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GEOMETIRCAL THEORY OF GHOST AND HIGGS FIELDS AND SU(2/1)

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Geometrical Theory of Ghost and Higgs Fields and SU(2/1)

DISCLAIMER

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Introduction

That a Principal Fiber Bundle provides a precise geometrical representation of Yang-Mills gauge theories has been known since 1963[1] and used since 1975[2]. The applications have consisted in the study of self-dual solutions to the Yang-Mills equation related to the choice of topologies in the base manifold (monopoles, instantons) and global properties of the bundle. In addition, homotopy was used in studying spontaneous symmetry breakdown, in the geometry of an Associated Vector Bundle corresponding to the Higgs field representation.

In our present work we shall present an entirely new domain of applications. We start with the identification of the Feynman-DeWitt-Fadeev-Popov ghost-fields[3] required in the renormalization procedure, with geometrical objects in the Principal Bundle. This will directly yield the BRS equations[4] garanteeing Unitarity and Slavnov-Taylor invariance[5] of the Quantum effective Lagrangian. Except for one ghost field (the "antighost") and its variation, this entire symmetry thus corresponds to "classical" notions, in that it is geometrical, and completely independent of the gauge-fixing procedure, which determines the Quantized Lagrangian.

Turning to Quantum Supergravity, we show how the above results may be used to fix the signs associated with the various ghost Loops of the theory[6]. The method has been applied in detail elsewhere to complete the proof of Unitarity. Our result is based upon the identification of a geometrical $Z(2) \otimes Z(2)$ double-gradation of the generalized fields in Supergravity: [physical/ghost] fields and [integer/half integer] spins.

We then consider the case of a supergroup as an internal symmetry gauge. We show how the ghosts geometrically associated to odd generators may be identified with the Goldstone-Nambu Higgs-Kibble scalar fields of conventional models with spontaneous symmetry breakdown. As an example, we realize the chiral $SU(3)_L \otimes SU(3)_R$ "flavour" symmetry[16] by gauging the supergroup Q(3).

Lastly, we recall the main results concerning Asthenodynamics (Weak-EM Unification) as given[15] by the ghost-gauge SU(2/1) supergroup.

1. Connections on a Principal Bundle: Gauge (Potentials) and Ghost Fields

We start by reintroducing[8] the concept of a Connection in a Principal Fibre Bundle $(P, M, \pi, G, \cdot)$. Previous authors used definitions in which the connection (a 1-form $\omega^A_{(YM)}$) was restricted to the base manifold M of dimension m=4, so that writing

$$\omega^A_{(YM)} = \omega^A_\mu \, dx^\mu \quad (A=1\ldots n, \ \mu=0,1,..3)$$

the ω^A_μ were identified with the Yang-Mills potentials. We denote the (vertical) projection by $\pi : P \to M$, the structure group by G and right-multiplication on by the dot $(\cdot) : P \times G \to P$, so that

$$\forall\, p \in P\ ,\ \forall\, g,g' \in G\ ,\ \begin{cases} \pi\,(\,p\cdot g\,) = \pi\,(p) \\ (\,p\cdot g\,)\cdot g' = p\cdot(\,gg'\,) \end{cases} \tag{1.1}$$

and for U_x a neighborhood of $x \epsilon M$, we get "local triviality" (a direct product) in P :

$$\left.\begin{aligned} &\pi^{-1}\,(\,U_x\,) \to U_x \times G \\ &p \to (\,\pi(p),\ \tau(p)\,)\ ,\ \text{where}\quad \tau(p\cdot g) = \tau(p)g \end{aligned}\right\} \tag{1.2}$$

(τ is a projection onto the fiber G).

