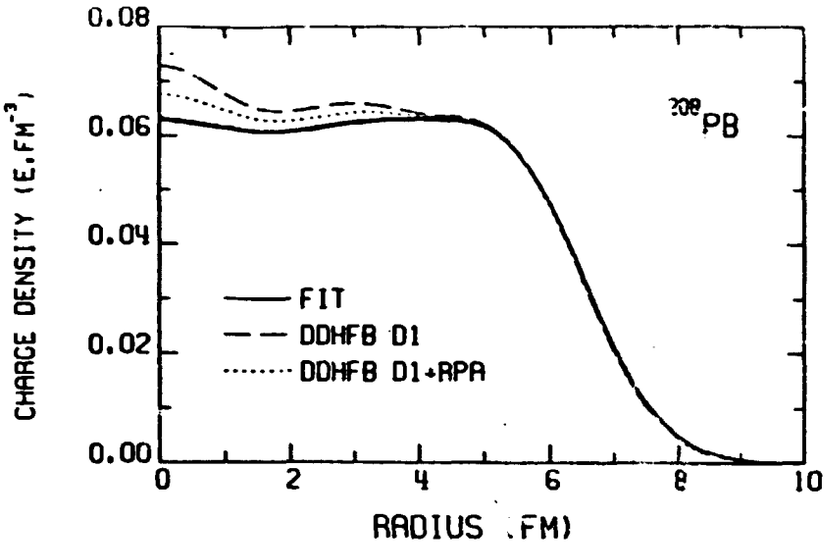

Fig. 3 - Same as Fig. 2 for ^{208}Pb.

IV. The charge differences of neighbouring nuclei

IV.1. Oxygen isotopes

A systematic study of oxygen isotopes ^{16,17,18}O by elastic and inelastic electron scattering is being performed at MIT. I will just discuss here the ground state charge distribution of these nuclei [15].

The charge density of ^{16}O is well known. In order to investigate the charge differences of oxygen isotopes, a very elegant solution has been found to measure accurately the ratios of cross-sections of ^{17}O and ^{18}O to ^{16}O. The experiment has used one of the advantages of the high energy resolution available at the Bates linear accelerator. The elastic cross-sections for the three isotopes have been measured simultaneously with isotopically mixed beryllium oxide targets. The elastic peaks were observed individually, separated by their difference in recoil energy (Fig. 4). The results are independent of charge collection and exact knowledge of target thickness. The magnetic contribution for ^{17}O has been measured at 160° and 180° [16], and has been subtracted. The C2 and C4 scattering from ^{17}O have been calculated using a deformed harmonic oscillator model, and subtracted.

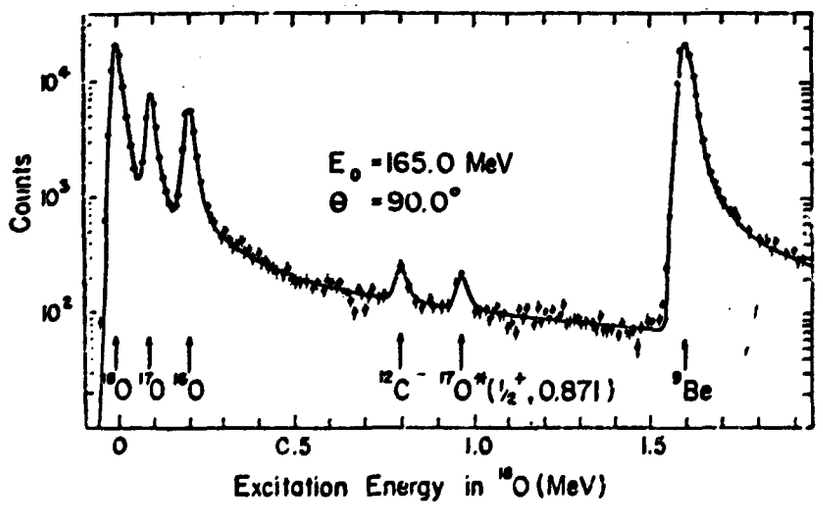

Fig. 4 - Typical spectrum for elastic scattering a from oxygen isotopes mixed in a BeO target.

