

THE INFLUENCE OF THE COULOMB-DISTORTION  
EFFECT ON PROTON-PROTON OBSERVABLES<sup>†</sup>

by

W. Plessas and L. Mathelitsch  
Institut für Theoretische Physik, Universität Graz  
Universitätsplatz 5, A-8010 Graz, Austria

and

F. Pauss  
Max-Planck-Institut für Physik und Astrophysik  
Föhringer Ring 6, D-8000 München 40, FRG

---

<sup>†</sup>Supported partly by Fonds zur Förderung der wissenschaftlichen Forschung, project 2900.

**Abstract**

The effect of the Coulomb distortion of the strong interaction is studied on the basis of nucleon-nucleon observables. In particular, cross sections, polarizations, spin-correlation parameters, and spin-transfer coefficients are considered for proton-proton as well as neutron-neutron scattering at laboratory kinetic energies  $E_{lab} = 10, 20,$  and  $50$  MeV. The calculations are performed for the meson-theoretical PARIS potential, the nonlocal separable GRAZ potential and also using the Arndt-Hackman-Roper parametrization of proton-proton scattering phase shifts. Important conclusions are drawn with respect to phenomenological phase-shift analyses.

## I. Introduction

In recent years much effort has been invested in order to obtain reliable phase-shift analyses of nucleon-nucleon (N-N) experimental data, cf. e.g. ref. <sup>1)</sup>-6). The situation looks quite satisfactory for the proton-proton (p-p) system (because of the large number of rather accurate data, which provide a solid basis for the analysis), but the neutron-proton (n-p) system still exhibits various insufficiencies, while the neutron-neutron (n-n) system remains totally unexplored in this respect. Because up till now not enough observables were measured, for n-p scattering, so that an unambiguous n-p phase-shift analysis was feasible, one was forced to supply the isospin-triplet part with p-p data; this way is followed e.g. in combined p-p and n-p phase-shift analyses. Within such attempts it is usually assumed that the p-p "nuclear" scattering amplitude, i.e. the one which is obtained from the total amplitude by subtracting the pure-Coulomb amplitude (and possibly other electromagnetic amplitudes, like vacuum polarization etc.) can also be considered as n-p amplitude and therefore can serve directly as isospin  $T = 1$  input for the n-p system. However, the "nuclear" p-p amplitude thus obtained is not purely nuclear, rather it contains further electromagnetic effects (for their discussion see e.g. ref. <sup>7) 8)</sup>), for which it has still to be corrected. Of these the Coulomb-distortion (CD) effect (or simply Coulomb correction as it is often called) of the nuclear interaction is commonly considered to be the most important

one. For two-hadron systems this effect has intensively been studied in the context of scattering phase shifts, cf. the review by Hamilton<sup>9)</sup>. The main purpose of the present paper is to investigate the influence of the CD effect on p-p observables. Such a study can be expected to shed light on the question of reliability of the assumption outlined above, namely, whether or not it is justified to substitute the  $T = 1$  part of the n-p scattering system by the Coulomb-subtracted p-p amplitude<sup>10)</sup>. In particular we calculate the influence on observables of the phase-shift difference

$$\Delta = \delta^{sc}(p-p) - \delta^s(p-p \text{ purely nuclear}),$$

i.e. of the CD effect as reflected by the phase parameters. Here  $\delta^{sc}(p-p)$  is defined by the difference of the total and the Coulomb phase shifts

$$\delta^{sc}(p-p) = \delta^{tot}(p-p) - \sigma^c(p-p \text{ purely Coulomb}).$$

Since in the framework of our approach we can employ a unique prescription for  $\Delta$ , we are in the position to make predictions about n-n observables under the assumption that other (less important) electromagnetic effects may be neglected.

We perform our investigation by means of two potential models for the strong N-N interaction, namely, the PARIS<sup>11)</sup> and the GRAZ<sup>12)</sup> potentials, as well as with a phenomenological phase-shift parametrization, namely, the Arndt-Hackman-Roper (AHR) p-p phase-shift analysis<sup>2)</sup>. Thus the second aim of our work is

to study p-p, likewise n-n, observables with respect to model differences. By comparing the results yielded by each one of the three models considered we can learn about their properties - most interestingly of the potential models - as reflected on the basis of experimental scattering observables.

In the following chapter we provide a short summary of the formalism that we used. Chapter III describes the various methods, how we obtained our results, and discusses in detail those observables that we decided to present in this paper. Chapter IV contains our conclusion.

## II. Proton-proton and neutron-neutron observables

Though a full account of the formalism for the evaluation of observables for both the p-p and n-n systems can be found in the literature (see e.g. refs. <sup>13)</sup>-<sup>17)</sup>), we like to summarize the most important formulae that we used, also because in some places of the cited literature we found a few of them to contain errors.

### 1. The proton-proton case

Denote by  $\vec{r}$  and  $\vec{k}$  the coordinate resp. wave number of the relative motion of the protons and let  $\{SM_z\}$  be their total spin  $S$  with z-component  $M_z$ . Their scattering wave function at large distances can be expressed in the form <sup>17)18)19)</sup>

$$\begin{aligned} \Psi_{SM_S}^{(+) }(\vec{k}, \vec{r}) = & (2\pi)^{-\frac{3}{2}} \exp\{i[\vec{k} \cdot \vec{r} + \nu \log(kr - \vec{k} \cdot \vec{r})]\} \chi_S^{M_S} + \\ & + (2\pi)^{-\frac{3}{2}} \frac{1}{r} \exp[i(kr - \nu \log 2kr + 2\sigma_0)] M \chi_S^{M_S} \end{aligned} \quad (2.1)$$

with  $\chi_S^{M_S}$  representing the spin state and

$$\nu = \frac{\mu c^2}{\hbar^2 k} \quad \mu \dots \text{reduced mass}$$

$$\sigma_0 = \text{arg } \Gamma(1 + i\nu).$$

Eq. (2.1) defines the spin-scattering operator  $M$ , which was first discussed by Wolfenstein and Ashkin<sup>13) 20)</sup>. Once we know it, we can calculate all relevant observables like differential cross section, polarization, depolarization, spin-transfer coefficients, spin-correlation, parameters etc.

Requiring invariance under time reversal, space reflections, and rotations the operator  $M$  is related to the usual transition operator  $T$  by<sup>21) 22)</sup>

$$M(k, \theta, \varphi) = -\frac{(2\pi)^2 \mu}{\hbar^2} T(\vec{k}_i, \vec{k}_f; k) \quad |\vec{k}_i| = |\vec{k}_f| = k \quad (2.2)$$

In terms of the scattering amplitude this relation reads

$$M(k, \theta, \varphi) = f(k, \theta, \varphi) = f^c(k, \theta, \varphi) + f^{sc}(k, \theta, \varphi) \quad (2.3)$$

with  $f^c$  and  $f^{sc}$  representing the pure Coulomb as well as Coulomb-distorted strong amplitudes.

The spin-scattering amplitude  $M(k, \theta, \varphi)$  of eqs. (2.2) and (2.3) is still to be considered as an operator in spin space acting on the initial spin state  $\chi_S^{M_S}$ . After performing a partial wave decomposition of the strong-interaction part  $f^{sc}$  of the total scattering amplitude  $f$  we can express the matrix elements

$\langle S' M_{S'} | M(k, \theta, \varphi) | S M_S \rangle$  via the following formulae<sup>15) 16) 23)</sup>;

1) in the spin singlet denoting  $M_{SS} = \langle 00 | M | 00 \rangle$ :

$$M_{SS} = f_s^c + \frac{2}{ik} \sum_{\text{even } L} P_L(\cos \theta) (2L+1) \frac{\alpha_L}{2} \quad (2.4)$$

11) in the spin triplet denoting  $M_{M_S M_S} = \langle 1 M_S | M | 1 M_S \rangle$ :

$$M_{11} = f_t^c + \frac{2}{ik} \sum_{\text{odd } L} P_L(\cos \theta) \frac{1}{4} \left[ (L+2) \alpha_{L,L+1} + (2L+4) \alpha_L + (L-1) \alpha_{L,L-1} - \sqrt{(L+1)(L+2)} \alpha^{L+1} - \sqrt{L(L-1)} \alpha^{L-1} \right] \quad (2.5a)$$

$$M_{00} = f_t^c + \frac{2}{ik} \sum_{\text{odd } L} P_L(\cos \theta) \frac{1}{2} \left[ (L+1) \alpha_{L,L+1} + L \alpha_{L,L-1} + \sqrt{(L+1)(L+2)} \alpha^{L+1} + \sqrt{L(L-1)} \alpha^{L-1} \right] \quad (2.5b)$$

$$M_{01} = \frac{2}{ik} e^{i\varphi} \sum_{\text{odd } L} P_L^1(\cos \theta) \frac{\sqrt{2}}{4} \left[ -\frac{(L+2)}{(L+1)} \alpha_{L,L+1} + \frac{2L+1}{L(L+1)} \alpha_L + \frac{L-1}{L} \alpha_{L,L-1} + \sqrt{\frac{L+2}{L+1}} \alpha^{L+1} - \sqrt{\frac{L-1}{L}} \alpha^{L-1} \right] \quad (2.5c)$$

$$M_{10} = \frac{2}{ik} e^{-i\varphi} \sum_{\text{odd } L} P_L^1(\cos \theta) \frac{\sqrt{2}}{4} \left[ \alpha_{L,L+1} - \alpha_{L,L-1} + \sqrt{\frac{L+2}{L+1}} \alpha^{L+1} - \frac{\sqrt{2}}{4} \sqrt{\frac{L-1}{L}} \alpha^{L-1} \right] \quad (2.5d)$$

$$M_{1-1} = \frac{2}{ik} e^{-2i\varphi} \sum_{\text{odd } L} P_L^2(\cos \theta) \frac{1}{4} \left[ \frac{1}{L+1} \alpha_{L,L+1} - \frac{2L+1}{L(L+1)} \alpha_L + \frac{1}{L} \alpha_{L,L-1} - \sqrt{(L+1)(L+2)} \alpha^{L+1} - \sqrt{L(L-1)} \alpha^{L-1} \right] \quad (2.5e)$$

The remaining elements can be obtained by the symmetry relations

$$\begin{aligned}
 M_{-l, -l}(\theta, \varphi) &= M_{ll}(\theta, -\varphi) \\
 M_{0, -l}(\theta, \varphi) &= -M_{0l}(\theta, -\varphi) \\
 M_{-l, 0}(\theta, \varphi) &= -M_{l0}(\theta, -\varphi) \\
 M_{-ll}(\theta, \varphi) &= M_{ll}(\theta, -\varphi).
 \end{aligned}
 \tag{2.5f}$$

In eqs. (2.4) and (2.5)  $P_L^M(\cos\theta)$  represents the associated Legendre function of the first kind<sup>20)</sup>;  $f_s^c$  and  $f_t^c$  are the Coulomb scattering amplitudes in the singlet and triplet states, respectively

$$f_s^c(\theta) = f^c(\theta) + f^c(\pi - \theta) \tag{2.6a}$$

$$f_t^c(\theta) = f^c(\theta) - f^c(\pi - \theta) \tag{2.6b}$$

$$f^c(\theta) = \frac{-\gamma}{2k \sin^2(\theta/2)} \exp[-2i\gamma \log \sin^2(\theta/2)]. \tag{2.7}$$

The quantities  $d$  occurring in eqs. (2.4) and (2.5) essentially consist of the on-shell elements of the Coulomb-distorted strong amplitude  $T^{sc} = T - T^c$  in the various partial-wave states. In terms of phase shifts they are given by

$$\left. \begin{aligned}
 S=0: \\
 S=1, L=j:
 \end{aligned} \right\} d_L = \exp(2i\eta_L) [\exp(2i\delta_L^{sc}) - 1] \tag{2.8}$$

$$S=1, L=j\pm 1: d_{j\pm 1, j} = \exp(2i\eta_{j\pm 1}) [\exp(2i\delta_{j\pm 1}^{sc}) \cos 2\epsilon_j^{sc} - 1] \tag{2.9a}$$

$$d^j = i \sin 2\epsilon_j^{sc} \exp i(\delta_{j-1}^{sc} + \eta_{j-1} + \delta_{j+1}^{sc} + \eta_{j+1}) \tag{2.9b}$$



Here  $\eta_L$  stands for the difference of pure Coulomb phases

$$\eta_L = \sigma_L - \sigma_0 = \arg \Gamma(L+1+i\eta) - \arg \Gamma(1+i\eta) = \sum_{n=1}^L \arctan \frac{\eta}{n} \quad (2.10)$$

whereas  $\delta^{sc}$  represents the Coulomb-distorted strong phase shift, which for some partial-wave state  $L$  is defined by

$$\delta_L^{sc} = \delta_L^{tot} - \sigma_L. \quad (2.11)$$

Note that in deriving eqs. (2.8) and (2.9) we used the Stapp parametrization of the scattering matrix<sup>23)</sup>. Therefore the phase parameters  $\delta^{sc}$  and  $\epsilon^{sc}$  represent the so-called "bar" phase parameters.

At this place a remark concerning the mixing parameter may be in order: while for the Coulomb-distorted nuclear phase shifts the relation (2.11) holds, we have for  $\epsilon^{sc}$ :

$$\epsilon_j^{tot} = \epsilon_j^{sc} \quad (2.12)$$

i.e. in the total  $\epsilon$ -parameter there is no pure Coulomb contribution. The only influence on it by the Coulomb potential comes in via the CD effect (and possibly other less important Coulomb effects, which will not be considered here, as we mentioned in the Introduction). The role of the distortion effect with respect to  $\epsilon$  is further discussed in the subsequent chapter.