The dot $(\cdot)$ induces a map t from the Lie algebra A of G into P_* , the tangent manifold to P . Thus,

$$\left.\begin{aligned} &\forall\, \lambda_a\ ,\ \lambda_b\ ,\ \lambda_e \leftarrow A \qquad (\,a,b,e = 1...n) \\ &\text{with} \\ &[\,\lambda_a\ ,\ \lambda_b\,] = C^e{}_{ab}\,\lambda_e \end{aligned}\right. \tag{1.3}$$

we have

$$t\ :\ A \to P_*\ ,\ \lambda \to \tilde{\lambda}\ \ \epsilon\ P_* \tag{1.4a}$$

By differentiation of (1.1) , one proves that t is an homomorphism of A , with the Lie Bracket operation realized on P_* as a Poisson Bracket

$$\widetilde{[\lambda\ ,\ \lambda']}_{L.B.} = [\tilde{\lambda}\ ,\ \tilde{\lambda}']_{P.B.} \tag{1.4b}$$

However, this map t has no inverse because the image of A (of dimension n) does not span P_* , of dimension $(\,n + m\,)$.

A linear mapping form P_* to A , the connection ω, is now chosen so as to provide the missing inverse

$$\left.\begin{aligned} &\omega\ :\ P_* \to A \\ &\forall\, \lambda \in A\ ,\quad \omega\,(\tilde{\lambda}\,) = \lambda \end{aligned}\right\} \tag{1.5}$$

ω is Lie-algebra valued, and belongs to the cotangent manifold $\overset{*}{P}$. It is thus a one-form. If z^R are local coordinates over P, one may explicitly write

$$\left.\begin{aligned}
&\forall v \in P_*\,, \quad v = v^R(z)\frac{\partial}{\partial z^R} \qquad\qquad (R,S = 1,2,\ldots n+m)\\
&\omega = \omega^a{}_S(z)\; dz^S\, \lambda_a\\
&\omega(v) = v \lrcorner\, \omega = \omega^a_R\; v^R\; \lambda_a \equiv \omega^a(v)\,\lambda_a\,.
\end{aligned}\right\} \qquad (1.6)$$

($\lrcorner$ denotes a contraction, $\frac{\partial}{\partial z^R} \lrcorner\; dz^S = \delta^S_R$)

As P_* is larger than A, there is a non-trivial kernel H of ω. In other words, to each point $p \in P$, ω associates a subspace $H_p \subset P_{*p}$. This is known as the "horizontal" tangent vector space at p, and defines an exact splitting of P_*

$$\begin{aligned}
&h \in H_p \iff \omega_p(h) = 0\\
&P_{*p} = V_p + H_p\\
&H_p = \mathrm{Ker}\,(\omega_p)\\
&V_p = \mathrm{Im}_t\,(A)\quad, \quad (\tilde{\lambda})_p \in V_p
\end{aligned} \qquad (1.7)$$

One also assumes an equivariance condition

$$H_{p\cdot g} = H_p \cdot g \qquad (1.8a)$$

We now introduce the Lie derivative ∇_v or convected derivative, along a vector field v in P_* (1.6). Its action on functions, vector-fields and one-forms reads:

$$\nabla_v\, f(z) = v^R \frac{\partial}{\partial z^R} f \qquad (1.9a)$$

$$\nabla_v\, v' = [v, v']_{P.B.} \qquad (1.9b)$$

$$\nabla v\; \omega = d\,(v \lrcorner\, \omega) + v \lrcorner\, d\omega \qquad (1.9c)$$

We now define the Curvature 2-form,

$$\Omega = d\omega + \frac{1}{2}\ [\omega,\omega] \tag{1.10}$$

and contract it with a vertical vector field $\tilde{\lambda}$

$$\tilde{\lambda} \lrcorner \Omega = \tilde{\lambda} \lrcorner d\omega + \frac{1}{2}\ [\tilde{\lambda} \lrcorner \omega, \omega] - \frac{1}{2}\ [\omega, \tilde{\lambda} \lrcorner \omega]$$

The first term is given by (1.9c), the last two by (1.5)

$$= \nabla_{\tilde{\lambda}}\, \omega + \frac{1}{2}\ [\lambda\ ,\ \omega] - \frac{1}{2}\ [\omega\ ,\ \lambda]$$

and using (1.8b)

$$= -\ [\lambda\ ,\ \omega] + \frac{1}{2}\ [\lambda\ ,\ \omega] - \frac{1}{2}\ [\omega\ ,\ \lambda] \quad = 0$$

The curvature 2-form is thus purely horizontal, (while (1.5) can be read to imply that ω is vertical)

$$\tilde{\lambda} \lrcorner \Omega = 0 \tag{1.11}$$

This equation is the Cartan-Maurer structural equation of a principal fiber bundle. Up to this point, we have just used textbook geometry. We can now identify the ghost fields.

