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I. INTRODUCTION

Most calculations of the implosion of spherical-shell inertial-confinement targets have been performed using elaborate hydrodynamic computer codes such as TRHYD. As with most complex models, these calculations have difficulty producing physical insight and in some cases, scaling information. In this report, we describe a simple model of the ablative implosion of a high-aspect-ratio (shell radius to shell thickness ratio) spherical shell. The model is similar in spirit to Rosenbluth's[1] snowplow model. We have determined the scaling of the implosion time in terms of the ablation pressure and the shell parameters such as diameter, wall thickness, and shell density, and compared these to complete hydrodynamic code calculations. We also have examined the energy transfer efficiency from ablation pressure to shell implosion kinetic energy and find it to be very efficient. It may be possible to "attach" a simple heat-transport calculation to our implosion model to describe the laser-driven ablation-implosion process.[2] The model may be useful for determining other energy driven (e.g. ion beam) implosion scaling.

II. THE MODEL EQUATIONS

Consider the implosion of a thin spherical shell which implodes due to the combination of the reactive force of ablating material (the "rocket-effect") and the force due to the external atmospheric pressure from the deposition of energy. We assume: 1) a high aspect ratio shell, 2) a constant ablation density (ρ_a), and ablation velocity (V_a) in a coordinate system moving with the shell and 3) that the shell remains thin, i.e. it can be represented by an infinitely thin, finite mass shell at R(t). Also, we consider the shell to be filled with an adiabatic fuel gas that is compressed and "turns-around" the imploding shell ($p_g V^\gamma$ = constant).

Newton's Law is written as,

$$\frac{d}{dt}(M_s\dot{R}) = (\dot{R} + V_a)\dot{M}_s + 4\pi R^2(p_g - p_a) \qquad (1)$$

where M_s is the shell mass, R is the (point) location of the shell, and $p_g (p_a)$ is the internal gas (ablation) pressure. The shell loses mass according to,

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$$\dot{M}_s = -4\pi R^2 \rho_a V_a$$

Making the above equations non-dimensional, we find the characteristic implosion time, τ is given by,

$$\tau = \left(\frac{M_o}{4\pi R_o \rho_a}\right)^{1/2} \tag{2}$$

and the differential equations become,

$$\eta\ddot{y} = -(M^2+1-\beta y^{-3\gamma})y^2 \tag{3}$$

$$\eta = 1 - \int_o^{t'} M\alpha y^2 dt' \tag{4}$$

where $\eta(t) = \frac{M_s(t)}{M_o}$, $y(t) = \frac{R(t)}{R_o}$, $t' = t/\tau$,

$$M_o = 4\pi R_o^{\,2}\delta\rho_{solid} \; , \; \alpha = \left(\frac{R_o}{\delta}\frac{\rho_a}{\rho_{solid}}\right)^{1/2} \tag{5}$$

and the Mach number $M = V_a/C_a$, where the atmospheric sound speed is $C_a^{\,2} = p_a/\rho_a$, and M_o is the original shell mass, γ is the adiabatic constant ($p_g V^{\gamma}$ = const. = $p_o V_o^{\,\gamma}$) ρ_{solid} is the shell density, and $\beta = p_o/pa$.

The initial conditions are,

$\eta(o) = 1$, $y(o) = 1$, $\dot{y}(o) = 0$

We have numerically integrated this system of equations for various choices of input parameters M and α. Figure 1 shows an example for which, $M = 0.2$, $\alpha = 3.48$, $\beta = .002$. Notice that the implosion time, t'_{imp}, depends on the parameters in τ and on the, parameters, M and α.

Figure 1 indicates that ~50% of the shell mass was ablated away up to $t' = t'_{imp}$ We have examined how the unablated shell mass and t'_{imp} depends on M,α in Figures 2 and 3. Notice that the parameter, α, contains the shell aspect ratio, R_o/δ.

