PREEQUILIBRIUM EFFECTS IN (n, 2n) CROSS SECTIONS AT 14.5 MeV

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The Griffin-Williams exciton model is used to calculate the preequilibrium contribution to the (n,2n) reaction around 14.5 MeV neutron energy for nuclei throughout the periodic table. The experimental cross sections for \(60 < A < 209\) are explained with an r.m.s. deviation of 0.31 by including a statistical evaporation and a preequilibrium component taking into account the competing proton emission. For \(A < 60\) the data is not reproduced very well.
1. Introduction

The cross-sections for \((n,2n)\) reactions are crucial to the neutron economy of both fission and fusion reactors. Statistical evaporation theory overestimates these cross-sections and an empirical factor is often used \(\frac{L}{\sigma_{0}}\) to fit the data. The systematic deviation of the evaporation theory from the experimental data indicates that direct or pre-equilibrium effects could be important \(\frac{L}{\sigma_{0}}\). In the present work a simple pre-equilibrium-cum-statistical model approach which takes into account the competing proton emission is shown to explain the data reasonably well. For the pre-equilibrium part of the calculation the average value of the squared matrix element \(|\mathbf{M}|^2\) is an important quantity. The values of \(|\mathbf{M}|^2\) used here which best explain the data are compared with those given in the literature \(\frac{L}{\sigma_{0}}\). In our earlier analysis \(\frac{L}{\sigma_{0}}\) of \((n,2n)\) cross sections we used a value of \(|\mathbf{M}|^2\) deduced from \((n,p)\) data \(\frac{L}{\sigma_{0}}\) while here \(|\mathbf{M}|^2\) is fitted to the \((n,2n)\) data.

2. Details of the Theory

a. Cross-section calculation

The \((n,2n)\) cross section is expressed as the sum of pre-equilibrium (PE) component and a statistical evaporation or equilibrium (EQ) component,

\[
\sigma(n,2n) = \left[\sigma_{\text{PE}}(n,2n) + (1-\delta)\sigma_{\text{EQ}}(n,2n)\right] F_n
\]

where \(\sigma_{\text{PE}}\) is the optical reaction cross section, \(\sigma_{\text{EQ}}\) is the PE fraction and \(F_n\) is the EQ fraction for the \((n,2n)\) events. The quantity \(\delta\) represents the depletion in EQ events due to the PE process and is given by

\[
\delta = \sum \delta_{n,x} \quad \text{(2)}
\]

where \(x\) denotes the dominant reaction modes which have been taken to be total neutron and proton emission. The factor \(F_n\) accounts for the competing proton emission after the emission of the first neutron. This second stage of the reaction is assumed to be purely statistical. In equation (1) \(\delta_{n,x}\) and \(\delta\) are set zero for a pure equilibrium model. The improvement resulting from including the PE contribution is discussed in section 3.

b.1 Pre-equilibrium decay

The absolute PE emission integrated over the solid angle in the Griffin-Williams exciton model as given by Gadioli and hillasco Coli \(\frac{L}{\sigma_{0}}\) is

\[
\frac{df_{\text{PE}}}{dx} = \frac{(2s+1)m_{n}\sigma_{\text{EQ}}(s)}{4\pi^{3/2}\left|\mathbf{M}\right|^{2}F^{3}} \sum_{t=2}^{t=3} \left(\frac{U}{E}\right)^{t-2} (t+1)^{t-1} \left[\frac{U}{E}\right]^{t+1} - \text{(3)}
\]

The last factor in the square brackets arises due to charge conservation and \(k = 1\) (or \(-1\)) for neutron (or proton) emission. Here \(s = \frac{1}{2} = \) neutron or proton spin, \(E\) is the kinetic energy of the outgoing nucleon and \(\sigma_{\text{EQ}}(s)\) is the corresponding inverse reaction cross-section; \(U\) is the excitation energy of the compound system; \(U\) is the excitation energy of the residual nucleus; \(g\) is the average single particle level density in the Fermi gas model, for
compound and residual nuclei; \( t \) is the number of excitons (particles + holes) in the compound system; \( m \) is the nucleon mass; \( |M|^2 \) is the average value of the squared matrix element in the intrainuclear cascade process \( t \to t + 2 \); \( \bar{t} \) is the value of \( t \) when equilibrium is reached. Using the optical model cross sections of Dostrovsky et al given by,

\[
\sigma_{opt}(E_x) = \mu_x + \nu_x E_x
\]  

(4)

where \( \sigma_{opt}(E_x) \) is the reaction cross-section for the particle \( x \) at energy \( E_x \). Equation (3) can be integrated in a simple closed form \[57\].

b.2 Average value of the squared matrix element.