Since we are in P_*, a gauge choice corresponds to defining a section, i.e. a surface Σ in P, locally diffeomorphic to the base manifold M. We fit the Z^R coordinates to Σ by lifting local x^μ coordinates from the base M, and α^i (group parameters) coordinates from G, using the maps (π^{-1}, τ^{-1}) of equation (1.2), to get the equation for Σ:

$$\Sigma: \quad \alpha^i (x) \doteq 0 \quad , \quad i = 1, \ldots n \tag{1.12}$$

We now express the vertical connection form ω in this basis

$$\left.\begin{array}{l} \frac{\partial}{\partial x^\mu} \lrcorner\, \omega = \Phi_\mu \quad , \quad \frac{\partial}{\partial \alpha^i} \lrcorner\, \omega = X_i \\ \\ \omega = X_i \, d\alpha^i + \Phi_\mu \, dx^\mu \end{array}\right\} \qquad (1.13)$$

It was originally suggested [9] to identify the ghost[3] fields C^a as

$$C^a \equiv X^a_i \, d\alpha^i$$

while Φ^A_μ is the Yang -Mills potential. More precisely, for C^a to have the dimensions of a field, we should redefine (ℓ is a constant length)

$$\ell C^a = X^a_i \, d\alpha^i \qquad (1.14)$$

According to (1.6), had we taken a topologically trivial P and a global flat section, $C^A_{(0)}$ would have coincided explicitly with the Cartan L.I. of the rigid group one-forms. It would then carry no x^μ dependence and would not be a true field. However, under a gauge transformation,

$$\delta\omega^a \, (x,\alpha) = D\varepsilon^a \, (x,\alpha) \qquad (1.15)$$

so that $C^a_{(0)} = \frac{1}{\ell} \, (\alpha^{-1} \, d\alpha)^a$ receives x^μ-dependent contributions,

$$\delta C^a = \frac{1}{\ell} \, d\alpha^i \, [\, \frac{\delta}{\delta\alpha^i} \, \varepsilon^a \, (x,\alpha)\,] - \frac{1}{\ell} \, C^a{}_{be} \, C^b \, \varepsilon^e \, (x,\alpha) \qquad (1.16)$$

similar to those of the Yang -Mills potential,

$$\delta\Phi^a_\mu = \partial_\mu \, \varepsilon^a \, (x,\alpha) - C^a{}_{be} \, \Phi^b_\mu \, \varepsilon^e \, (x,\alpha) \qquad (1.17)$$

We now rewrite Ω of (1.10) in component form, applying what we learned from the Cartan - Maurer equation. Defining

$$df = sf + \bar{d}f \quad ; \quad sf = d\alpha^i \, \frac{\partial}{\partial\alpha^i} \, f \quad ; \quad \bar{d}f = dx^\mu \, \frac{\partial}{\partial x^\mu} \, f \qquad (1.18)$$

Cohomology implies

$$\bar{d}^2 = s\bar{d} + \bar{d}s = s^2 = 0 \qquad (1.19)$$

$\bar{d}$ is our "ordinary" horizontal d which depends on the section Σ, s is the exterior differential normal to the section. Ω can be broken into 3 pieces, i.e. terms in $d\alpha^i \wedge d\alpha^j$, in $d\alpha^i \wedge dx^\mu$ and in $dx^\mu \wedge dx^\nu$:

$$\frac{1}{2}\,\Omega^a_{ij}\, d\alpha^i \wedge d\alpha^j = sX^a + \frac{1}{2}\,[X,X]^a \tag{1.20}$$