The ratios of the experimental cross-sections is shown in Fig. 5. Best fits are also represented . In agreement with previous results for

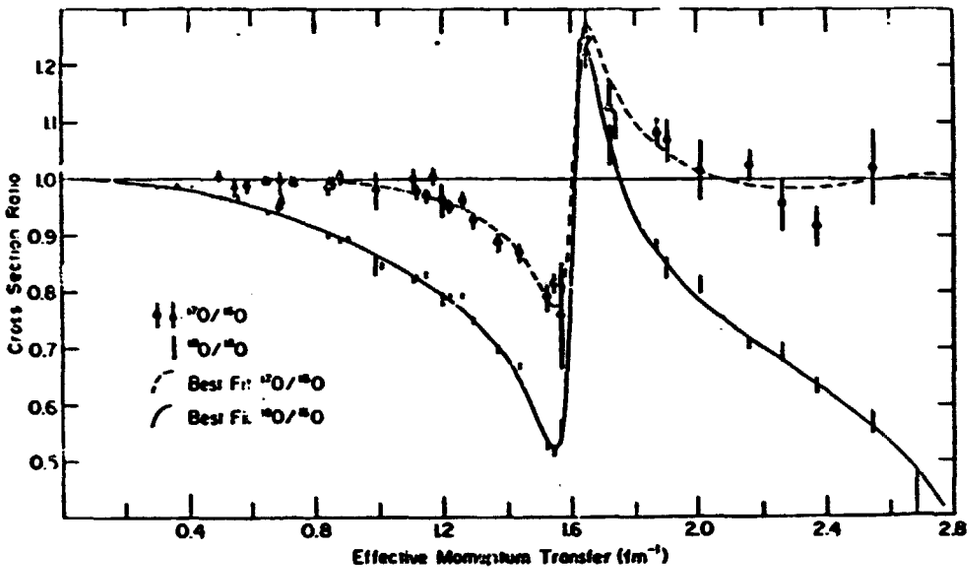

Fig. 5 - Cross-section ratios for oxygen isotopes.

other nuclei an odd-even staggering effect is observed : the ^{17}O has almost the same radius $\Delta r(^{17}O - ^{16}O) = -0.008 \pm 0.007$ fm, while the addition of two neutrons to ^{16}O increases the radius by 0.074 ± 0.005 fm. Fig. 6 shows the charge difference of oxygen isotopes together with theoretical predictions. Curve 1 is a semi self-consistent calculation using closed 1s and 1p shell configurations, while curve 2 takes into account the occupation probabilities for the Rechal-Wildenthal wave functions allowing for up to four holes in the $p_{1/2}$ orbit, within a space of $1p_{1/2}$, $2s_{1/2}$ and $1d_{5/2}$. Both calculations have been made by B.A. Brown et al. [17]. Curve 3 is a DDHFB calculation [12]. The charge difference between ^{17}O and ^{16}O is correctly reproduced by DDHFB, but not the charge difference of ^{18}O and ^{16}O. Curve 2 is in better agreement in this case, which shows that it is desirable to include nonclosed shell configurations for ^{18}O. This is also shown in Fig. 7 by the curves representing the deformation energy of ^{16}O and ^{18}O as a function of the quadrupole moment. The calculations have been done by Girod et al. [18] in the DDHFB framework. ^{16}O has a sharp and deep minimum