## 2. The neutron-neutron case

For the n-n system the formalism simplifies considerably. Loosely speaking we get the appropriate formulae from the ones in the preceding subsection by switching off the Coulomb potential; then all major (direct) Coulomb effects on the nuclear interaction vanish. For the n-n system the well-known formulae for scattering from short-range interactions apply. Thus e.g. the analogue of eq. (2.1) reads

$$\psi_{S M_S}^{(+)}(\vec{k}, \vec{r}) = (2\pi)^{-\frac{3}{2}} e^{i\vec{k}\cdot\vec{r}} \chi_S^{M_S} + (2\pi)^{-\frac{3}{2}} \frac{1}{r} e^{ikr} M \chi_S^{M_S} \quad (2.13)$$

where M now only contains the strong scattering amplitude <sup>22)</sup>

$$M(k, \theta, \varphi) = f(k, \theta, \varphi) = f^s(k, \theta, \varphi).$$

For n-n scattering the matrix elements of the spin-scattering amplitude are obtained from eqs. (2.4)-(2.5) by omitting the pure Coulomb amplitudes. Equivalently the quantities  $d_L$  now contain only the purely nuclear phase shifts

$$\left. \begin{array}{l} S=0: \\ S=1, L=j: \end{array} \right\} d_L = \exp(2i\delta_L^s) - 1 \quad (2.14)$$

$$S=1, L=j\pm 1: d_{j\pm 1, j} = \exp(2i\delta_{j\pm 1}^s) \cos 2\varepsilon_j - 1 \quad (2.15)$$

$$d^j = i \sin 2\varepsilon_j \exp i(\delta_{j-1}^s + \delta_{j+1}^s). \quad (2.16)$$

In this section our aim was to find the matrix elements of the spin-scattering amplitude for both the p-p and n-n systems once the phase parameters are given. Then their notion allows us to calculate all observables that we are interested in. The formulae expressing them in terms of M-matrix elements together with a complete list of all possible observables can e.g. be found in refs.<sup>15)17)</sup> or in the book by Moravcsik<sup>25)</sup>.

### III. Results

#### 1. Basis and scope

Before describing the results of our study in detail let us first outline the methods, how we obtained them. As was already mentioned in the Introduction we calculated observables for three cases, namely, two potential models and one phenomenological phase-shift analysis:

- PARIS potential<sup>11)</sup>
  - GRAZ potential<sup>12)</sup>
  - p-p scattering analysis of Arndt, Hackman, and Roper (AHR)<sup>2)</sup>.
- We performed the calculations at three energies, namely, at  $E_{\text{Lab}} = 10, 20, \text{ and } 50 \text{ MeV}$ .

1) Full p-p observables were obtained by using the theoretical (for GRAZ and PARIS) resp. phenomenological (for AHR) Coulomb-distorted nuclear phase shifts  $\delta^{\text{sc}}$  (cf. their definition in eq.(2.11)). We inserted them into eqs.(2.8) and (2.9) and

computed the quantities  $\delta$ . They were used together with the pure Coulomb amplitudes (2.6) and (2.7) for evaluating the matrix elements of the spin-scattering amplitude via eqs. (2.4) and (2.5). Once the M-matrix elements were known, the p-p observables could easily be deduced.

ii) The influence of the Coulomb-distortion effect on full p-p observables was investigated in the next step. For this purpose we used the purely nuclear phase shifts  $\delta^S$  instead of  $\delta^{sc}$  and followed the same procedure as under i). Note that for the two potential models the phase shifts  $\delta^S$  are defined unambiguously by the solution of the nuclear problem alone; for AHR we had to look for a prescription to subtract the CD effect in the phase shifts  $\delta^{sc}$ . Such a prescription was already given by Plessas et al.<sup>26)</sup> and later on as an improved version by Streit et al.<sup>27)</sup>:

$$\Delta_L = \delta_L^{sc} - \delta_L^S = -\frac{\mu e^2}{k^2} a_L \left[ \frac{d}{dk} \delta_L^S(k) + \frac{1}{2k} \sin 2\delta_L^S(k) \right] \quad (3.1)$$

with  $a_L$  being some numerical constant for each partial wave. Though this formula is only approximative to first order in  $e^2$ , it yields reasonable results at least at those energies where we performed our calculations, namely at  $E_{\text{Lab}} = 10, 20,$  and 50 MeV. We remark that eq. (3.1) applies only to uncoupled partial waves. An analogous formula holds also for the Coulomb-distortion effect in scattering parameters for coupled partial-wave states, in particular for the mixing parameters  $\xi_j^{sc}$ . In the course of our study it turned out that with respect to the latter quantities the Coulomb-distortion effect is quite

negligible;<sup>28)</sup> thus only the phase shifts had to be treated according to eq. (3.1) for coupled as well as for uncoupled partial-wave states.

We remind the reader that in this step of our study we retained the pure Coulomb interaction; merely the CD effect was switched off in order to investigate its importance with respect to p-p observables.

iii) Coulomb-distorted nuclear observables for p-p scattering were obtained by setting the Coulomb amplitude  $f_c$  in eq. (2.7) equal to zero, but following the same procedure as under i) in all other respects. By this way we calculated nuclear observables corresponding to the Coulomb-distorted nuclear phase shifts  $\delta^{sc}$ .

iv) Purely nuclear observables for p-p scattering were calculated in the last step. For this purpose the pure Coulomb interaction as well as the CD effect were removed. Neglecting all other less important Coulomb effects the observables we found by this way can equally well be regarded as n-n scattering observables. (only the purely nuclear phase shifts  $\delta^p$  served as input).

For the calculations according to each one of the foregoing cases we took into account partial-wave states and phase parameters as appearing in Table 1; whenever needed the pure Coulomb phases as well as the pure Coulomb amplitude were in-

corporated as following from eqs. (2.10) and (2.7) respectively.

All the results that were obtained by the procedures outlined above on the one hand provide the basis for comparing the properties exhibited by the meson-theoretical PARIS potential, the phenomenological separable GRAZ potential and the AHR experimental data within each one of the cases i)-iv); on the other hand they allow for a study of the influence of Coulomb effects, in particular and most interestingly of the CD effect. For the latter purpose we only need to contrast cases i) and ii) as well as cases iii) and iv) to each other. By examining the situation for AHR we additionally get an idea of the relevance of the approximation formula (3.1).

## 2. Discussion of results

In the course of our study we computed the whole manifold of p-p observables. For many of them, especially for the more complicated ones like some triple-scattering parameters, it turned out that the CD effect, whose study was our prior aim, played only a minor role; these observables are not considered in the present paper. In the following we are going to discuss in detail only those observables, which are considerably sensitive to this effect; for their explicit definition see e.g. refs. <sup>15) 17)</sup>, a schematic picture of them can be found in Fig. 6.

a) Differential cross section  $I_0$  (Fig. 1)

Let us first outline the gross features of the p-p differential cross section. It can nicely be read off from the lower bunch of curves marked "p-p" in Figs. 1. a-c. They correspond to the cases i) and ii) of Sec.1., namely where the pure Coulomb interaction was included. The solid line, the long-dashed line, and the long-dashed dotted line represent the results for resp. the AHR, GRAZ, and PARIS models according to case i), while the dotted line, the short-dashed line, and the short-dashed dotted line correspond to the same models treated according to case ii). Evidently all six curves exhibit the well-known Coulomb singularity at forward scattering angles. The influence of the strong interaction comes into play only at larger scattering angles. But it gains importance with increasing energy thus forcing the Coulomb dip back to smaller and smaller angles (cf. Figs.1.a-c).

The differences between curves corresponding to case i) and ii) stem from the different input of "strong" phases ( $\delta^{sc}$  or  $\delta^s$ ). They exhibit the role of the CD effect on p-p observables. When considered as a function of the scattering angle, it can be seen that the effect shows up, whenever the strong interaction gathers relative importance; of course the effect is primarily correlated to the strong interaction. It results in shifting the curves without changing their shapes in exactly those domains, where they are governed by the strong potential. (this shift is even much better reflected by the "n-n" curves,

which we are going to discuss in a moment). In these very domains the effect is quite independent of the scattering angle. But the CD effect depends on the energy; in particular, it is most important at lower energies (Figs.1.a and b). At  $E_{\text{Lab}} = 10$  MeV the differences caused by it even exceed the differences between the various models (AHR, GRAZ or PARIS), at  $E_{\text{Lab}} = 20$  MeV they are still of the same magnitude, only at  $E_{\text{Lab}} = 50$  MeV the Coulomb effect seems to become negligible.

Next we examine in detail the upper bunch of curves marked "n-n". They correspond to the cases iii) and iv) of Sec.1., i.e. where the pure Coulomb interaction was left out<sup>29)</sup>. The relation of the curves to each other and the various models is the same as before (cf. also the figure caption). Again the differences of curves corresponding to case iii) and iv) exhibit the CD effect, this time on Coulomb-subtracted nuclear observables. What was already said before about its characteristics, remains true. Only the singular structure of the differential cross section at forward angles is wiped out. Thus the CD effect results in shifting the curves over the whole range of scattering angles: switching it off causes a uniform rise in  $I_0$ , which is the greater the lower the energy.

The CD effect is seen to be of the same order of magnitude for all three models, the two potentials as well as for AHR, though for obtaining the scattering phase shifts  $\delta''$  resp.  $\delta'$  in each one of these cases a different method was followed:



for the PARIS potential the Schrödinger equation was solved whereas for the GRAZ potential the Lippmann-Schwinger formalism, was employed. For the AHR case the Coulomb-distortion effect  $\Delta$  in phase shifts was treated (approximatively) according to formula (3.1); since the corresponding results are in keeping with the (exact) ones of the potential models, we can conclude that the approximation (3.1) is practicable.

Now we compare p-p with n-n cross sections. By doing so we get an idea of the alternate play of the Coulomb and the strong interaction. The Coulomb interaction dominates at the lower energies also in the non-forward scattering range. Only at higher energies the nuclear potential succeeds in balancing the Coulomb interaction: then the n-n curves approach the p-p curves at least for scattering angles around  $\frac{\pi}{2}$  (cf. Fig. 1.c). If we include the CD effect into the present consideration, we see that it results in weakening the action of the strong potential - a fact which is reflected by the common rise of all curves, when the effect is switched off.

Let us now say a few more words about model differences, in particular about differences in the various cross sections themselves as yielded by AHR, GRAZ, and PARIS. In general these differences are relatively small, though by no means unimportant. The deviations of the GRAZ potential from AHR and the PARIS potential at 50 MeV are remarkable. With respect to

the nuclear contribution to  $I_0$ , the GRAZ potential yields too low a result and does not quite well agree with the data. The reason for this behaviour obviously lies in the  $^1S_0$  partial wave, where the GRAZ potential is not enough attractive at the energies in question, above all at 50 MeV; we will come back to this point lateron. The PARIS potential in turn seems to fit the experimental data equally well or even better than the AHR phase parametrization.

At this place we would also like to make the following remark with respect to phase-shift analyses: A comparison of the magnitude of the CD effect (on p-p observables) with error bars shown by the experimental data impressively suggests the inclusion of this type of Coulomb corrections into phase-shift analyses especially into combined p-p and n-p analyses. Concerning  $I_0$  this statement holds for all three energies considered here, particularly also for  $E_{\text{Lab}} = 50$  MeV, since there the error bars are so small. A way taking into account this fact was e.g. followed in a recent phase-shift analysis by Bugg et al.<sup>3)</sup>. On the other hand the same comparison performed for n-n observables gives an idea of the desirable order of magnitude for the error in possible n-n cross section measurements that would still allow to decide on the question of charge symmetry (and/or independence).

Finally we note that most of the features of the CD effect described above can be traced back to the situation in the phase parameters mainly in the  $^1S_0$  state, which of course plays the most important role for  $I_0$ ; at  $E_{\text{Lab}} = 10$  MeV the differences

$\Delta = \delta^{sc} - \delta^s$  coming from the Coulomb-distortion effect are of considerable magnitude, they are larger than model differences (cf. the first column of Table 1). In the energy range considered the strong interaction is predominantly attractive; again it is seen that the CD effect results in weakening this attractive property of the nuclear potential: when it is switched off, i.e. for  $\delta^s$ , the phase shift is much larger (positive). With increasing energy the difference diminishes, the Coulomb effect becomes negligible. Also reflected by the  $^1S_0$  phase shift is the deviation of the GRAZ potential from ANR and PARIS in particular at 50 MeV. The phase shift appears to be too small, the potential too less attractive and as a consequence the differential cross section curves happen to lie at too small values.

#### b) Polarization P (Fig.2)

The next observable where we found Coulomb effects to be important is the polarization. Corresponding results are shown in Fig. 2 in the same manner as before for  $I_0$ . Let us begin the discussion again with the p-p curves. At  $E_{\text{Lab}} = 10$  MeV the polarization is governed by the Coulomb interaction, the influence of the strong potential is still rather weak; therefore we see the pronounced dip at small scattering angles. With increasing energy the strong interaction comes into play more and more, the polarization goes to positive values beginning from  $\frac{\pi}{2}$ , and the depth of the Coulomb dip is reduced.

Relative to the magnitude of  $P$  the CD effect for the p-p case is most important at 20 MeV, there coming up to about 30% of the polarization itself; at 50 MeV it still amounts to about 10%, while at the lower energy of  $E_{\text{Lab}} = 10$  MeV it shows up rather insignificantly. With respect to the latter energy point, however, a comparison with the n-n case tells us that the CD effect on the strong interaction is by no means unimportant also at this energy. For the n-n curves it again amounts to about 30% of  $P$ , only for the p-p case it is veiled by the predominant Coulomb interaction. A further examination of the n-n curves confirms a finding that we already obtained with  $I_0$ , namely that the CD effect weakens the action of the strong potential. For the polarization this fact is reflected by a reduction of its magnitude: when the CD effect is switched off, the polarization is raised by some multiplicative factor which is again quite independent of the scattering angle. This factor grows smaller with increasing energy (being about  $\frac{1}{3}$ ,  $\frac{1}{5}$ , and  $\frac{1}{10}$  at  $E_{\text{Lab}} = 10, 20$  resp. 50 MeV).

