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ESCAPE AND TRANSMISSION PROBABILITIES IN CYLINDRICAL GEOMETRY*

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This paper describes an effort to apply an efficient technique for the generation of escape and transmission probabilities in cylindrical geometry to an existing resonance cross section processing code, ROLAIDS. The technique applied was that developed by Hwang and Toppel[1] at Argonne National Laboratory (ANL). After important modifications of this technique were established, the probabilities generated were found to be as accurate as those given by the method previously applied in ROLAIDS, while requiring much less computer core storage and CPU time. These savings are particularly significant to the development of a new doubly-heterogeneous ROLAIDS geometry model in which coupled cylindrical and spherical systems are to be treated simultaneously.

The ROLAIDS code, a part of the AMPX[2] system at ORNL, is an interface currents integral transport multigroup cross section generation code which operates from a point-wise cross section set. The energy point mesh is the union of the point cross section files for each nuclide in the problem. This can amount to a large number of points, with 20-30,000 neutron flux distribution determinations in an average problem and as high as 60,000 in a large problem.

At each energy point, a set of escape and transmission probabilities must be calculated for every zone in the problem. The major quantities calculated are T^{OI}, the probability of transmission from the inner to the outer surface, and T^{OO}, the probability of transmission from the outer surface back to the outer surface. The transmission from outer to inner, T^{IO}, and the two escape probabilities, P^+ and P^-, are simply related to T^{OI} and T^{OO}. Assuming an isotropic flux, or cosine-shaped current, on the surface of a zone, T^{OI} and T^{OO} are given by the following exact integral expressions:

$$T^{OI} = \frac{4}{\pi}\int_0^1 dx \; Ki_3\left[\frac{b}{2}\left(\sqrt{1-a^2x^2} - a\sqrt{1-x^2}\right)\right]$$

$$T^{OO} = \frac{4}{\pi}\int_0^{\alpha} dx \; \frac{x}{\sqrt{1-x^2}} \; Ki_3\;(bx) \quad .$$

In these equations, Ki_3 is the third order Bickley function, a and α are geometric quantities, and b depends on the total cross section. In the original version of ROLAIDS, the method for calculating these probabilities was that applied in the RABBLE[3] program. A Newton-Cotes numerical integration (Weddle's Rule with 48 intervals) was used to evaluate the probabilities at various values of the parameters a, the ratio of inner to outer radius of the zone, and τ, the minimum optical path length. A double interpolation in each array of probabilities then yielded the actual probability.

Each of these two probability arrays, T^{OI} and T^{OO}, was dimensioned 182x54, a mesh fine enough to ensure accuracy after interpolation. The preparation of these arrays required approximately 80K bytes of computer core storage and two minutes of CPU time on an IBM 360/91. It was the dual problem of large core storage and long running times that prompted the implementation of a more efficient algorithm.

The method proposed by Hwang and Toppel treats the exact integral expressions for T^{OI} and T^{OO} with a combination of analytical approximations and numerical quadrature integrations. The parameters a and τ are used to define the regions of applicability of each approximation. The analytical approximations take advantage of the limits of the equations to give reasonably simple expressions which are a function of only one variable. One then needs to store only a few singly-dimensioned, precalculated arrays and perform a single interpolation.

In those regions where a limiting expression cannot be found, a 3-point Gauss-Jacobi quadrature scheme is applied. The low order of this quadrature scheme leads to a very fast algorithm, and also requires only a few singly-dimensioned, precalculated arrays.

The elimination of the doubly-dimensioned arrays for T^{OI} and T^{OO} decreases the computer core storage by about 70K bytes, while the simple analytical expressions and low-order quadrature reduces the CPU time from about 2 minutes to about 10 seconds. The CPU time required for interpolations under either method is insignificant.

Several modifications were made to the Hwang and Toppel method in order to produce accurate results. The accuracy required was 0.05% in all regions of interest, where a region of interest is one in which the probability is greater than 10^{-3}. For the calculation of T^{OI}, The Hwang and Toppel method utilizes four regions:

1) A nearly transparent region, where the optical path length is small,
2) An asymptotic region, where the path length is large,
3) A Gauss-Jacobi quadrature region, which lies between regions 1 and 2, and
4) A region which lies between 1 and 3 and combines the expressions of each.

In applying this method, region 4 was discarded altogether because the region 1 part of the expression was found not to apply. The straight Gauss-Jacobi quadrature of region 3 was extended to region 4 and found to work quite well. Because the low-order quadrature is such an efficient scheme, it was extended down into region 1 as far as the accuracy limit would allow.

For the calculation of T^{00}, the Hwang and Toppel method utilizes three regions:

1) An asymptotic region for large optical path lengths,
2) A Gauss-Jacobi quadrature region, below region 1 and for a ≤ 0.7, and
3) A region below region 1 and for a >0.7, combining a quadrature integration and a small path length expression.

The T^{00} regions were used as presented by Hwang and Toppel, and only some reformulation of the equations for efficient computer calculation was necessary.

In summary, with some modification the algorithm of Hwang and Toppel for determining the transmission probabilities through cylindrical shells has been implemented in the ROLAIDS program. Significant computational advantages over an earlier method have been demonstrated.

References

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