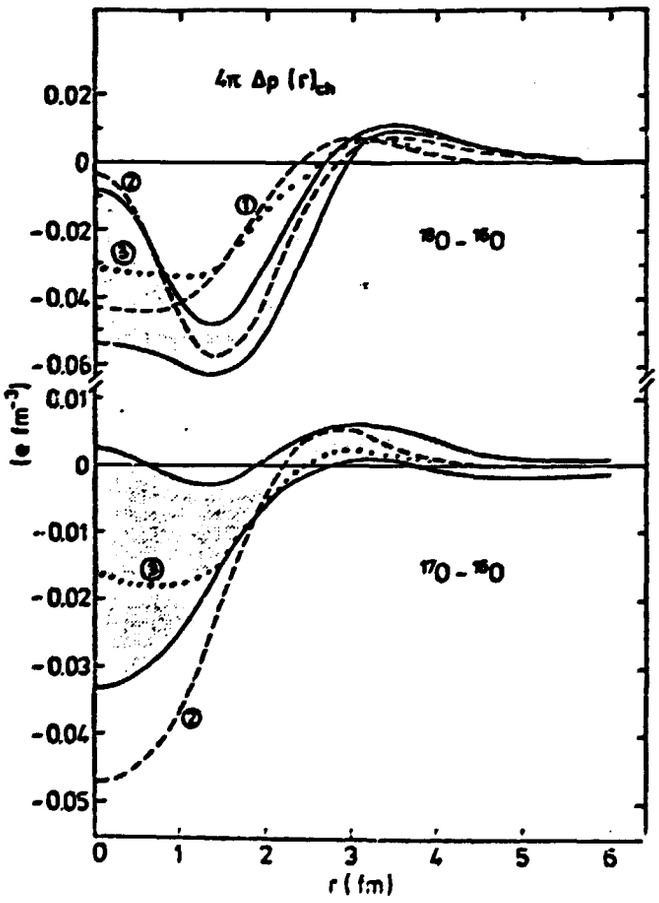

Fig. 6 - Oxygen isotopes charge differences. The shaded area is the experimental uncertainty. 1 and 2 are theoretical predictions of A. Brown, 3 is a self consistent of Dechargé and Gogny.

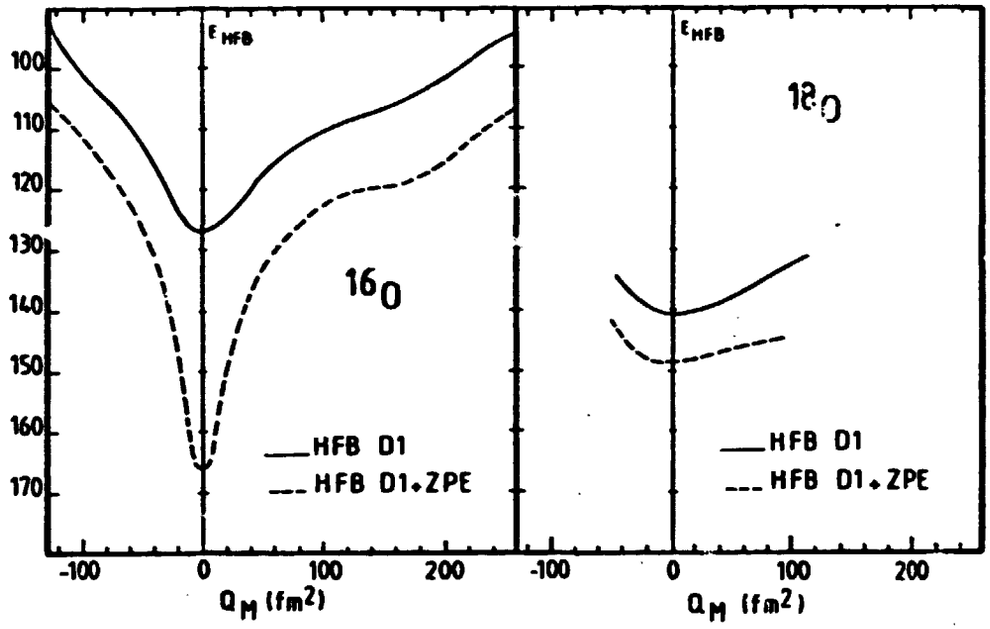

Fig. 7 - Deformation energy curves for ^{16}O and ^{18}O.

at the equilibrium configuration, characteristic of a good closed shell nucleus, spherical and very rigid, especially when the zero point energy correction is made. (This does not agree with the calcualtion of Negele [19] who finds an almost flat curve). ^{18}O is found to be very soft against deformation, and cannot be described correctly by Hartree-Fock theory. It is a superposition of deformed admixtures, which must treated in a better theory such as the generator coordinate method.

4.2. The charge difference of ^{40}Ca and ^{48}Ca

Using the new result for th ^{40}Ca charge density one has now a better determination of the ^{40}Ca and ^{48}Ca charge difference. Fig. 8 shows the error band determined by Sick [9] with the sum of gaussian model. The theoretical predictions are the same as the one discussed in section 3. It is clear that for the charge difference between two nuclei with different closed shells, the self consistent field theory is not able to reproduce the experimental result even with the inclusion of RPA correlations.