Let us further have a look at model differences. As is seen especially from the n-n curves all models yield similar results except for the GRAZ potential, which slightly deviates from PARIS and AHR again at 50 MeV. For the p-p curves the agreement with experimental data may be regarded as satisfactory for all models. We remark that most of the properties of the polarization, which we just discussed, may again be traced back to phase shifts, but this time of the P-wave states. They are

known to play quite an important role for the polarization, cf. e.g.<sup>33)</sup>. In particular the behaviour of the CD effect can be correlated to the  $\Delta$ 's of P-wave phase shifts, likewise the deviation of the GRAZ model is redetected there.

For further detailed studies especially of the influences of potential properties on the polarization we refer the reader to the recent paper by Bittner and Kretschmer<sup>34)</sup> and also by J.Coté et al.<sup>35)</sup>, where the properties of the PARIS potential with respect to the polarization are discussed also at  $E_{\text{Lab}} = 6.141$  and  $E_{\text{Lab}} = 16.0$  MeV. Let us now go on to the description of triple-scattering parameters.

c) Spin-correlation parameters  $C_{\text{NN}}$  and  $C_{\text{KP}}$  (Figs.3 and 4)

The observables  $C_{\text{NN}}$  and  $C_{\text{KP}}$  we decided to present in figures only for the most interesting cases. In particular Fig. 3 shows the parameter  $C_{\text{NN}}$  at  $E_{\text{Lab}} = 50$  MeV. Only at this energy the strong-interaction influence on  $C_{\text{NN}}$  shows up remarkably<sup>40)</sup>; at the lower energies of 10 resp. 20 MeV the shape of  $C_{\text{NN}}$  for p-p scattering is determined mainly by the pure Coulomb interaction (i.e. a pronounced peak at forward angles followed by a rather deep minimum at larger scattering angles), while for the n-n case it does not exhibit much structure. The reason for this behaviour lies in the fact that the spin-correlation parameter  $C_{\text{NN}}$  is predominantly governed by the tensor and spin-orbit ingredients of the strong interaction - a fact which is

quite manifested for the n-p system<sup>41)42)</sup>. For the p-p system, however, the tensor and spin-orbit parts come into play only at higher energies, a property, which, becomes evident by noticing that coupled partial waves occur only for higher angular momenta ( $^3P_2$ - $^3F_2$ ,  $^3F_4$ - $^3H_4$ , ...) and that their mixing is weak and grows only slowly with increasing energy (cf. the columns for  $\xi_2$  and  $\xi_4$  in Table 1.)

A comparison of the corresponding curves of Fig. 3. clearly shows that the CD effect is rather unimportant for  $C_{NN}$  in both the p-p and n-n systems. In particular it is smaller than model differences and also smaller than experimental errors at about that energy (cf. the compilation of N-N data by Bystricky and Lehar<sup>43)</sup>). Together with the arguments given before this latter finding, when traced back to phase parameters, also suggests that the Coulomb-distortion may be considered as weak in coupled p-p partial waves and especially in the appertaining mixing parameters.

The description of the CD effect just given for  $C_{NN}$  in more or less the same way also applies to the observable  $C_{KP}$  represented in Fig.4. But there we detect a remarkable model difference, namely, the GRAZ potential deviates considerably from AHR and PARIS especially at  $E_{Lab} = 10$  and 20 MeV. In order to explain this property we have drawn the n-n curves for  $C_{KP}$  at all three energies (Figs.4.b-d). It is seen that the results for GRAZ begin with negative values (10 MeV), go to positive ones, which are still too small at 20 MeV, and finally (at 50 MeV) exceed the AHR resp. PARIS values. This

behaviour can be traced back to the phase shifts of particularly the P-waves. While the GRAZ potential is too less repulsive (too much attractive) in the  ${}^3P_1$  ( ${}^3P_0$ ) state at  $E_{\text{Lab}} = 10$  and  $20$  MeV it turns out to be too much repulsive (too less attractive) in  ${}^3P_1$  ( ${}^3P_0$ ) at  $E_{\text{Lab}} = 50$  MeV. Thus we can argue that  $C_{\text{KP}}$  is rather sensitive to P-waves.

The action of the pure Coulomb interaction on  $C_{\text{KP}}$  is seen from Fig. 4.a. Its influence simply consists in preventing the curves from rising at forward scattering angles, this effect being less pronounced at the higher energies thus indicating the strengthening of the nuclear interaction with increasing energy.

d) Spin-transfer coefficient  $A$  resp.  $A_{\text{T}}$  (Fig. 5)

Though we have calculated three observables of that type, namely  $A, R,$  and  $D$  (resp.  $A_{\text{T}}, R_{\text{T}},$  and  $D_{\text{T}}$ ), at all of the three energies, we decided to present only the parameter  $A$  at  $E_{\text{Lab}} = 50$  MeV; the reason being that all observables of that type practically revealed the same behaviour. Above all the CD effect turned out to be unimportant for both the p-p and n-n systems at all energy points considered. In Figs. 5a and b the corresponding curves are given for  $E_{\text{Lab}} = 50$  MeV. Only at this energy the nuclear potential gets strong enough to force back the influence of the pure Coulomb potential for the p-p case; only at the same energy the observable  $A$  exhibits some structure for the n-n case. At the lower energies the pure Coulomb influence becomes predominant, thus leading to broader

minima of  $A$  at forward and backward scattering angles.

Note that because of the symmetry relation between  $A$  and  $A_T$  the latter quantity can be deduced from the former one by reversing the angular dependence ( cf. Fig.5). The same is true for  $D$  and  $R$  as well.

All model predictions for  $A$  excellently agree with the experimental data. Differences between the models themselves also appear to be insignificant, thus suggesting that spin-transfer coefficients are rather insensitive to details of the nuclear potential at least in the energy range considered here.

#### IV. Conclusion

The discussion of the foregoing chapter clearly revealed the role of the CD effect both in full p-p observables as well as in Coulomb-subtracted nuclear observables. We noticed its importance particularly with respect to the differential cross section  $I_0$  and also the polarization  $P$ . Though the effect is predominant at the lower energies this statement holds for all three energy points considered, i.e. at  $E_{\text{Lab}} = 10, 20, \text{ and } 50 \text{ MeV}$ . The magnitude of the influence of the Coulomb distortion considerably exceeded model differences and also experimental errors (of nowadays feasible p-p experiments). The CD effect turned out to be rather insignificant for more complicated observables like  $C_{NN}$ ,  $C_{KP}$ , and  $A(A_T)$  and also further ones which



were not presented. Nevertheless our results impressively suggest the CD effect to be taken into account in combined p-p and n-p phase-shift analyses. For such an attempt the approximation eq.(3.1) for  $\Delta_L$  may be used; in our study it worked quite well in handling the AHR case.

Slight model differences appeared in about all observables that we considered especially with respect to the GRAZ potential. A comparison of the results of the two potential models with the experimental data favours the PARIS potential; it predicts all observables with satisfactory accuracy.

The various results we quoted for the n-n system will probably not before long be tested by experiment. Hitherto only a few measurements of n-n differential cross sections were performed (at higher energies). Under these circumstances our n-n curves may seem somewhat academic. But they can serve as a measure for the required accuracy in n-n experiments, which will possibly be undertaken in order to decide on the question of charge symmetry on the level of scattering observables; in particular the experimental errors would have to fall below the magnitude of the CD effect in Coulomb-subtracted observables.

Ack

The

com

pri

for

in

**Acknowledgement**

The authors thank Prof. R. Vinh Mau and his group for communicating to them the data of the PARIS potential prior to publication. They are grateful to Prof.H.F.K.Zingl for many encouraging discussions and his constant interest in their work.

is

ly

ed

es

bles.

## Figure Captions

Fig. 1. Differential cross sections  $I_0$  for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. III.1. is as follows:

—————	AHR	} case 1) for p-p and case iii) for n-n, i.e. with the CD effect included
—————	GRAZ	
— . — .	PARIS	
.....	AHR	} case ii) for p-p and case iv) for n-n, i.e. with the CD effect excluded.
-----	GRAZ	
-----	PARIS	

Fig.

For the differential cross section the symmetry relation  $I_0(\pi - \theta) = I_0(\theta)$  holds. Experimental data at 10, 20 and 50 MeV were taken from refs.<sup>30)-32)</sup>; they refer particularly to energies  $E_{\text{Lab}} = 9.918 \pm 0.020$  MeV,  $E_{\text{Lab}} = 19.8$  MeV, and  $E_{\text{Lab}} = 49.41 \pm 0.06$  MeV, respectively.

Fig.

In Fig. 1.c the curves for PARIS practically coincide with those of AHR.

Fig. 2. Polarization  $P$  for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec. III.1. is as in Fig. 1. For the polarization the symmetry relation  $P(\pi - \theta) = -P(\theta)$  holds. Experimental data at  $E_{\text{Lab}} = 10, 20$ , and 50 MeV were taken from refs.<sup>36)-39)</sup>; they refer particularly to

Fig.

energies  $E_{\text{Lab}} = 10.00 \pm 0.05$  MeV,  $E_{\text{Lab}} = 20.2 \pm 0.02$  MeV,  $E_{\text{Lab}} = 49.9 \pm 0.1$  MeV, and  $E_{\text{Lab}} = 52.34$  MeV, respectively. In Figs. 2.a and b case-ii) curves are only given for AHR, since the influence of the Coulomb-distortion on the p-p polarization is analogous and very similar in shape for the other models. In Fig.2.a the n-n curves for PARIS practically coincide with those of AHR.

**Fig. 3.** Spin-correlation parameter  $C_{\text{NN}}$  for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec.III.1. is as in Fig. 1. For  $C_{\text{NN}}$  the symmetry relation  $C_{\text{NN}}(\pi-\theta) = C_{\text{NN}}(\theta)$  holds. The experimental datum was taken from ref.<sup>44</sup>; it refers particularly to the energy  $E_{\text{Lab}} = 52.3$  MeV.

**Fig. 4.** Spin-correlation parameter  $C_{\text{KP}}$  for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation to the case studies of Sec.III.1. is as in Fig.1. For  $C_{\text{KP}}$  the symmetry relation  $C_{\text{KP}}(\pi-\theta) = C_{\text{KP}}(\theta)$  holds.

**Fig. 5.** Spin-transfer coefficients  $A$  and  $A_T$  for p-p (with) and n-n scattering (without inclusion of the pure Coulomb interaction). The description of the curves in relation

to the case studies of Sec.III.1. is as in Fig. 6. For  $A$  and  $A_T$  the symmetry relation  $A_T(\pi-\theta) = A(\theta)$  holds. Therefore the angular dependence of  $A_T$  has to be read off from the upper scale in the figures. Experimental data were taken from ref.<sup>45)</sup>. They refer particularly to the energy  $47.5 \pm 2.5$  MeV. In Fig.5.a the curves for PARIS practically coincide with those of AHR.

- Fig. 6. Schematic representation of N-N observables, which are considered in detail in the present paper. Circles mark spin orientations normal to the scattering plane, whereas arrows correspond to spin directions lying in the scattering plane:
- a) differential cross section  $I_0$
  - b) polarization  $P$
  - c) spin-correlation parameters  $C_{NN}$  and  $C_{KP}$
  - d) spin-transfer coefficients  $A$  and  $A_T$

**Table Caption**

Partial-wave states and phase parameters (in degrees) used for the calculation of observables according to the various procedures described in Sec.III.1. The states  ${}^3H_5$  and  ${}^3H_6$  as well as higher partial waves were neglected.

- a) The purely nuclear phase shifts  $\delta^S$  for the AHR-case were deduced as explained in Sec.III.1., namely, by employing the approximation formula (3.1).
- b) Phase parameters for partial-wave states not covered by the GRAZ potential were substituted by the corresponding AHR data.