$$\begin{aligned}\Omega^a_{i\mu}\, d\alpha^i \wedge dx^\mu &= s\Phi^a + \bar{d}X^a + \frac{1}{2}\,([X,\Phi]^a + [\Phi,X]^a)\\ &= s\Phi^a + \bar{d}X^a + [\Phi,X]^a\\ &= s\Phi^a + \bar{D}X^a\end{aligned} \tag{1.21}$$

$$\frac{1}{2}\,\Omega_{\mu\nu}\, dx^\mu \wedge dx^\nu = \bar{d}\,\Phi^\alpha + \frac{1}{2}\,[\Phi,\Phi]^a \tag{1.22}$$

Applying (1.11) and identifying the field and ghost we have

$$sC^a = -\frac{\ell}{2}\,[C,C]^a \tag{1.23}$$

$$s\Phi^a_\mu = \ell\bar{D}_\mu C^a \tag{1.24}$$

These are the BRS equations[4] for Φ^a_μ and C^a. s is thus the BRS operator.

One of us (J. T-M) ha shown[9] how the covariant quantization path-integral, used in summing over all configurations of the potential satisfying BRS, can be given a geometrical form. In this representation, Feynman diagrams involve non-integrated exterior forms (the ghosts) together with anticommuting Legendre multipliers (the antighosts). One can then check that the minus sign required by ghost loops, which led to the assignment of Fermi statistics to spin-zero fields $C^a(x)$, is indeed just the sign due to self anticommutation of one-forms.

Note that this identification may in some special cases imply different commutation relations than those derived by naive generalization of the Yang-Mills situation. In Supergravity, for instance, the ghost of the $J = \frac{3}{2}$ gravitino Ψ^α_μ is a $J = \frac{1}{2}$ field Ψ^α. This field is a boson, since the one-forms ω^α or Ψ^α ($= \Psi^\alpha_\beta\, d\Theta^\beta$ in superspace)

$$\omega^{\alpha} = dx^{\mu}\, \psi^{\alpha}_{\mu} + \gamma^{\alpha}$$

correspond to anticommuting generators S_{α} in a GLA are commutations. They thus have two sign flips, one due to the dx^{μ} and one to the index α in the bracket $\{S_{\alpha}, S_{\beta}\}$. This represents a $Z(2) \otimes Z(2)$ gradation.

We may write[13]

$$\eta^{pa} \wedge \xi^{qb} = (-1)^{pq+AB}\, \xi^{qb} \wedge \eta^{pa} \tag{1.25}$$

where η^{pa} and ξ^{qb} are respectively a p-form and a q-form, the indices a and b represent a basis of a GLA, and A,B are their respective gradings. For Supergravity, our GLA is the graded Poincare algebra when working in the first order formalism. Denoting the gravitaton (vierbein) by e^{a}_{μ}, the "gravitino" spin $\frac{3}{2}$ field by ψ^{α}_{μ} (we put in spinor indices for clarity) and the Lorentz connection by Γ^{ab}_{μ}, their ghosts will be C^{a} (spin one), C^{α} (spin half) and C^{ab} (spin 1) respectively. We can fill up a table

bilinear	p	A	q	B	$(-1)^{pq+ab}$	
$e^{a}_{\mu}\ e^{b}_{\nu}$	0	0	0	0	+	[,]
$e^{a}_{\mu}\ C^{a}$	0	0	1	0	+	[,]
$C^{a}\ C^{b}$	1	0	1	0	-	{ , }
$\psi^{\alpha}_{\mu}\ \psi^{\beta}_{\nu}$	0	1	0	1	-	{ , }
$\psi^{\alpha}_{\mu}\ C^{\beta}$	0	1	1	1	-	{ , }
$C^{\alpha}\ C^{\beta}$	1	1	1	1	+	[,]
$\Gamma^{ab}_{\mu}\ \Gamma^{cd}_{\nu}$	0	0	0	0	+	[,]
$\Gamma^{ab}_{\mu}\ C^{cd}$	0	0	1	0	+	[,]
$C^{ab}\ C^{cd}$	1	0	1	0	-	{ , }

Note for example the departure from "intuitive" choices in the fifth row: ψ^{α}_{μ} is a fermion (4th row), C^{β} a boson (6th row) but they anticommute!