The dependence of \( |M|^2 \) on mass number and excitation energy has been given in the form

\[
|M|^2 = \kappa, A^{-3} \epsilon^{-1}
\]  

(5)

in ref. 4. Braga-Marcuzzan et al \[5\] have fitted the data on \((n,p)\) reactions for \( A > 100 \) at 14 MeV with

\[
|M|^2 = \kappa_2 A^{-3}
\]  

(6)

In the present work the data \[2\] on \((n,2n)\) reactions around 14.5 MeV has been fitted with both the above forms for \( |M|^2 \). The excitation energy \( E \) of the compound system varies somewhat over the range of the data owing to the variation in neutron binding energy from 19.0 to 24.4 MeV. However, the forms (5) and (6) give nearly equivalent fits to the \((n,2n)\) data.

c. Equilibrium Decay and Second Stage Emission.

In the statistical evaporation theory, neglecting the \((n,np)\) process, the \((n,2n)\) fraction is calculated \[6\] using the level density formula of Kataria et al \[9\].

In eqn. (1) the factor \( F_n \) corrects for the \((n,np)\) process. A simple approximate expression for \( F_n \) has been obtained in a closed form \[6\].

\( F_n \) ranges from 0.8 to 1 from light to heavy nuclei and indicates that the \((n,np)\) process is important for \( A < 65 \) particularly for targets with odd \( Z \). For many targets usually with even \( N \)-odd \( Z \), \( S_n \) is about 4 MeV larger than \( S_2 \) so that the second stage proton emission for lighter targets competes favourably with neutron emission in spite of the Coulomb barrier. The \((n,p)\) process which has also been accounted for reduces the calculated cross-sections by hardly 5% except in a few cases for \( A < 50 \). The contribution due to the \((n,\alpha)\) process, which has been neglected is still less. The \((n,3n)\) process is energetically allowed in only 15 cases and its effect is less than 10%.

3. Results of the calculation

In Fig. 1, a comparison of the calculation, using the \( |M|^2 \) of eqn. (5) and \( \kappa_1 = 108.3 \text{ MeV}^{-1} \) with the \((n,2n)\) data is shown. The experimental cross-sections have been taken from the compilation of Kondaiah \[27\]. A value of \( \kappa_1 = 108.3 \text{ MeV}^{-1} \) in eqn. (5) and \( \kappa_2 = 5.07 \text{ MeV}^{-1} \) in eqn. (6) gave the minimum r.m.s. deviation defined as,

\[
R = \left[ \frac{1}{N} \sum \left( \frac{\sigma_{exp} - \sigma_{calc}}{\sigma_{exp}} \right)^2 \right]^{1/2}
\]  

(7)

This definition of \( R \) ignores the experimental errors which range between 5 and 20%. The discrepancy in different measurements of the same cross-section is often more than these errors. Taking 88 data points with 60 < \( A < 209 \) we find \( R = 0.31 \) with the Kalbach-Cline
form (eqn.(5)) for $|M|^2$ and $R = 0.36$ with the Braga-Marcazzan form (eqn.(6)). The same quantity in the case of a pure EQ calculation has a value of 0.86. The improvement of the PE-EQ model over the pure EQ calculation is further illustrated by the histograms of Fig. 2. In 2(a) the histogram of $\sigma_{\text{tot}}/\sigma_d$ is seen to peak at around 0.75. The histogram $\sigma_{\text{tot}}/\sigma_d$ of Fig. 2(b) peaking near 0.95 is with $|M|^2 = 7.64/A^2$ MeV$^2$ which is the value obtained by Braga-Marcazzan et al by fitting $(n,p)$ data. Figs. 2(c) and (d) show the histograms arising from selecting the values of $|M|^2$ which best explain the $(n,2n)$ data assuming dependences of the form of eqns.(5) and (6) respectively. These histograms are more strongly peaked at and centred around the value $|M|^2 = 1.0$. The fitted value of $|M|^2$ is smaller by a factor 1.5 as compared to that in ref. 5, while for the energy dependence of the Kalbach-Cline type it is comparable to the value of $|M|^2 = 25/(A^2/\epsilon)$ MeV$^2$ given in ref. 4. In comparison to our earlier work [5] the r.m.s. deviation is reduced from 0.48 to 0.31 and 0.36.

References


Figure 1: Here different symbols are used depending upon the excess residual energy, $U_e = B_n - B_n$. For small $U_e$ and low mass numbers the calculated values are discrepant.

Figure 2 (a): Histogram of the ratio of the experimental cross-section to the EQ model cross-section.  
(b) to (d): Histograms for the ratio of the experimental cross-section to the PE-EQ model cross-section with different values of $|M|^2$. 

Cross-sections for the $(n, 2n)$ Reaction at 14.5 MeV using PE-EQ model with $|M|^2 = 108.3 \text{ A}^2 \text{ E}^1 \text{ MeV}^2$. 

Figure 1: Cross-sections for the $(n, 2n)$ Reaction at 14.5 MeV using PE-EQ model with $|M|^2 = 108.3 \text{ A}^2 \text{ E}^1 \text{ MeV}^2$. 

Figure 2 (a): Histogram of the ratio of the experimental cross-section to the EQ model cross-section. (b) to (d): Histograms for the ratio of the experimental cross-section to the P5-EQ model cross-section with different values of $|M|^2$. 