References and Footnotes

- 1) M.H.MacGregor, R.A.Arndt, and R.M.Wright, Phys.Rev. 182, 1714 (1969).
- 2) R.A. Arndt, R.H.Hackman, and L.D.Roper, Phys.Rev. C9, 555 (1974).  
R.A.Arndt, R.H.Hackman, and L.D.Roper, Phys.Rev. C15, 1002 (1977).
- 3) D.V.Bugg, J.A.Edgington, W.R.Gibson, N.Wright, N.M.Stewart, A.S. Clough, D.Axen, G.A.Ludgate, C.J.Oram, L.P.Robertson, J.R.Richardson, and C.Amsler, Rutherford Lab.Preprint RL-79-023 (1979); Phys.Rev.C, to be published.
- 4) R.Bryan, R.B.Clark, and B.VerWest, Phys.Rev. C18, 371 (1978).  
R.Bryan, R.B.Clark, and B.VerWest, Phys.Lett. 74B, 321(1978).  
R.Bryan, in Few-Body Systems and Nuclear Forces II (Lecture Notes in Physics Vol.87) ed.by H.Zingl, M.Haftel, and H.Zankel (Springer-Verlag, Berlin-Heidelberg-New York, 1978), p.2.
- 5) N.Hoshizaki, Kyoto Preprint NEAP-18 (1978).
- 6) J.Bystricki, C.Lechanoine, and F.Lehar, CEN-Saclay Preprint DPhPE 79-01 (1979).
- 7) E.M.Henley, in Isospin in Nuclear Physics ed. by D.H. Wilkinson (North-Holland Publishing Company, Amsterdam, 1969), p.14.
- 8) M.S.Sher, P.Signell, and L.Heller, Ann. Phys. 58, 1 (1970)
- 9) J.Hamilton, Fortschr. Physik 23, 211 (1975)
- 10) A similar study of electromagnetic effects was carried out on the n-p system by Gersten: A.Gersten, Phys. Rev. C18, 2252 (1978).
- 11) R.Vinh Mau, in Mesons in Nuclei, Vol.I ed. by M.Rho and D.Wilkinson (North-Holland Publishing Company, Amsterdam-New York - Oxford, 1979), p.151.
- 12) L.Crepinsek, H.Oberhammer, W.Plessas, and H.F.K.Zingl, Acta Physica Austr. 39, 345 (1974).  
L.Crepinsek, C.B.Lang, H.Oberhammer, W.Plessas, and H.F.K.Zingl, Acta Physica Austriaca 42, 139 (1975):

13) L.  
14) M.  
Nuc  
15) N.  
16) G.  
Ed  
P.  
17) J.  
1  
18) A.  
Pu  
19) We  
me  
Ha  
ca  
wa  
in  
ex  
th  
di  
20) L.  
21) C.  
Pu  
22) In  
th  
al  
Th  
mo  
23) H.  
30  
24) M.  
Fu  
25) M.  
Ox

- 13) L. Wolfenstein, *Ann. Rev. Nucl. Sci.* 6, 43 (1956).
- 14) M.H. MacGregor, M.J. Moravcsik, and H.P. Stapp, *Ann. Rev. Nucl. Sci.* 10, 291 (1960).
- 15) N. Hoshizaki, *Progr. Theor. Phys. Suppl.* 42, 107 (1968).
- 16) G. Breit and R.D. Haracz, in High-Energy Physics Vol. I, Ed. E.H.S. Burhop (Academic Press, New York-London, 1967), p.21.
- 17) J. Bystricky, F. Lehar, and P. Winternitz, *J. Physique* 39, 1 (1978).
- 18) A. Messiah, Quantum Mechanics, Vol. I (North-Holland Publishing Company, Amsterdam, 1961).
- 19) We remark that in the context of a rigorous quantum-mechanical treatment of the Coulomb interaction H. van Haeringen just recently pointed out the difficulties caused by such a splitting with respect to a partial-wave decomposition: *Nuovo Cimento* 34B, 163 (1976). Since in our approach we will be able to avoid a partial-wave expansion of the pure-Coulomb scattering amplitude throughout, we will not be encountered in these difficulties.
- 20) L. Wolfenstein and J. Ashkin, *Phys. Rev.* 85, 947 (1952).
- 21) C.J. Joachain, Quantum Collision Theory (North-Holland Publishing Company, Amsterdam, 1975).
- 22) In eq. (2.2) the dependence of M on  $\{k, \theta, \varphi\}$  stems from the fact that as usual the quantization axis was chosen along the direction of motion of the incoming particle. Therefore we have for the initial and final relative momenta  $\vec{k}_i = (k, 0, 0)$ ,  $\vec{k}_f = (k, \theta, \varphi)$ .
- 23) H.P. Stapp, T.J. Ypsilantis, and N. Metropolis, *Phys. Rev.* 105, 302 (1957).
- 24) M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions (Dover Publications Inc., New York, 1972), p.332.
- 25) M.J. Moravcsik, The Two-Nucleon Interaction (Clarendon Press, Oxford, 1963).



- 26) W.Plessas, L.Streit, and H.Zingl, Acta Physica Austr. 40, 272 (1974).
- 27) L.Streit, J.Fröhlich, H.Zankel, and H.Zingl, J.Phys.G to be published.
- 28) Coupling itself is already weak for p-p states, i.e. for  $^3P_2$ - $^3F_2$  and  $^3F_4$ - $^3H_4$ , especially in the energy range we were interested in. Often it is justified to neglect it at all, as is frequently done in the literature. However, we were not content with this attitude, but rather made a separate study of the possible influence of the Coulomb-distortion effect in mixing parameters. For this purpose we examined the observable  $C_{NN}$ , which is regarded to be most sensitive on  $\epsilon_2$  and  $\epsilon_4$ , (cf. also the discussion in Subsec.III.2.c) under varying conditions, each time attaching a different (arbitrary) Coulomb effect (within some reasonable margins) to these phase parameters. The changes in  $C_{NN}$  turned out to be insignificant.
- 29) For this reason we have chosen the label "n-n", though these curves rather represent Coulomb-subtracted nuclear observables. Only the case iv) curves are to be considered as n-n observables (under the assumption that all Coulomb effects other than the CD effect can be neglected).
- 30) N.Jarmie, J.H.Lett, J.L. Detch, Jr., and R.L.Hutson, Phys. Rev. Lett. 25, 34 (1970).
- 31) J.W.Burkig, J.R.Richardson, and G.E.Schrank, Phys. Rev. 113, 290 (1959).
- 32) C.J. Batty, T.C.Griffith, D.C.Imrie, G.J. Lush, and L.A. Robbins, Nucl. Phys. A98, 489 (1967).
- 33) F.Pauß, Dissertation, Univ. Graz (1975), unpublished.
- 34) G.Bittner and W.Kretschmer, Phys.Rev.Lett. 43, 330 (1979).
- 35) J.Coté, P.Pirès, R.de Turreil, M.Lacombe, B.Loiseau, and R.Vinh Mau, IPN Orsay-Preprint TH 80-01 (1979).

- 36) J.D. Hutton, W.Haeberli, L.D. Knutson, and P.Signell, Phys. Rev.Lett. 35, 429 (1975) and also Contributed Paper A6 to the IV. International Symposium on Polarization Phenomena in Nuclear Reactions, Zurich, 1975, ed. by W.Gruebler and V.König (Birkhäuser Verlag, Basel, 1976).
- 37) P.Catillon, J.Sura, and A.Tarrats, Phys. Rev. Lett. 20, 602 (1968).
- 38) C.J.Betty, R.S.Gilmore, and G.H.Stafford, Nucl.Phys. 45, 481 (1963).
- 39) N.Tamura et al., in Proc. Int. Conf. Nuclear Structure, Tokyo, 1977, J.Phys. Soc. Japan 44, 289 (1978).
- 40) A further reason, why we omitted a representation of  $C_{NN}$  at the other energies, lies in the fact that there no experimental data are available for a comparison.
- 41) J.Binstock and R.Bryan, Phys.Rev. D9, 2528 (1974).
- 42) F.Pauß and H.F.K.Zingl, Phys.Rev. C15, 1231 (1977).
- 43) J.Bystricky and F.Lehar, Nucleon-Nucleon Scattering Data I, II; Physics Data 11-1 (Fachinformationszentrum Karlsruhe, Karlsruhe, 1978).
- 44) K.Nisimura, J.Sanada, P.Catillon, K.Fukunaga, T.Hasegawa, H.Hasai, N.Ryn, D.C.Worth, and H.Imada, Progr. Theor. Phys. 30, 719 (1963).
- 45) A.Ashmore, M.Devine, B.Hird, J.Litt, W.H.Range, M.E. Shepherd, and R.L.Clark, Nucl. Phys.65, 305 (1965).