In their treatment of Ward identies in Supergravity, Sterman Townsend and Van Nieuwenhuizen[6] have dealt with the various ghost fields, though they did not specify the signs to be attached to the relevant closed ghost loops (see lines e and f in ref.[6]). The explicit minus sign which may easily be derived from their study (figs. 5 and 10) is indeed compatible with our geometric $Z(2) \times Z(2)$. In another publication[7] we have been able to complete their proof of the Unitarity of Supergravity, using this method.

2. Nambu - Goldstone and Higgs - Kibble Fields

The last remarks form a good introduction to our next subject. We have seen that when the Lie Group G is replaced by a Lie Supergroup[11] and the Lie algebra A by a Graded Lie Algebra (GLA)[12], some connection 1-forms commute instead of anticommuting.

For an internal GLA, the one-forms

$$\omega^i = G^i_\mu \ dx^\mu + \phi^i \tag{2.1}$$

commute when i represents an odd-grading (using (1.25)) and ϕ^i is thus a Lorentz-scalar physical Bose field. One of us (YN) has recently conjectured[15] that these fields be identified with Nambu-Goldstone Higgs-Kibble fields, when the Weinberg-Salam model's $SU(2) \times U(1)$ gauge group is embedded in the Supergroup $SU(2/1)$. The internal supergroup represents a Ghost-Symmetry (i.e. a symmetry between physical and ghost fields). The Higgs fields thus become in this approach the appropriate gauge fields for the odd part of the ghost symmetry.

In the following, we shall first take as an example the Chiral flavour symmetry $SU(3)_L \otimes SU(3)_R$. This is a "global" group with Nambu-Goldstone realization through a (pseudo)scalar field octet. We show how replacing this conventional approach by a local phenomenological supergauge reproduces the observed physical picture and yields the predictions of SU(6) and its extensions.

In the study of Goldstone-type realizations of global symmetries, the Goldstone field corresponded to that part of the Invariance group which was not a symmetry of the vacuum and could thus not be realized linearly on single-particle-state multiplets. It is indeed instructive to choose as an example the one case of that type we understood between 1960 and 1967: the pion's (and 0^- octet) role[16] as the zero-mass Goldstone particle in chiral $W(3)_{ch} = SU(3)_L \otimes SU(3)_R$.

In the non-linear picture[17], the vacuum is invariant under the positive parity $SU(3) \subset W(3)$ charges X^+. The remaining $\underline{8}$ of generators (under that SU(3)) corresponding to the axial-vector charges X^- is realized non-linearly. The $\underline{8}$ of 0^- mesons η acts as realizer,

$$\exp\left(-i\, \eta \cdot X^-\right) \ (0 \ , \ \psi) \ = \ (\eta \ , \ \psi) \tag{2.2}$$

The are in fact parameters of the axial generators. We denote the more common parameter of the (linear) vector subgroup by α.

For a generic element g of W(3) we get

$$g^{-1} \exp(-i\, \eta\cdot X^-) = \exp(-i\, \eta'\cdot X^-)\ \exp(-i\, \alpha\cdot X^+) \tag{2.3}$$

where $\eta \to \eta'$ is caused by the positive parity part of g^{-1}, whereas α is produced by the negative parity element acting on η, which is itself such an element. The resulting group action is given by

$$g^{-1}(\eta\, ,\, \psi) = (\eta',\, D\,(\exp -i\, \alpha\cdot X^+)\ \psi) \tag{2.4}$$