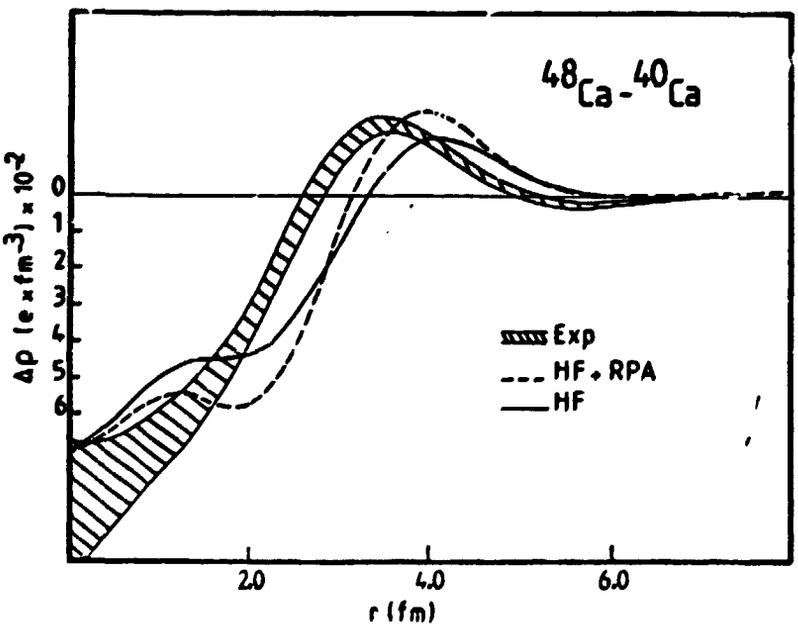

Fig. 8 - ^{40}Ca - ^{48}Ca charge difference

4.3. The charge difference of ^{116}Sn and ^{124}Sn

We have performed at Saclay an experiment on both isotopes at an incident energy of 500 MeV. Our data have been fitted together with previous data of Ficenec et al. [20] and muonic X-rays [2]. Fig. 9 shows the ratio of cross-sections together with the best fit, using a sum of gaussian to expand the charge density of both isotopes. Fig. 10 shows the charge density difference deduced from this analysis [21] and self consistent field calculations [12,22,24]. The very surprising thing is that all calculations reproduce quite well the increase of 8 neutrons, the one of Negele being almost a best fit to our data. However for each nucleus the theory predicts too much structure (Fig. 11). The best agreement is found for the calculation of Dechargé and Gogny [12] where the pairing field is self-consistent.

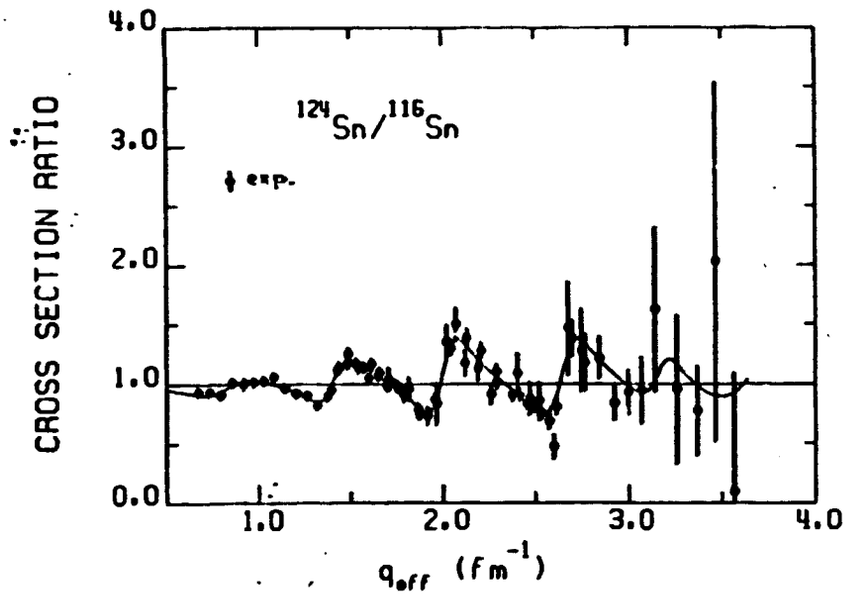

Fig. 9 - Ratio of $^{124}Sn/^{116}Sn$ (e,e) cross-section as a function of momentum transfer.