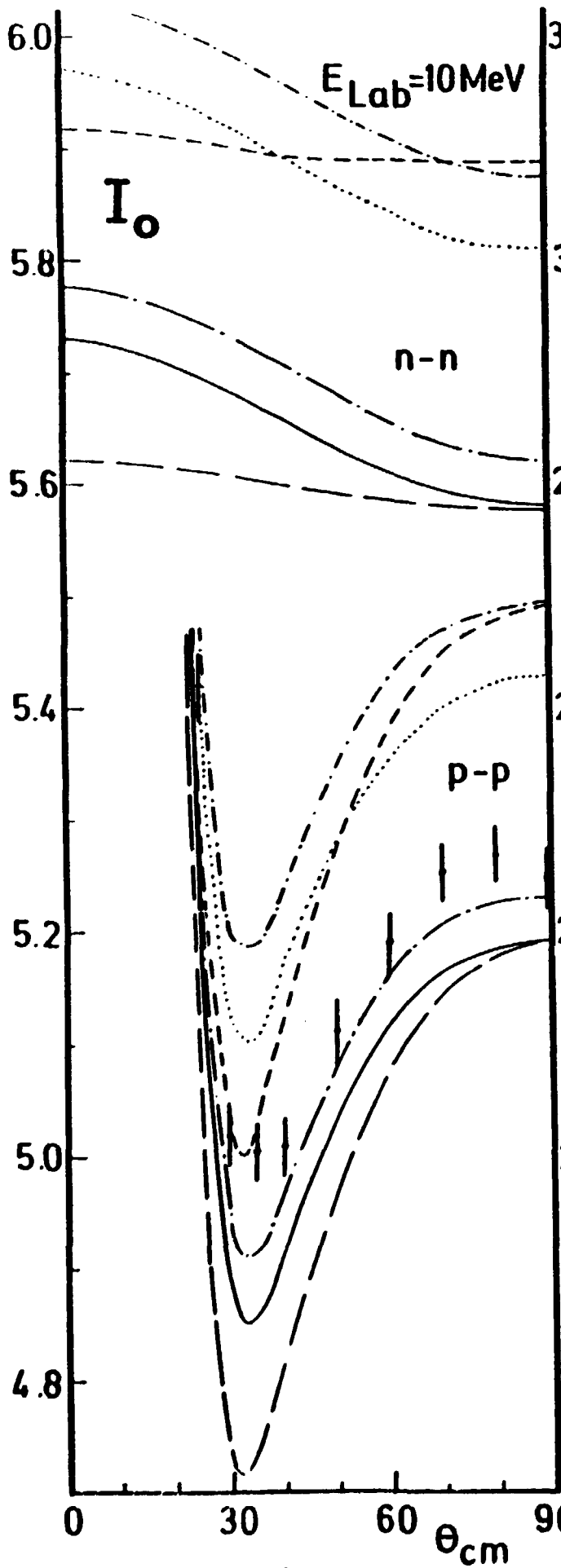


Fig. 1.a

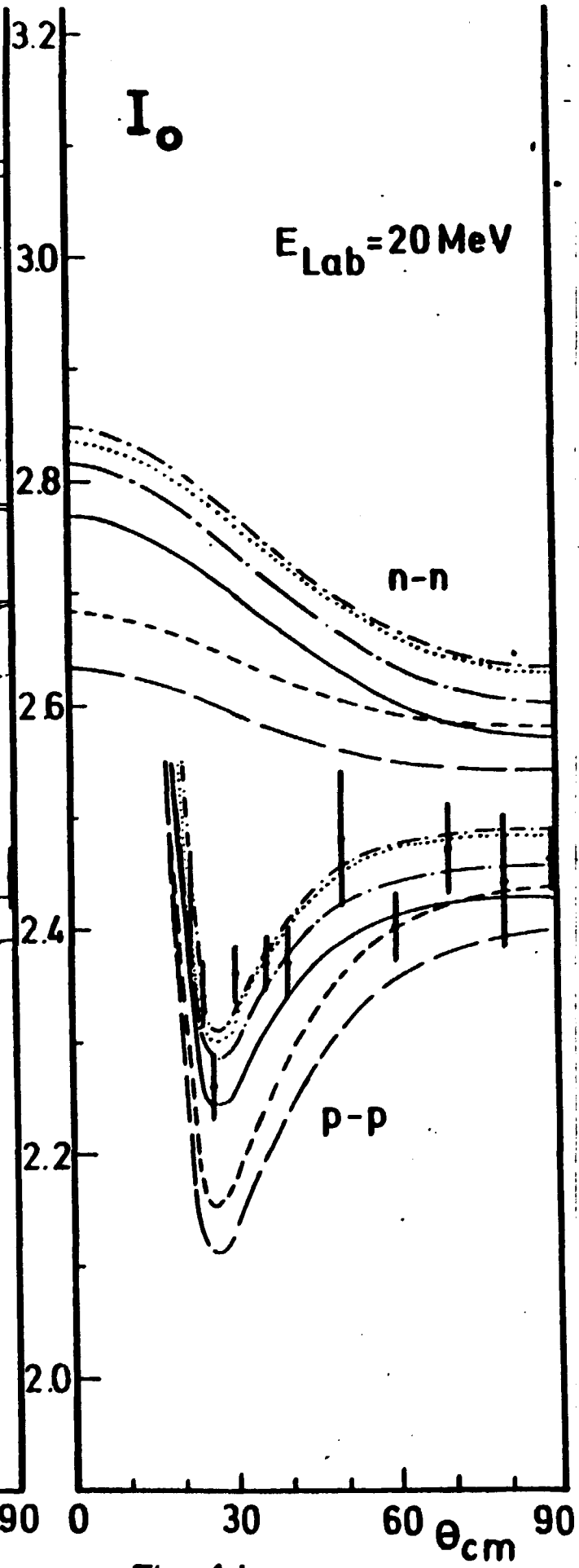


Fig. 1.b

TeV

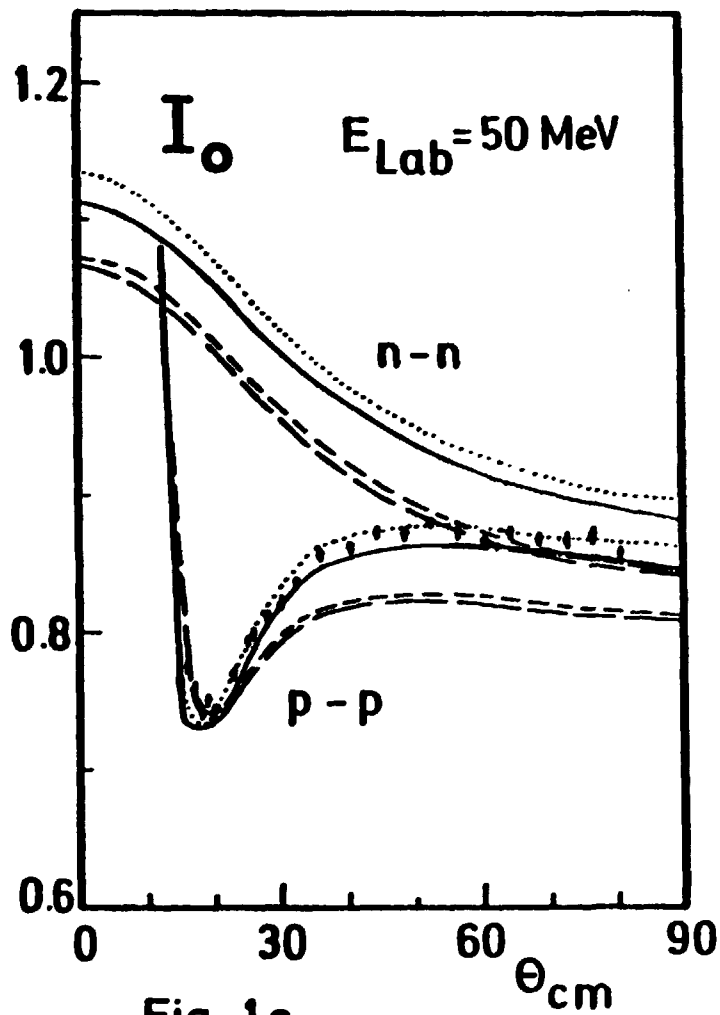


Fig. 1.c

$\theta_{cm}$  90

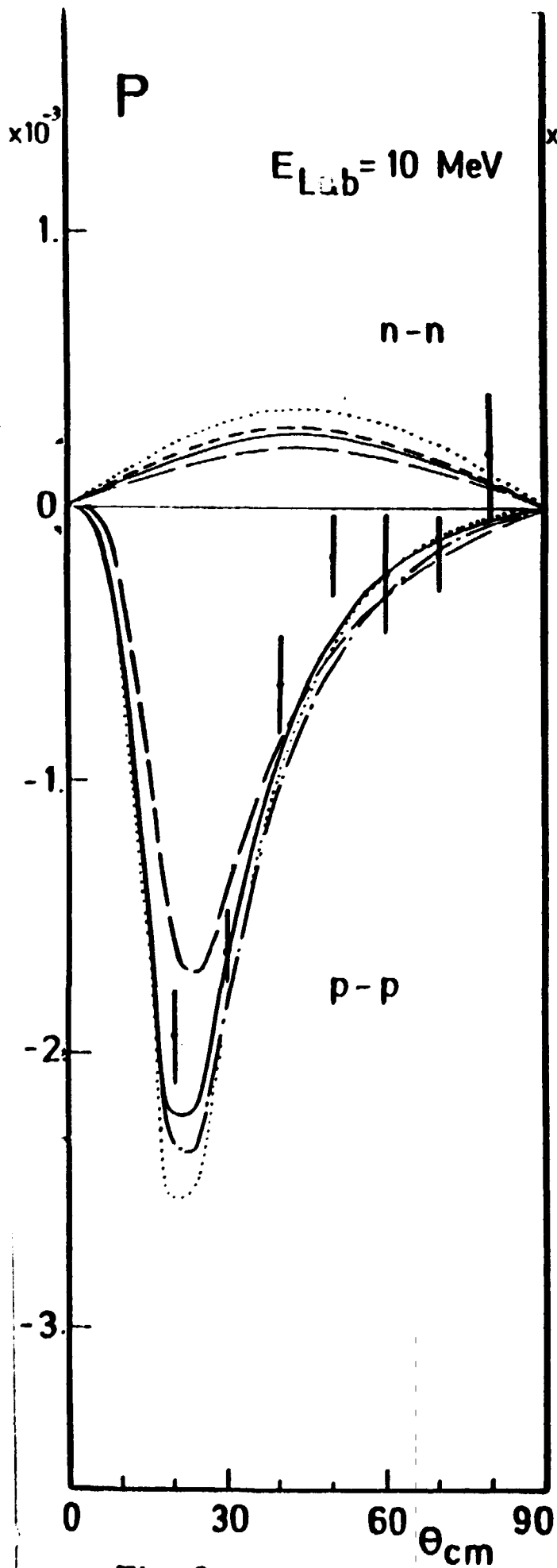


Fig. 2 a

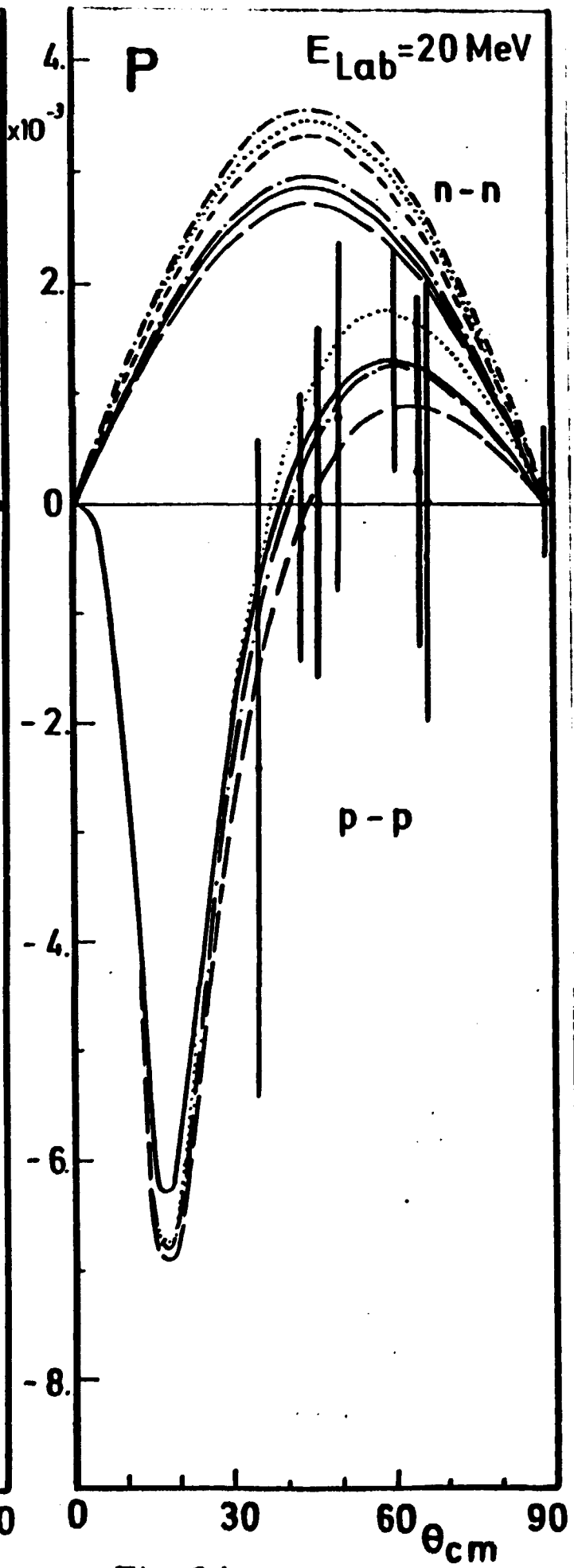
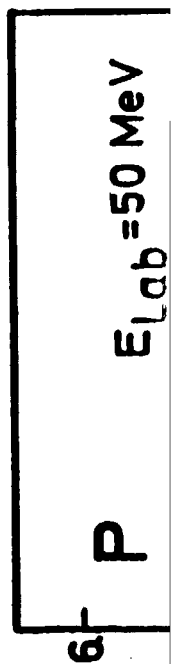
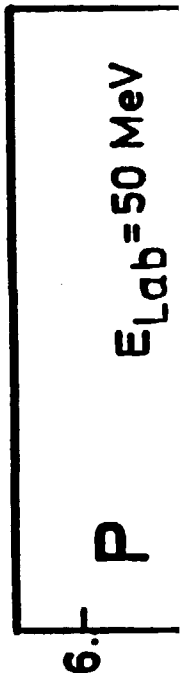


Fig. 2 b



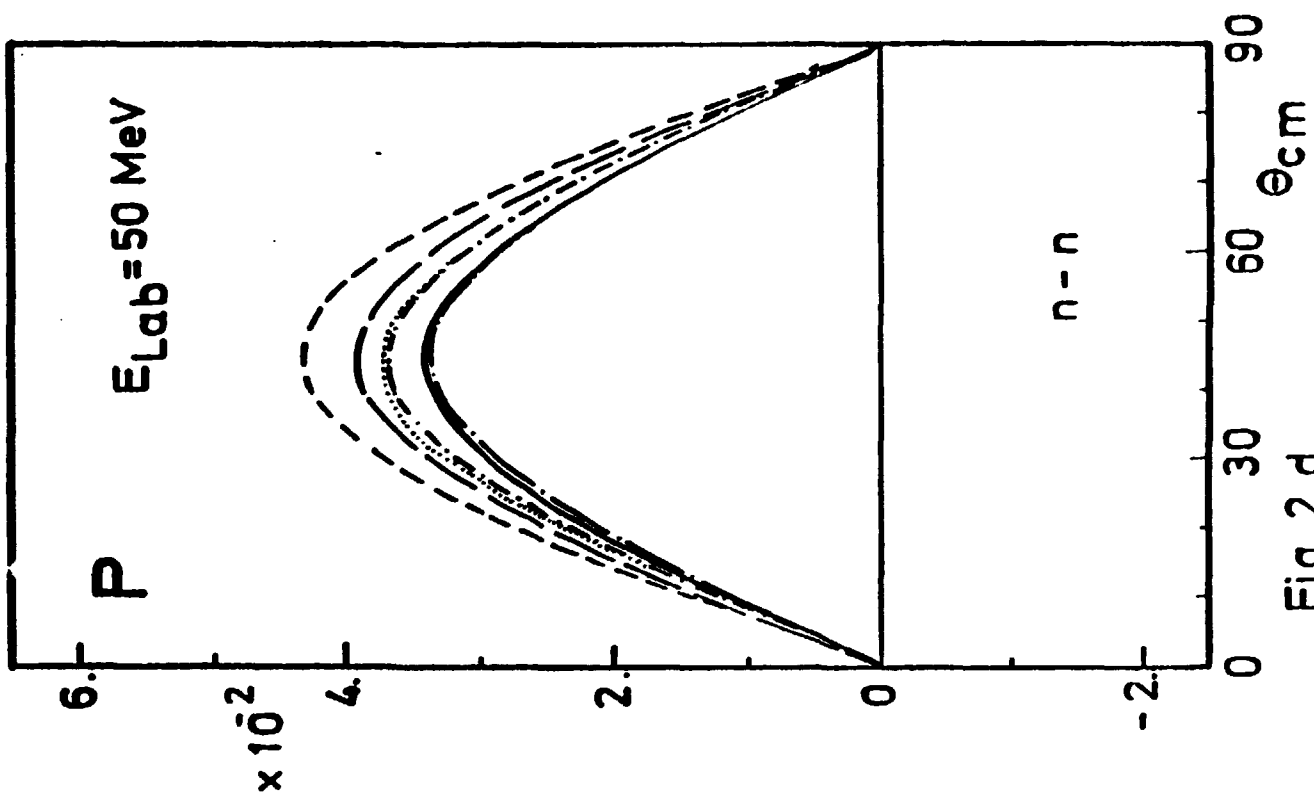
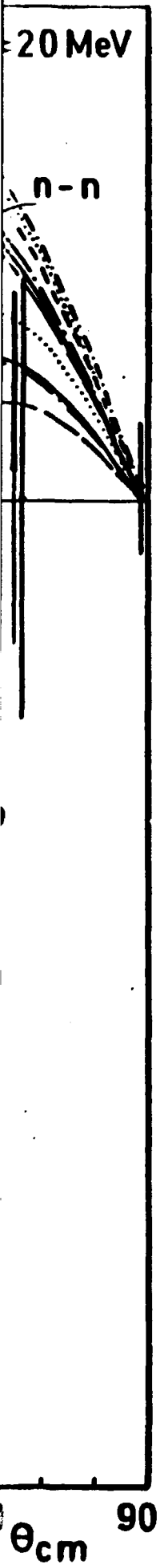