This action clearly exhibits a $\mathbb{Z}(2)$ grading provided by parity. Can we represent it linearly by a supergroup? In ref.12 we had indeed constructed the relevant superalgebra explicitly, in trying to discover a variety of examples for GLA. It now appears as (Q(2) in the Kac classification[18] and, more clearly) Q(3) in ref.11. For $Q(3)_{ch}$, take a set of sixteen (6 x 6) matrices,

$$X^+ : \begin{pmatrix} \lambda_m & \\ & \lambda_m \end{pmatrix} \qquad\qquad X^- : \begin{pmatrix} & \lambda_n \\ \lambda_n & \end{pmatrix} \gamma_5 \tag{2.5}$$

(λ_m , λ_n are SU(3) matrices[15]) and define the brackets,

$$\begin{aligned} [X^+_m\, ,\, X^+_n] &= i\, f_{mn\ell}\ X^+_\ell \\ [X^+_m\, ,\, X^-_n] &= i\, f_{mn\ell}\ X^-_\ell \\ [X^-_m\, ,\, X^-_n]_D &:= X^-_m X^-_n + X^-_n X^-_m - \tfrac{2}{3}(\mathrm{Tr}\, X^-_m X^-_n)\, I \\ &= 2\, d_{mn\ell}\ X^+ \end{aligned} \tag{2.6}$$

where the $d_{mn\ell}$ are SU(3) totally symmetric Clebsch-Gordan coefficients for $\underset{\sim}{8} \times \underset{\sim}{8} \to \underset{\sim}{8}_{sym}$ and are defined [15] by

$$\{\lambda_m\, ,\, \lambda_n\} = \tfrac{4}{3}\, \delta_{mn}\, 1 + 2\, d_{mn\ell}\, \lambda_\ell \tag{2.7}$$

The symmetric bracket between two odd elements thus differs from an anti-commutator (in this defining representation) by a trace. In the adjoint representation, it will again be an anticommutator.

We now take this G = W(3) in P and study the connections. Under the X^+ generated SU(3) subgroup, we have two octets,

$$\omega^a = \ell\, C^a(x) + dx^\mu\ \phi^a_\mu(x)$$
$$\omega^i = \ell\, \eta^i(x) + dx^\mu\, G^i_\mu(x) \tag{2.8}$$

In the even subgroup, $\phi^a_\mu(x)$ is a J = 1 octet, and $C^a(x)$ the corresponding ghost $\underset{\sim}{8}$. In the odd piece, the ω^i also form an $\underset{\sim}{8}$ under X^+, but they are commutative as we have seen in (1.25) and (2.1). Thus $\eta^i(x)$ is a 0^- octet of bosons, i.e. physical fields! In fact, <u>we can identify these "exorcized ghosts" as the Goldstone-Higgs multiplet of the theory</u>! They are accompanied, however, by a new type of ghost, the J = 1 Fermi-statistics $G^i_\mu(x)$. The role of the latter is perhaps not entirely understood at this point, but one can already see them in action in one-loop renormalization group equations: they provide a relatively heavier weighted contribution of ghost type. Notice that the entire counting system for such internal supergauges has to be reordered, since the Higgs fields η^i will be coupled universally, thus providing new diagrams of order g^3, etc...

We may solve (1.24) and write

$$G^i_\mu = -\ell\, s^{-1}\, \bar{D}_\mu\, \eta^i \quad := \quad \varepsilon_\mu\, \eta^i$$
$$\phi^a_\mu = -\ell\, s^{-1}\, \bar{D}_\mu\, C^a \quad := \quad \varepsilon_\mu\, C^a \tag{2.9}$$

For a matter field ψ, an SU(3) triplet in representation (2.5) the BRS equation is

$$s\psi^n = \ell\,[C\,,\,\psi]^n \quad := \quad r^n \tag{2.10}$$

where we have defined an <u>effective</u> ghost-field r^n. If ψ^n is a Lorentz-spinor <u>fermion</u> (the β-positive components in fact), r^n will be a Lorentz-spinor <u>boson</u>, i.e. a ghost. However, since G is a supergroup, the ψ^n fill up only half of a representation like (2.5). The other half consists of a triplet t^u of Lorentz-spinor opposite parity <u>bosons</u> (i.e. ghosts). Thus BRS equation becomes

$$st^u = \ell\,[C\,,\,t]^u \quad := \quad \psi^u \tag{2.11}$$

thus relating them to Lorentz-spinor <u>fermions</u> ψ^u which complement the r^n in making a six-dimensional (and Dirac β diagonalized) representation of W(3).