III. THE ENERGY INTEGRALS AND PULSE SHAPE

It is useful to compute a few energies associated with this simple implosion system. We can multiply Newton's Law (1) by $\dot{R}$ and integrate to find the following energies (we have set $p_g = 0$). The shell kinetic energy,

$$E^{shell}_{k.e.} = \frac{3}{2} E_o\left(\frac{dy}{dt}\tau\right)^2 \eta = 1/2 M_s \dot{R}^2$$

The "blow-off" kinetic energy,

$$E_{k.e.}^{blow-off} = \frac{1}{2}\int_0^t \dot{M}_s(\dot{R}+V_a)^2 = \frac{3}{2}E_o\int_0^{t'}\dot{n}(\dot{y} + \frac{M}{\alpha})^2 dt' ,$$

and the energies which can be considered as <u>input</u> energies (they are negative). The mechanical energy produced by the ablation pressure,

$$E_{mech} = -\int_0^t 4\pi R^2 p_a \dot{R} dt = -3E_o\int_0^{t'} y^2\dot{y}\, dt'$$

and the "rocket-exhaust" energy,

$$E_{exh} = 1/2\int_0^t \dot{M}_s V_a{}^2 dt = \frac{3}{2}E_o\, M^2/\alpha^2\int_0^{t'}\dot{n} dt' .$$

In the above, the scale energy E_o given by,

$$E_o = \frac{4\pi}{3}R_o{}^3 p_a ,$$

is just the energy needed to fill the original shell volume at the ablation pressure, p_a. The energy conservation equation reads,

$$E_{k.e.}^{shell} + E_{k.e.}^{blow-off} = E_{mech} + E_{exh}$$

It is interesting, and perhaps unexpected, to find that the transfer of energy from the "mechanical" input i.e. ablation pressure, to the kinetic energy of the shell is rather high in almost all cases. Physically, this means that the fraction of energy which it turned into atmospheric pressure at the ablation surface is effectively utilized in the shell implosion process. If we define the energy transfer efficiency, relative to the ablation surface as,

$$\varepsilon = \frac{E_{k.e.}^{shell}}{E_{mech} + E_{exh}} ,$$

we can compute this efficiency for various parameter choices, M, α. Figure 4 shows plots of the various dimensionless energies and the efficiency as a function of time. In these figures, we also present the power input pulse shape which is required to produce our assumed constant ablation pressure, p_a. This pulse shape is simply computed from $P = \dot{E}_{mech} + \dot{E}_{exh}$. By allowing $p_a = p_a f(t)$ it would be possible to determine the ablation pressure pulse shape which would produce a perfectly isentropic compression[3] of an imploding shell.

IV. COMPARISONS TO HYDRO-CODE CALCULATIONS

To compare this simple model to complete heat-transport hydrodynamic code calculations, we need to specify the values of M, α. It is interesting to note that the value of the Mach number of the ablating material, M is a measure of the relative contributions of the "rocket-reaction" force and the ablation-pressure forces driving the shell. This can be seen by examining equation 3. For smaller values of M (which are expected on the basis of steady-state spherical hydro considerations,[4]) the ablation-pressure force dominates; at M = 1, both forces contribute about equally. In many cases of interest, it is found from hydro-code runs that about 50% of the mass is ablated from an imploding shell at the peak compression time if the energy deposition pulse is longer than the implosion time. We, therefore, are most interested in the M, α points for $\eta = 0.5$ at $t' = t'_{imp}$. Figure 5 is the plot of these points found from the model. We might expect different values of M for different transport models of the ablation process, for example strong or weak heat flux-inhibition.[5] Also, shown in Figure 5 is the implosion time in units of τ.

We have chosen to fit our simple model to a hydrocode calculation of the strongly flux-limited (f = .05) implosion of a 80-μm diameter by 2-μm thick wall spherical glass shell, which absorbed 50 joules of 1.06-μm laser light in 250 psec. X-ray preheat was "turned-off" for this calculation as it tends to further expand the shell. Figure 6 shows the agreement between the simple model (dots) and the complete hydrocode run (straight lines). The lines from the hydro code indicate the location of the inward moving shell material. The simple model parameters where $M = 0.2$, $a = 3.48$ and $\beta = 0.002$. Perhaps more important is the ablation pressure comparison. The x's give the peak values of the ablation pressure versus time from the hydro code whereas the simple model had <u>constant</u> ablation pressure shown by the straight line. Notice that the peak pressure from the code increases (~1/R) at the later stages of the implosion. This pressure increase due to spherical convergence is not contained in our simple calculation.
The ablation density in the model is also constant and is compared in Figure 6 to the ablation density from the code calculation. The "fitting-point" in the hydro-code was found to be a best match with the simple model if we choose the peak pressure point in radius as the effective ablation point. Figure 7 shows the hydro-code pressure, density, sound speed ($C_a = \sqrt{P/\rho}$) and flow mach number (in the frame moving with the shell). It appears that the R vs t trajectory from the simple model can be made to agree rather well with the full hydrocode calculation, for our chosen values of the parameters.