Fig. 2.d

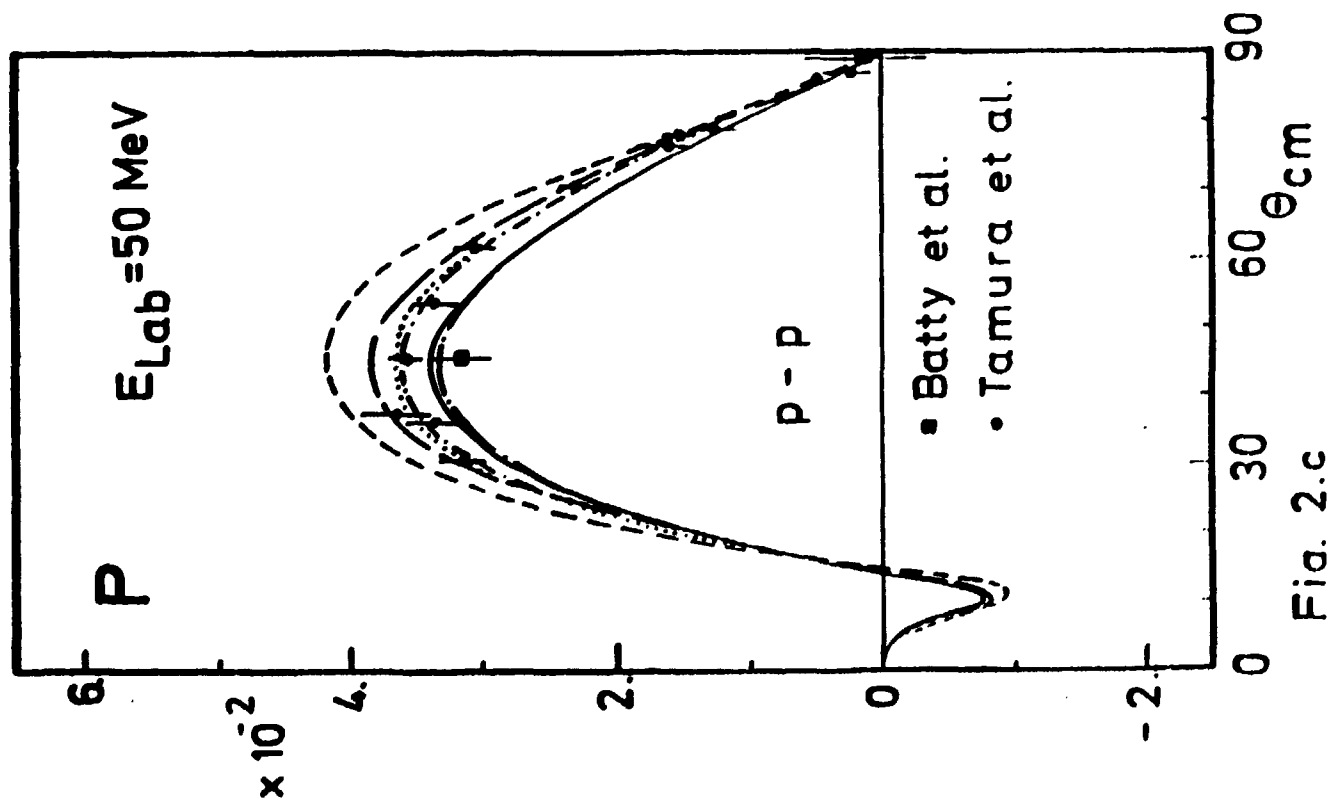


Fig. 2.c

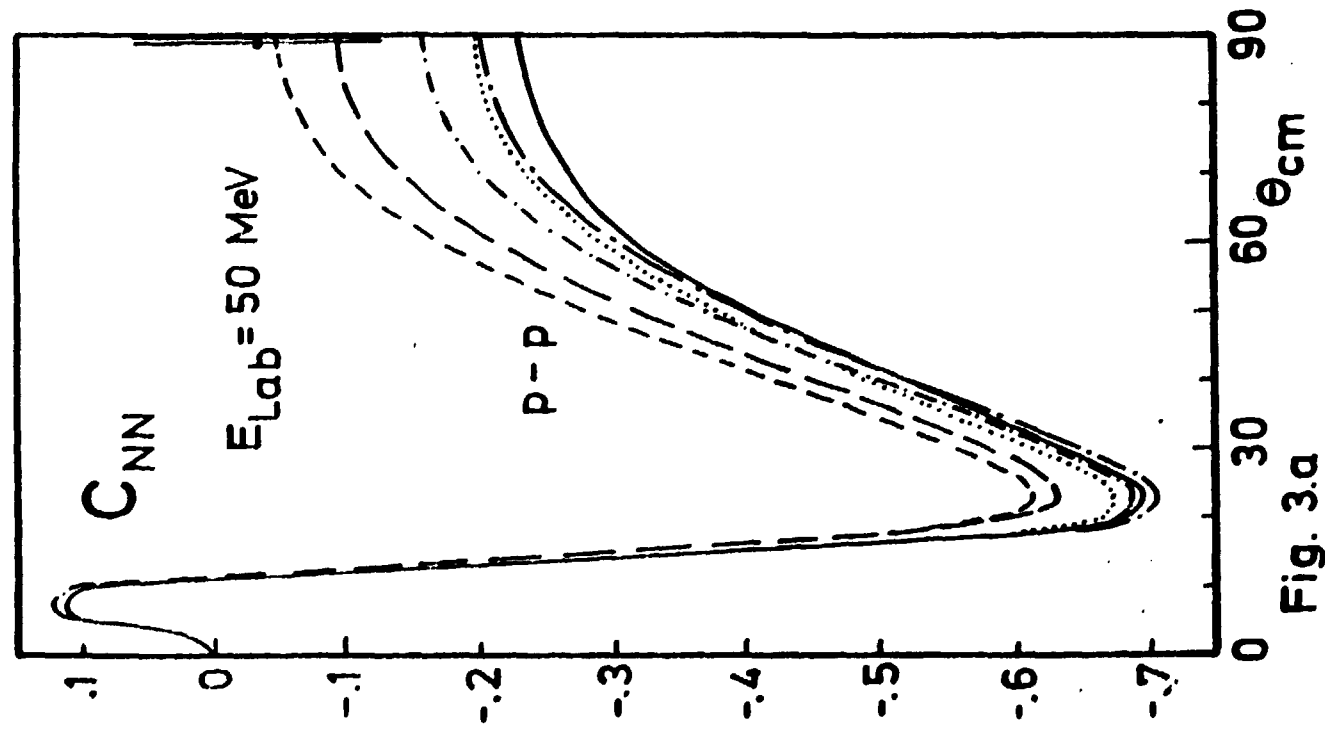


Fig. 3.a

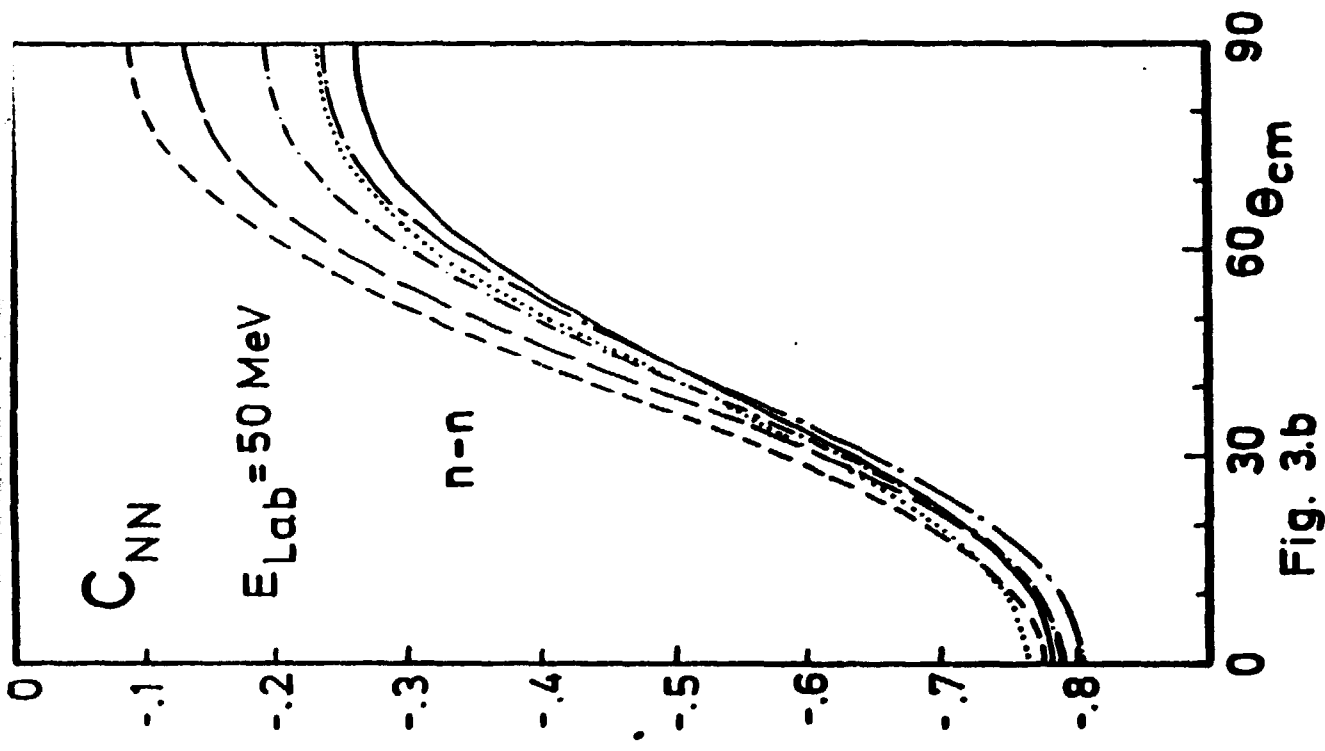


Fig. 3.b

6-  $C_{KP}$   $E = 10 \text{ MeV}$