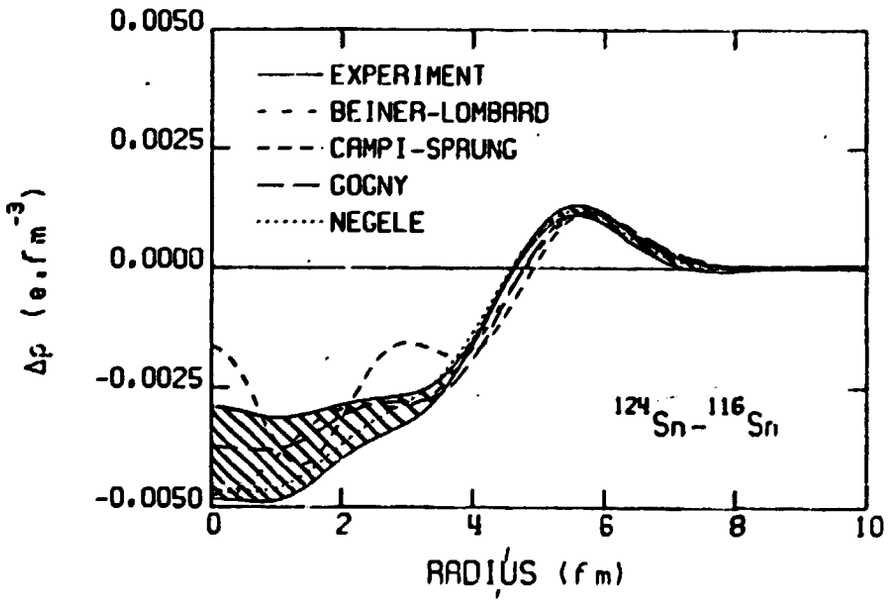

Fig. 10 - ^{124}Sn - ^{116}Sn charge differences together with self consistent field predictions.

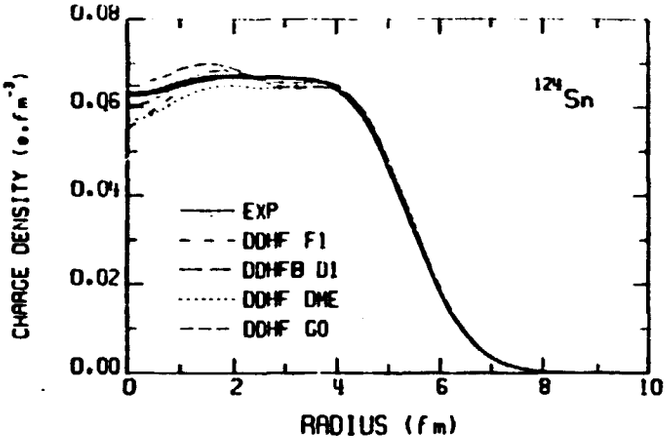

Fig. 11 - ^{124}Sn charge density together with mean field predictions (F1 = Beiner-Lombard, D1 = Gogny, DME = Negele, GO = Campi-Sprung).

In the case of ^{204}Pb and ^{203}Tl charge difference, Euteneuer et al. [25] have found a structure compatible with the shape of a $3s_{1/2}$ proton wave function predicted by mean field theory. However, the experiment has not reached a maximum momentum transfer sufficiently large to determine accurately the shape of this very structured wave function. In the case of $^{206,207,208}Pb$ the new results from MIT are shown in Fig. 12 [26]. For ^{207}Pb the magnetic contribution has been measured and subtracted. The effect observed in the ^{207}Pb - ^{208}Pb charge difference is due to the $3p_{1/2}$ neutron hole. On can guess that the charge difference seem to be modulated by this $3p_{1/2}$ wave function from the wiggles determined experimentally. However, all the structure information is given by only 3 data points at high momentum transfer (Fig. 13). It would be very desirable to extend this experiment to higher q to get a definitive conclusion about this polarization due to the effective n-p force.