V. IMPLOSION TIME SCALING

The scaling of the implosion time depends on τ as well as α and M. For small M (which seems close to the full hydro simulations) it is interesting to extract the scaling of t'_{imp} with α at fixed M. Figure 3 shows the plot of t'_{imp} as a function of α for M = 0.2. Also plotted is the shell mass at peak compression, $n(t'_{imp})$. The resulting scaling gives

$$t'_{imp} = \tau(1.7 - 0.8\alpha)$$

The implosion time scaling with aspect ratio, ρ_a/ρ_{solid}, C_a and δ, is

$$t'_{imp} = \frac{\delta}{C_a}\left(\frac{\rho_{solid}}{\rho_a}\right)\alpha[1.7 - 0.8\alpha]$$

This equation shows that the implosion time is a multiple of the acoustic transit-time (at the ablation surface sound speed) through the shell thickness.

VI. COMPRESSION SCALING

The simple model can be used to examine the scaling of peak compression upon the amount of preheat in the core gas. In equation (1), we set $p_g = p_o\left(\frac{R_o}{R(t)}\right)^{3\gamma} = p_o y^{-3\gamma}$, and $\beta = p_o/p_a$. Figure 8 shows the peak compression, i.e. $C = \left(\frac{1}{y_{min}}\right)^3$ as a function of the initial gas pressure. If the gas is preheated at close to t = 0, or before the implosion has proceeded very far, this curve shows how the compression scales with the preheat pressure. The curve shows that $C \sim \beta^{-1.4}$ for an assumed adiabatic index of $\gamma = 5/3$. If we take an isothermal index, $\gamma = 1$, we find the compression scaling given in Figure 9 which is not a simple power law, but indicates smaller compressions for the same preheat pressure ratio.

VII. CONCLUSIONS

We have found that this simple spherical ablative-compression model can reproduce some aspects of the implosion of thin walled spherical shell targets. In particular, the implosion time scaling with shell parameters, the R-t trajectory and shell velocity agree well with the more complete hydro-code calculations. Also, it is clear from the model that the ablation pressure most strongly influences the pellet implosion and the energy delivered to the shell from the mechanical albation pressure is the efficient energy transfer mechanism.

Future work on a "heat-transport" model from which the ablation pressure is determined from the corona conditions may lead to further understanding of the implosion of thin shell targets.

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FIGURES

1. Normalized radius vs. time plot y vs. t', η vs. t' for M = 0.2 α = 3.48, β = 0.002.

2. A plot of t'_{imp} and $\eta(t'_{imp})$ vs M for α = 1.

3. A plot of t'_{imp} and $\eta(t'_{imp})$ vs α for M = 0.2

4. A plot of ε, E_{exh}, E_{ke}^{pusher}, $E_{k.e.}^{blow\text{-}off}$, E_{mech} and the pulse shape P vs. t' for M = 0.2, α = 3.48.