6-  $C_{KP}$   $E = 10 \text{ MeV}$

Fig. 3.a

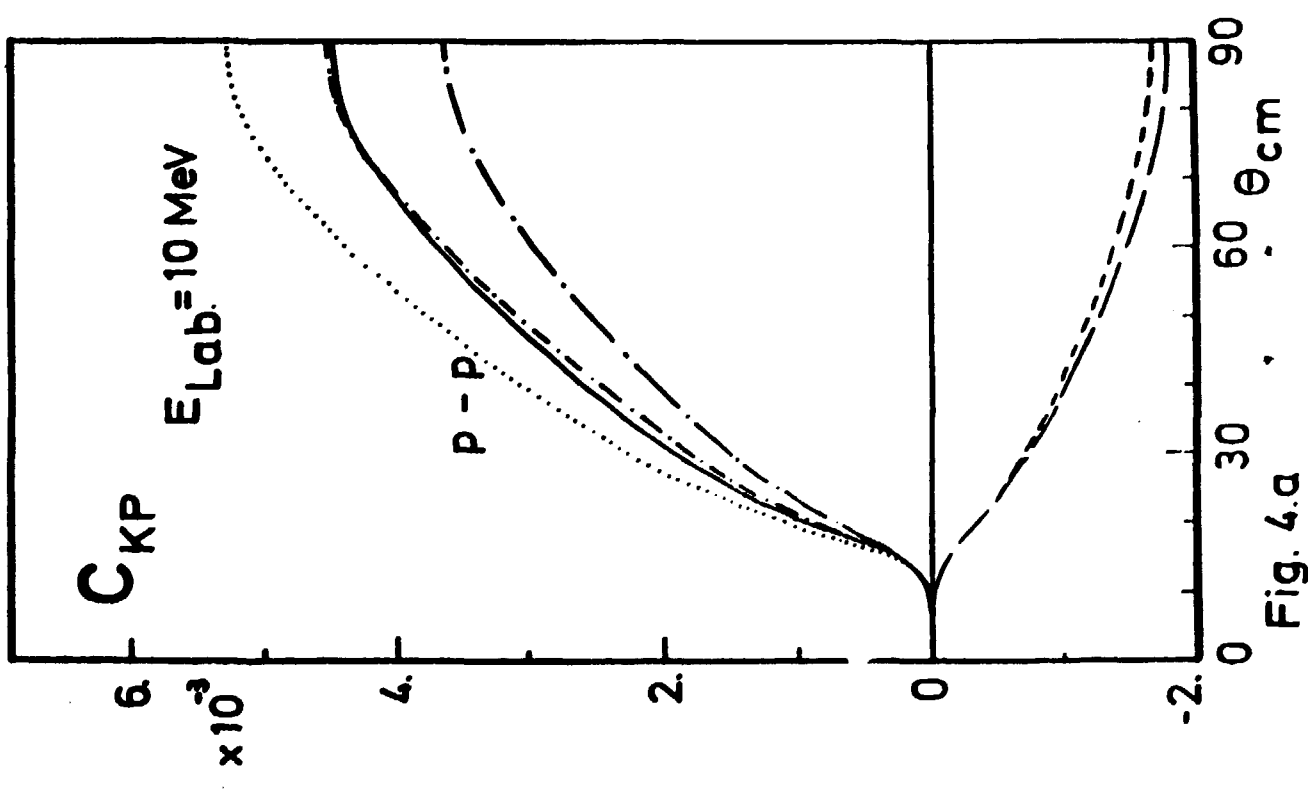


Fig. 3.b

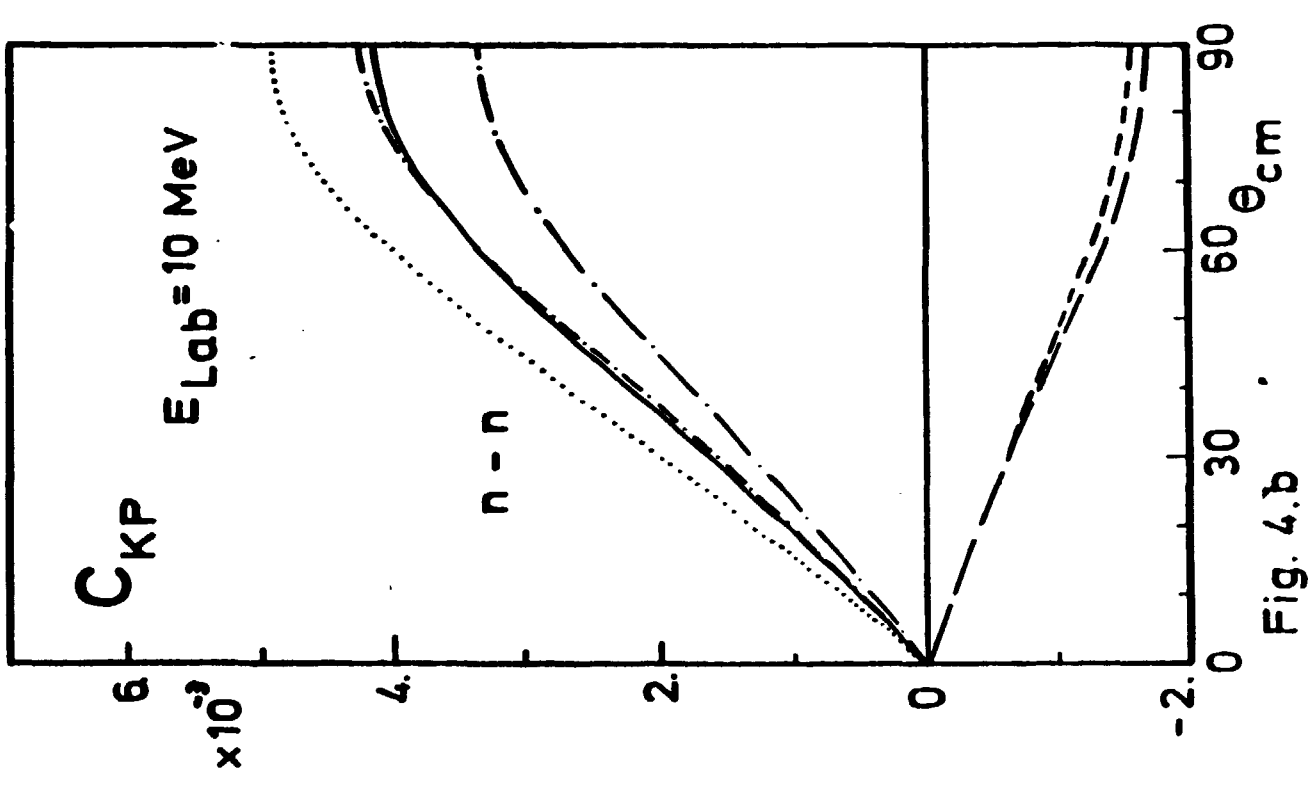


Fig. 4.a

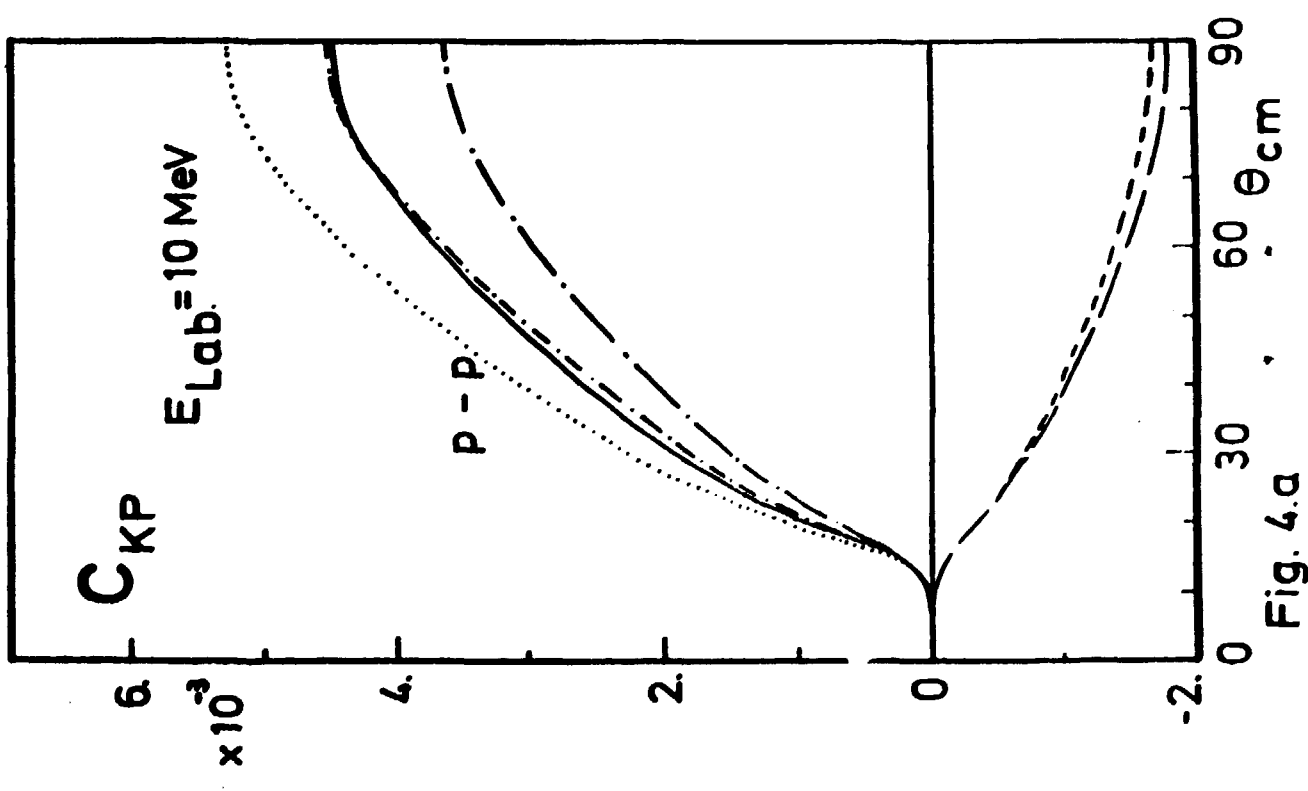
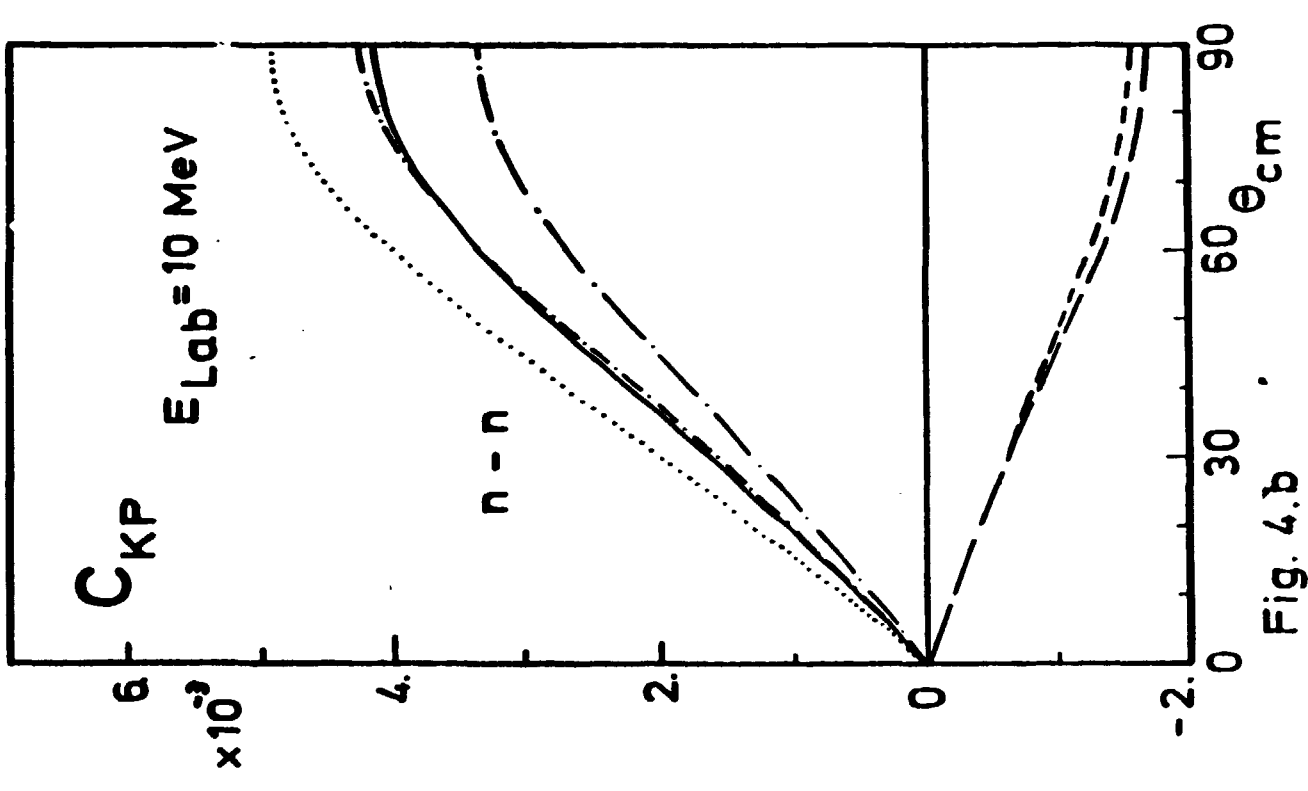