V. Multipole charge distribution of ^{59}Co

This is an example of the complexity arising from a nucleus with spin $\neq 0$. In this case $J = 7/2$. All the individual multipoles add coherently. Recently data were obtained by O. Schwentker [27] at Mainz for a momentum range up to 2.3 fm^{-1}. Magnetic scattering measured at Saclay was subtracted and the electric multipole scattering has been analyzed with

4.4. The region of Pb

Elastic electron scattering cross sections and cross-section ratios have been recently studied at Mainz [25] and MIT [26]. The results have been combined with previous data and muonic X-rays in order to determine the contribution of single nucleons to $\rho(r)$. I will just examine two typical results.

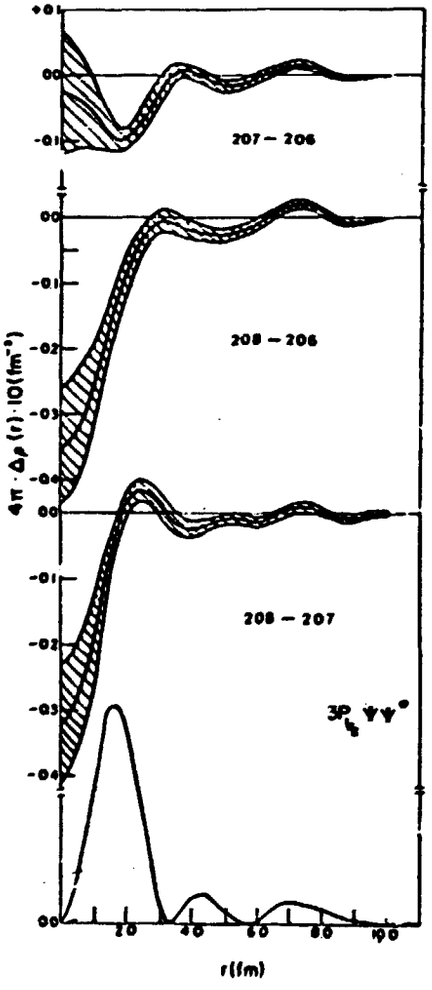

Fig. 12 - 208-207Pb, 208-206Pb, 207-206Pb charge differences together with a $3p_{1/2}$ neutron wave function predicted by a self consistent theory.

the Tassie Model. The results are shown in Fig. 14.

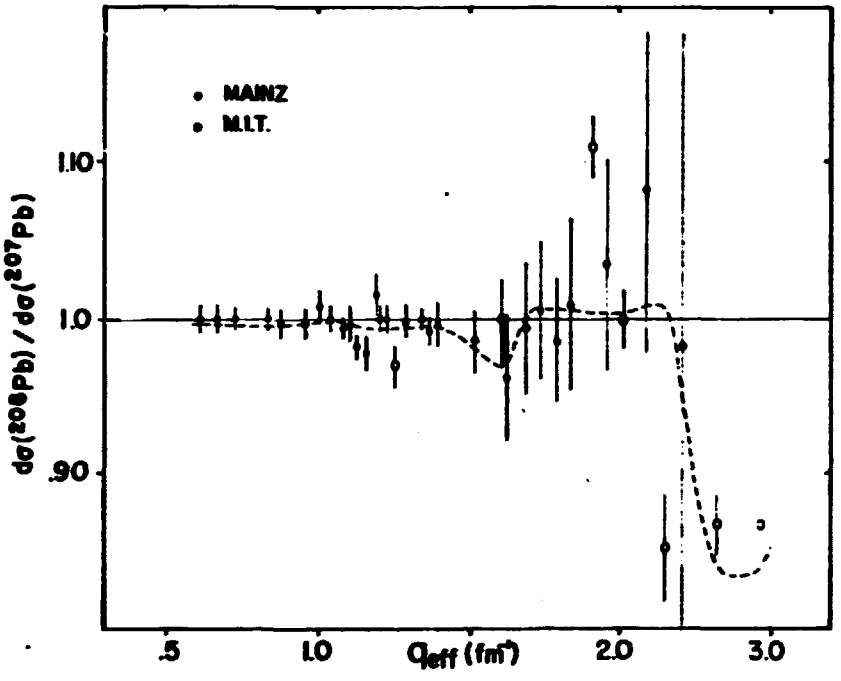

Fig. 13 - Ratio of cross-section for $^{208}Pb/^{207}Pb$.