so as to have all new ghosts (r^n, t^u) appear as composite, and the ψ^u as additional (inverse parity) matter fields, we may write

$$r^n = s\,\psi^n \quad , \quad t^u = s^{-1}\,\psi^u \quad . \qquad (2.12)$$

Summing up, we have seen that gauging a supergroup G produces as gauge fields both the vector-mesons ϕ^a_μ coupled to the even subgroup G^+ and a Goldstone-Higgs multiplet η^i behaving as $A(G^-)$ under $A(G^+)$ itself. At the same time, the theory contains the renormalization ghosts C^a and a new set of vector-ghosts G^i_μ. Matter fields ψ^n, ψ^u are split between two analogous representations of G, even though when taken together they fit exactly the quantum numbers of one such representation. In their split assignment, they are accompanied by composite ghost fields which complete the two representations.

Notice that the resulting gauge Lagrangian (in its physical part) is exactly that of the "flavour" SU(3) of the sixties with phenomenological constituent quark fields and with the 1^- mesons ρ, K*, ϕ^0/ω^0 as gauge fields, plus a universally coupled 0^- meson multiplet π, K, η. This is just the Lagrangian postulated by Gursey and Radicati, which gave rise to SU(6) as its static symmetry[14]!

SU(2/1) as the Ghost Theory of Asthenodynamics (the Weak-Electromagnetic Interactions).

The idea of a supergroup as an internal gauge group involving the ghosts of renormalization was first suggested[15] in the context of a basic theory of the unified Weak-Electromagnetic Interaction. It reproduces the Salam-Weinberg model[20] in an extremely constrained form, imposed by $SU(2/1) \supset SU(2)_L \times U(1)$. The kinematics of SU(2/1) are astonishingly precise in fitting just the observed particle representations of $SU(2)_L \times U(1)_U$:

	Particles	Ghosts
$\underset{\sim}{8}$, J=1	ϕ^1_μ, ϕ^2_μ, ϕ^3_μ, ϕ^8_μ	G^4_μ, G^5_μ, G^6_μ, G^7_μ
$\underset{\sim}{8}'$, J=0	η^4, η^5, η^6, η^7	C^1, C^2, C^3, C^8
$\underset{\sim}{3}$, J=½	ν^0_L , e^-_L	r^-_R (composite)
$\underset{\sim}{3}'$, J=½	e^-_R	t^0_L , t^-_L (composite)
$\underset{\sim}{4}$, J=½	$u^{2/3}_L$, $d^{-1/3}_L$	$r^{-1/3}_R$, $r^{2/3}_R$ (composite)
$\underset{\sim}{4}'$, J=½	$d^{-1/3}_R$; $u^{2/3}_R$	$t^{2/3}_L$, $t^{-1/3}_L$ (composite)

A "family" is thus $(\underset{\sim}{3} + \underset{\sim}{3}') + 3 \times (\underset{\sim}{4} + \underset{\sim}{4}')$.

Note that SU(2/1) predicts[21] that the $I_L = ½$ multiplet is in a $\underset{\sim}{4}$ representation when the charges are fractional and in a $\underset{\sim}{3}$ when they take on integer values! Similarly, it predicts that the Higgs-Goldstone multiplet is an isodoublet $I_L = \frac{1}{2}$, $U = \pm 1$. Also, $\theta_W = 30^o$, and $m_\eta \sim 245$ GeV. We refer the reader to the original article[15] and to a recent discussion at the classical level[21] (the full quantum Lagrangian is under study). The $\lambda\phi^4$ self-coupling of the Higgs-Goldstone multiplet is $\lambda \sim \frac{4}{3} g^2$.

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