Fig. 4.b





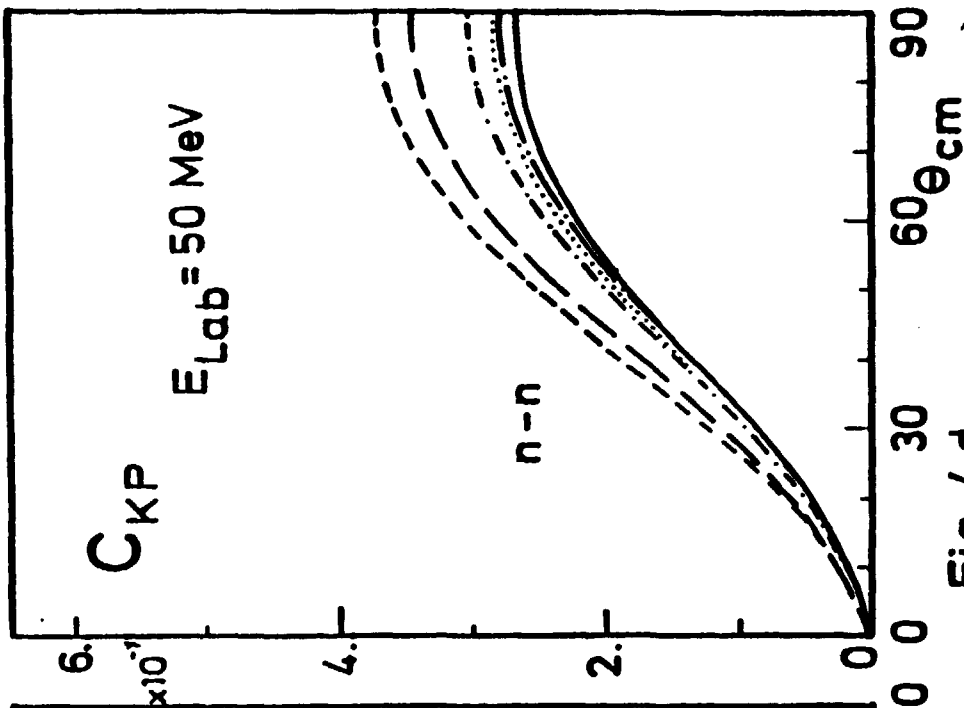


Fig. 4.d

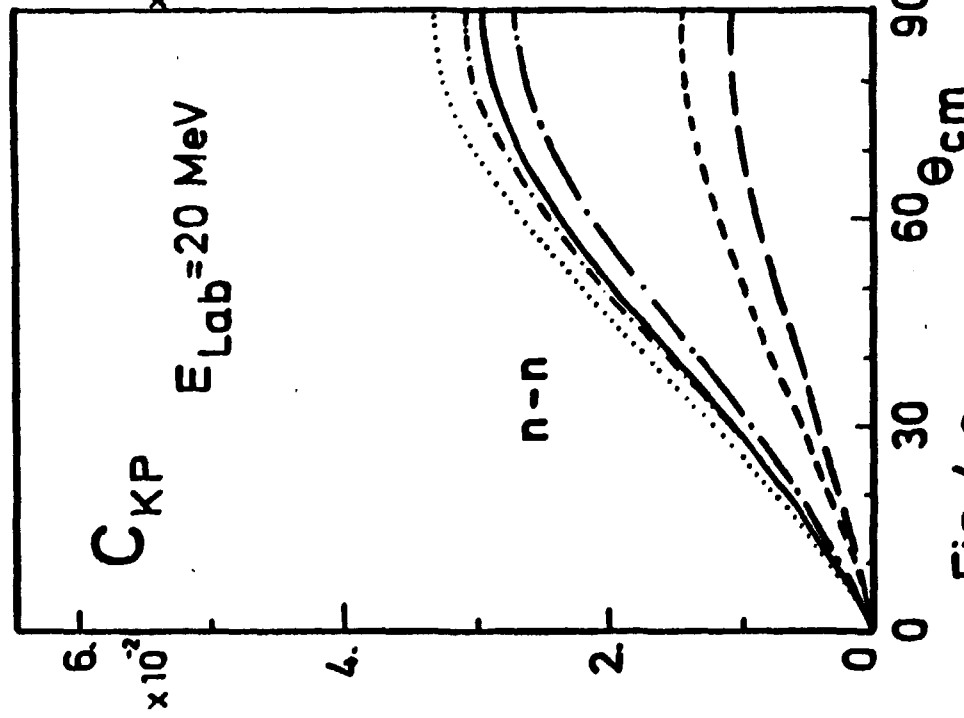
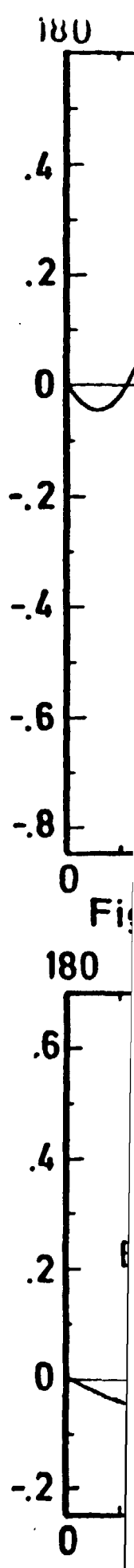
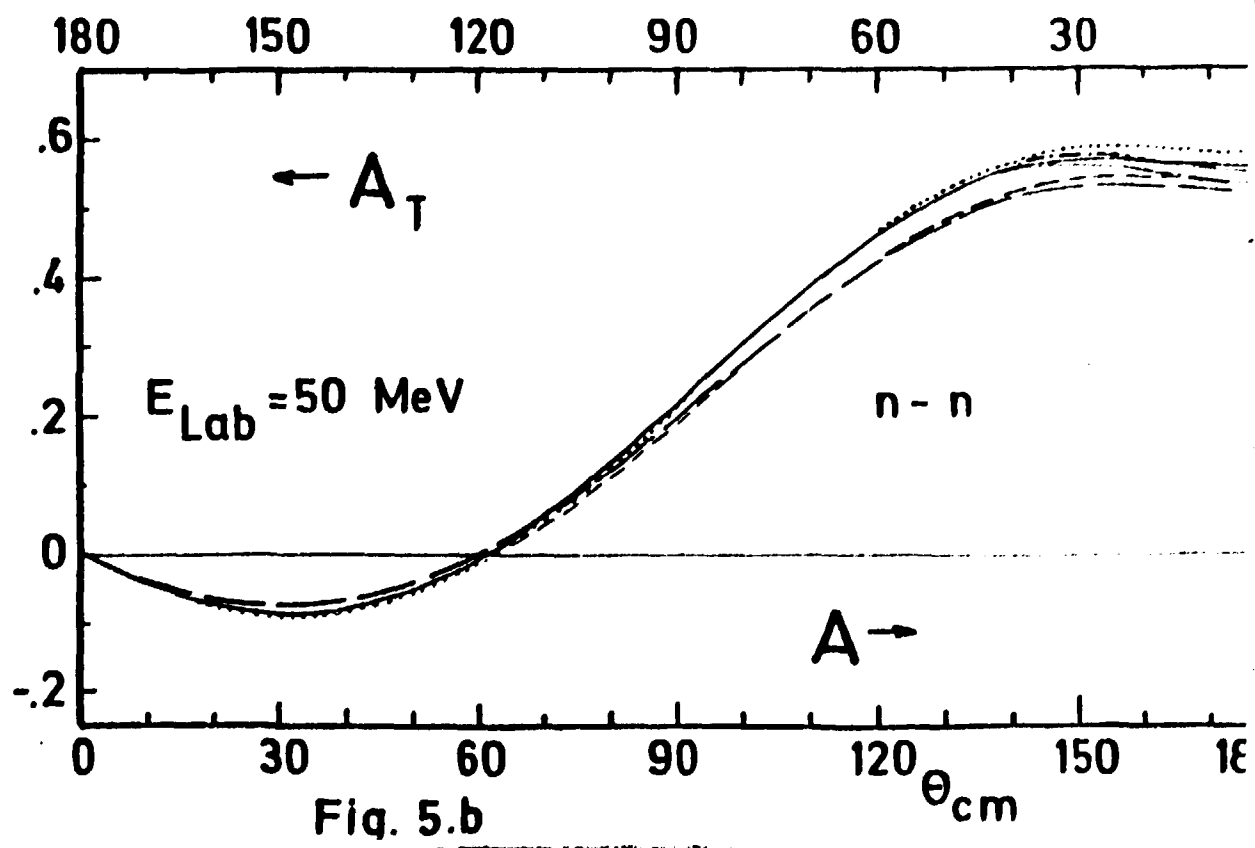
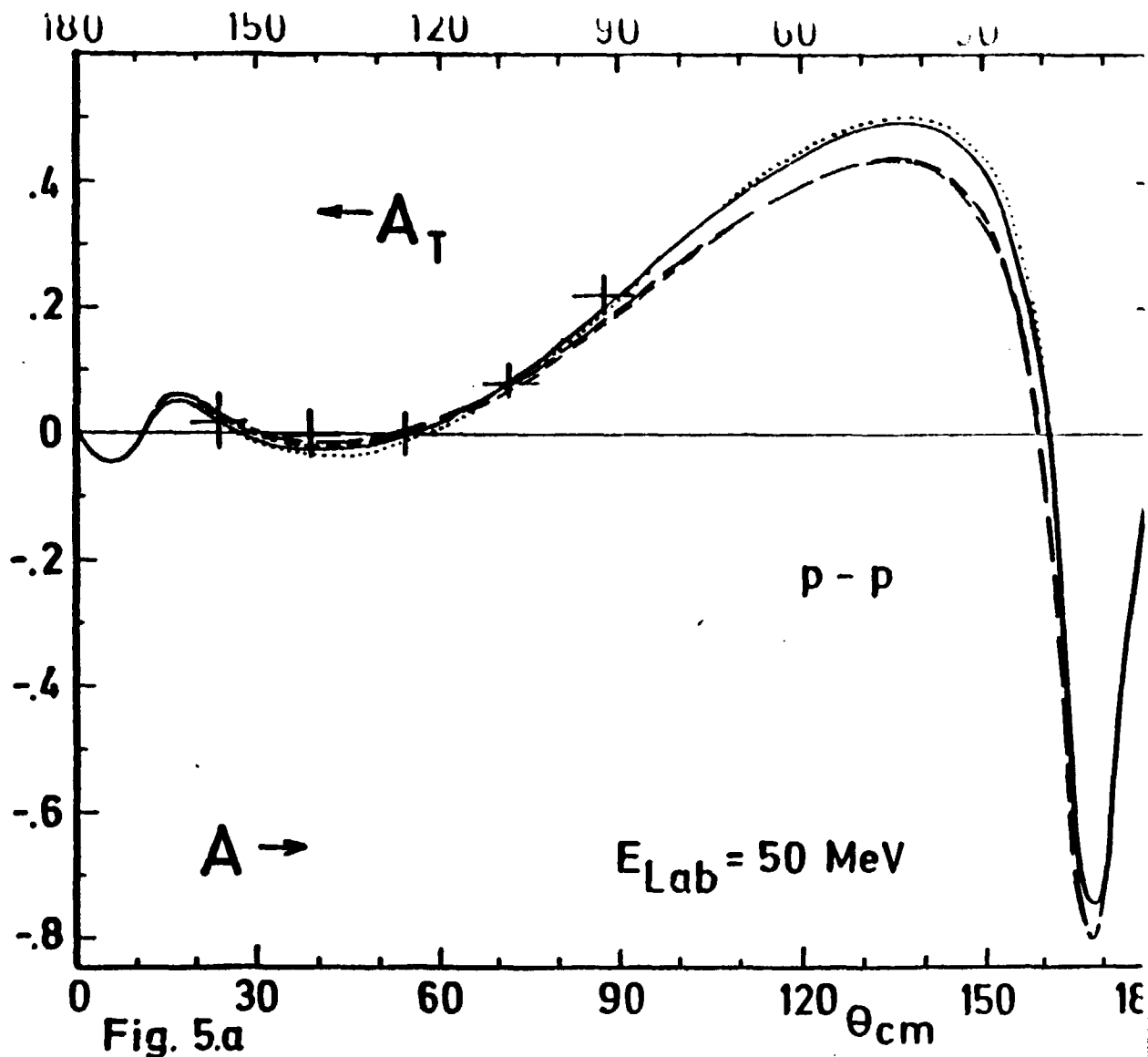


Fig. 4.c





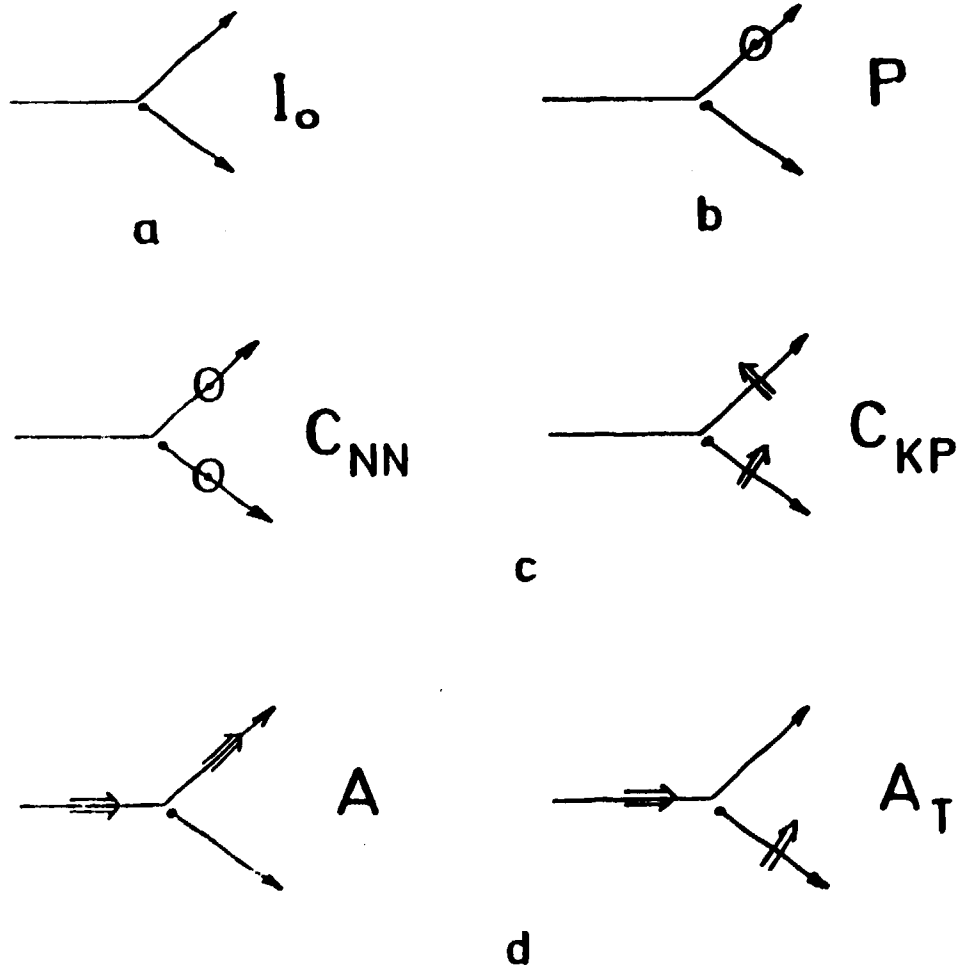


Fig. 6

Energy	Model	Phase	$^1S_0$	$^1D_2$	$^1G_4$	$^3P_1$	$^3F_3$	$^3P_0$	$^3P_2$	$\epsilon_2$	$^3F_2$	$^3F_4$	$\epsilon_4$	$^3H_4$
$E_{\text{Lab}} = 10 \text{ MeV}$	AHR	$\delta^{SC}$	54.91	0.1656	0.00353	-2.056	-0.03247	3.614	0.6753	-0.1984	0.01275	0.002039	-0.004071	0.000167
		$\delta^{sa}$	56.51	0.1803	0.00381	-2.265	-0.03520	3.984	0.7605	-0.1984	0.01385	0.002257	-0.004071	0.000180
	PARIS	$\delta^{SC}$	55.18	0.18	0.0	-2.12	-0.03	3.89	0.64	-0.21	0.01	0.0	0.0	0.0
		$\delta^S$	56.95	0.19	0.0	-2.31	-0.04	4.26	0.74	-0.23	0.02	0.0	0.0	0.0
	GRAZ	$\delta^{SC}$	54.71	0.05994	b)	-1.408	b)	4.066	0.5805	b)	b)	b)	b)	b)
		$\delta^S$	56.78	0.06781	b)	-1.579	b)	4.482	0.6652	b)	b)	b)	b)	b)
$E_{\text{Lab}} = 20 \text{ MeV}$	AHR	$\delta^{SC}$	50.51	0.5043	0.02447	-4.077	-0.1513	7.090	1.850	-0.5914	0.06358	0.01663	-0.02863	0.002028
		$\delta^{sa}$	51.07	0.5312	0.02564	-4.331	-0.1583	7.496	2.003	-0.5914	0.06675	0.01802	-0.02863	0.002170
	PARIS	$\delta^{SC}$	50.81	0.55	0.03	-4.18	-0.16	7.56	1.80	-0.63	0.07	0.01	-0.03	0.0
		$\delta^S$	51.00	0.57	0.03	-4.39	-0.17	7.92	1.96	-0.66	0.07	0.02	-0.03	0.0
	GRAZ	$\delta^{SC}$	49.66	0.2975	b)	-3.541	b)	8.043	1.630	b)	b)	b)	b)	b)
		$\delta^S$	50.04	0.3215	b)	-3.796	b)	8.438	1.782	b)	b)	b)	b)	b)
$E_{\text{Lab}} = 50 \text{ MeV}$	AHR	$\delta^{SC}$	38.98	1.652	0.1619	-8.323	-0.6735	11.88	5.720	-1.662	0.3136	0.1624	-0.1884	0.02455
		$\delta^{sa}$	38.83	1.700	0.1654	-8.605	-0.6888	12.13	5.975	-1.662	0.3212	0.1679	-0.1884	0.02508
	PARIS	$\delta^{SC}$	38.75	1.81	0.17	-8.42	-0.73	11.83	5.74	-1.78	0.35	0.14	-0.21	0.03
		$\delta^S$	38.13	1.86	0.17	-8.65	-0.74	11.94	5.98	-1.80	0.36	0.14	-0.21	0.03
	GRAZ	$\delta^{SC}$	36.42	1.845	b)	-9.748	b)	10.97	5.587	b)	b)	b)	b)	b)
		$\delta^S$	35.92	1.915	b)	-10.07	b)	11.05	5.855	b)	b)	b)	b)	b)

Table 1