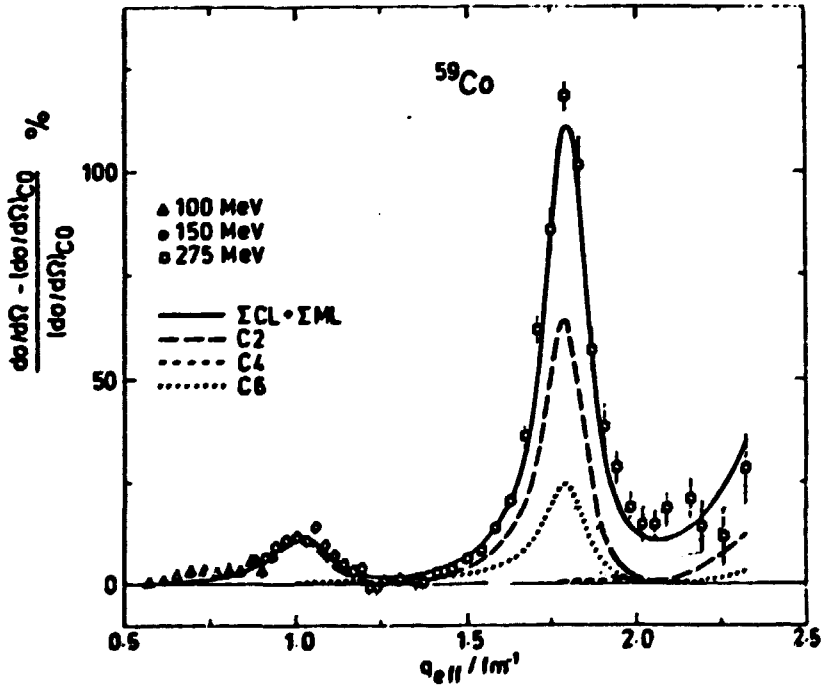

Fig. 14 - Deviation of the total cross-section for ^{59}Co from the monopole scattering.

The C4 component is almost suppressed, the C2 dominates at q = 1 fm^{-1} and at 1.8 fm^{-1} one observes a mixture of C2 and C6. One can extract with some confidence separately the constribution of these different multipoles.

VI. Radial distribution of the $1f_{7/2}$ orbit

It has been shown that the magnetization density which is measured unambiguously by electron scattering can in selected cases be related to single particle orbits for both protons and neutrons. We have previously found at Saclay a significant discrepancy with density dependent Hartree-Fock theory in the $g_{9/2}$ shell [28]. We have recently performed a similar experiment in the $f_{7/2}$ shell. We have measured at 155° the magnetic cross-sections for ^{49}Ti [29] and ^{51}V [30]. These cross-sections are interpreted in terms of the radial distribution of the neutron hole in the closed shell $f_{7/2}$ for ^{49}Ti and of a $(1f_{7/2})$ proton distribution in the case of ^{51}V. The experimental results shown on Fig. 14 and Fig. 15. The DDHFB [12] form factor falls off too quickly, showing that the theory predicts a radial extension too large. A quantitative estimate for this difference has been obtained by fitting the data with Woods-Saxon wave functions. The r.m.s. radius of the $1f_{7/2}$ valence orbits determined by electron scattering are r_n = 4.011(40) fm and r_p = 4.006(40). DDHFB theory predicts r_n = 4.159 and r_p = 4.107.

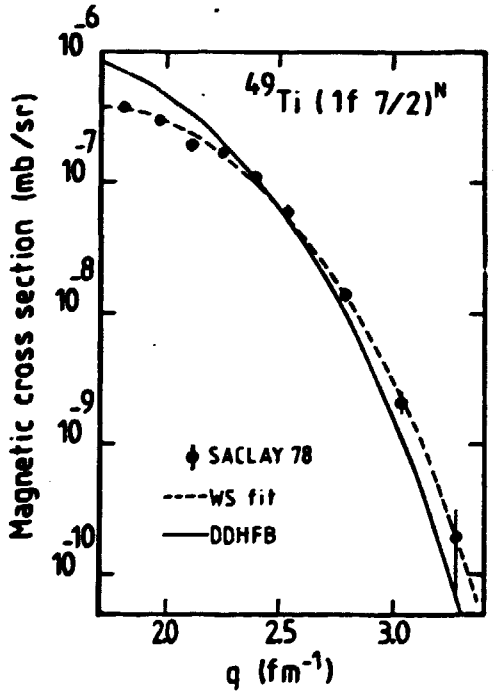

Fig. 15 - Experimental cross-section for magnetic scattering from ^{48}Ti as a function of momentum transfer. The dashed curve represents the Woods-Saxon fit, the solid one the Hartree-Fock prediction of Gogny.

In order to explain the disagreement observed in the measurement of valence radii by magnetic elastic electron scattering, recent calculations by Dubach [31], and Suzuki [32], have evaluated the contributions of meson exchange corrections. These calculations show that the amplitude of the form factor is changed and also its shape. However tensor correlations have not yet been included, and the contribution of the πNN form factor not been determined accurately. It is therefore difficult to make a very precise correction, but it is of the order of 1 % to 2 %. However MEC are not sufficient to remove the disagreement with the mean field prediction. A recent calculation by Lejeune and Mahaux [33] has evaluated the correction due to the fact that the mean field is energy dependent. This is neglected in the Hartree-Fock approximation. The single particle spectrum is compressed near the Fermi surface and in the case of ^{51}V the r.m.s. radius of the valence orbit is squeezed by about 0.5 % only.

The difference between neutron and proton radii can be also extracted directly from the comparison of the magnetic form factors of ^{49}Ti and ^{51}V (Fig. 16). This gives $r_n - r_p = -0.4 \pm 0.8$ % fm the theoretical values is 1.3 %, about 2 % too large. The

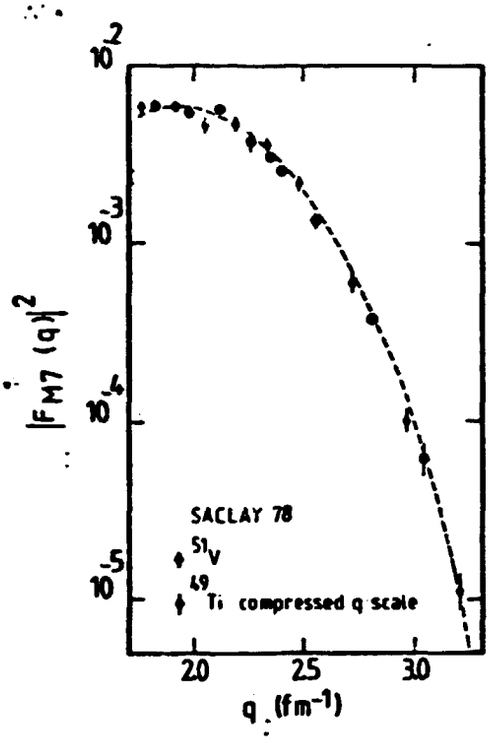

Fig. 16 - Direct comparison of the ^{49}Ti and ^{51}V form factors.

advantage of this direct comparison is that the result is almost insensitive to experimental errors and model assumptions. Meson exchange corrections are also largely cancelled in this comparison.

These measurements are extremely sensitive to the shape of the single particle wave functions, and they are the most accurate determination of neutron-proton valence orbit radii. They confirm that the theory predicts valence orbit too large by a few percents. But they definitely exclude the anomalous contraction suggested by Nolen and Schiffer [34] to explain the experimental Coulomb energy differences for mirror nuclei.

VII. Conclusions

Elastic electron scattering has now determined extremely fine details of the shape of the nuclear ground state. The combination of (e,e) and muonic X-rays data are giving informations that are among the most precise on nuclear structure. This enables to see all the limitations of existing theories. However, we begin to have a very coherent description of nuclei with the self consistent field theories to a few percent . A very significant progress has been achieved with the calculations of RPA correlations in the ground state in a self consistent way. However some improvements of the many body theory are needed, in particular in the case of odd even nuclei where the particle vibration coupling is probably a large effect.

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