5. A plot of the locus of points M,α for which half the shell mass is ablated η = 0.5 at t = t_{imp}.

6. A comparison of the simple model R vs t and the complete hydro-code calculation.

7. Hydro-code density, pressure, sound speed, and flow velocity profiles at t = 60 psec showing the model fitting point.

8. Peak compression, C vs. $\beta = p_o/p_a$, γ = 5/3.

9. Peak compression, C vs. $\beta = p_o/p_a$, γ = 1.

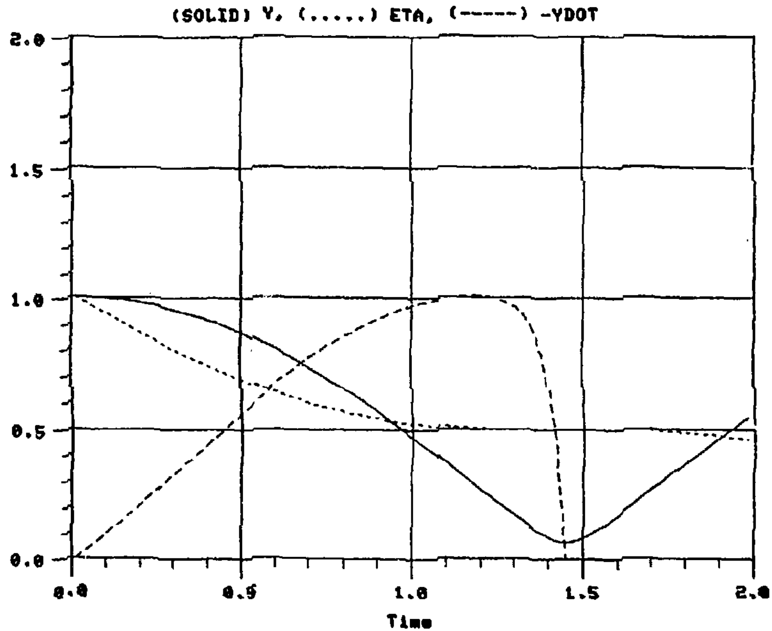


FIGURE 1

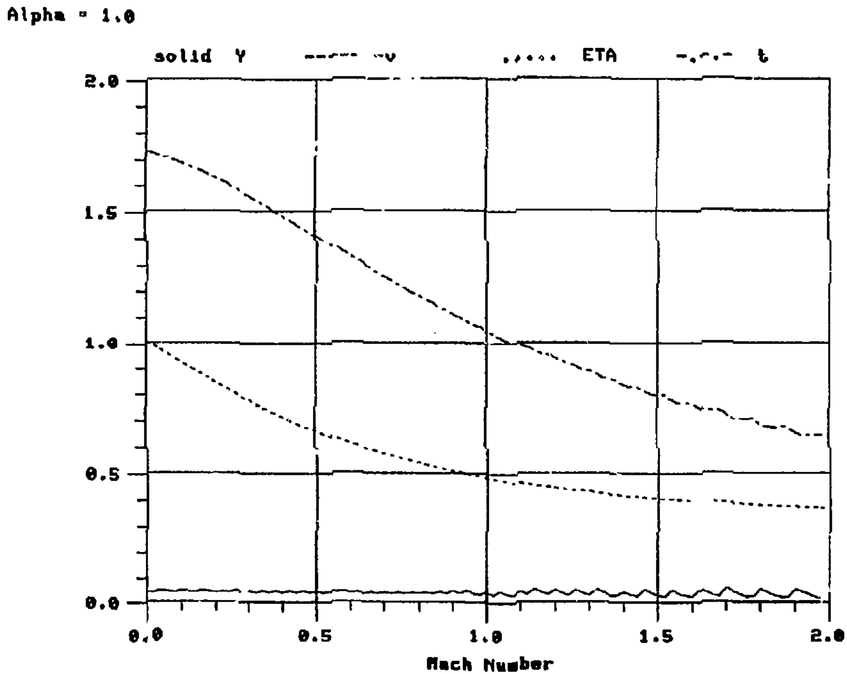


FIGURE 2

READY

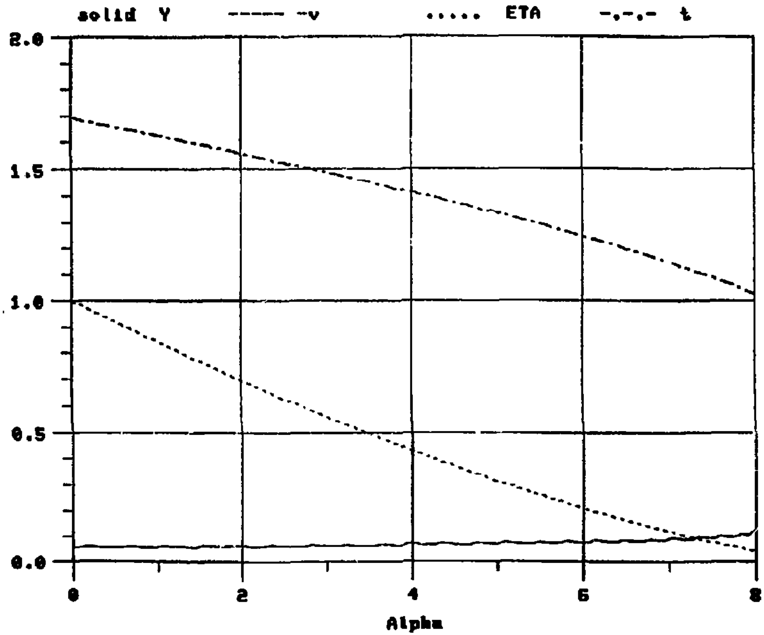


FIGURE 3

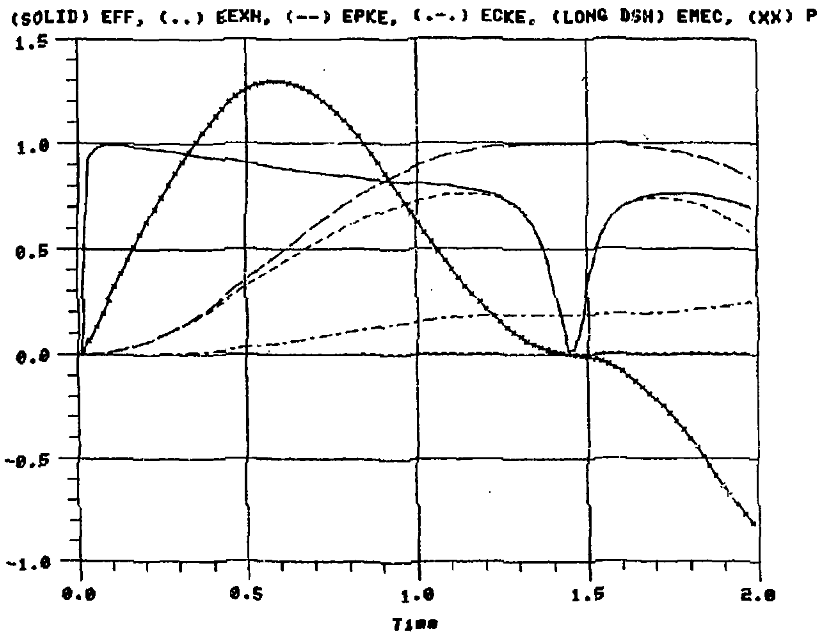

(SOLID) EFF, (..) EEXH, (--) EPKE, (.-.) ECKE, (LONG DSH) EMEC, (XX) P
1.5
1.0
0.5
0.0
-0.5
-1.0
0.0
0.5
1.0
1.5
2.0
Time

FIGURE 4

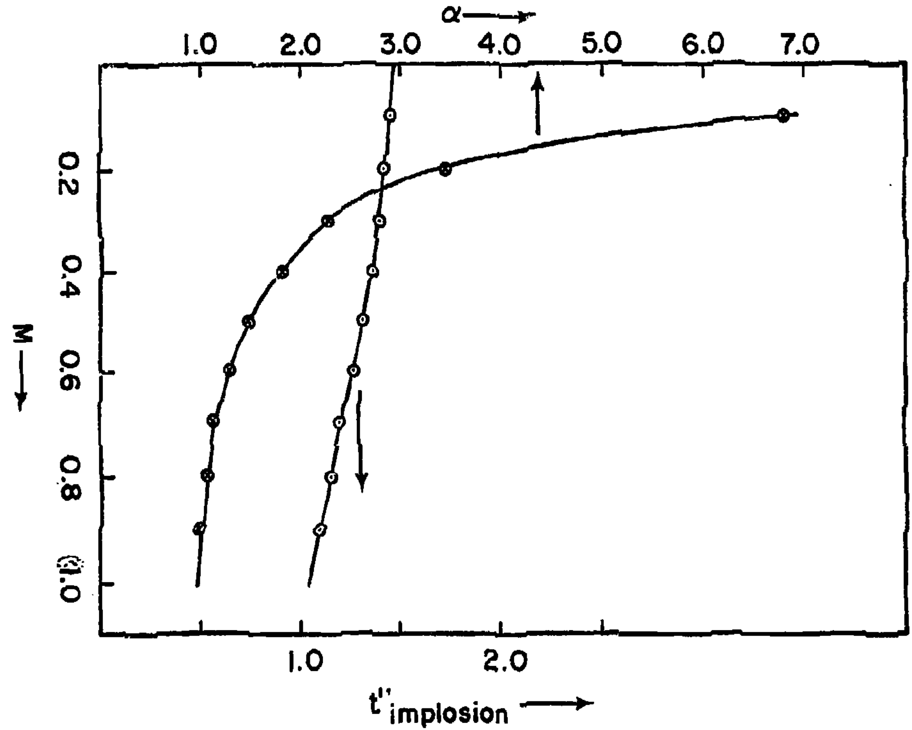
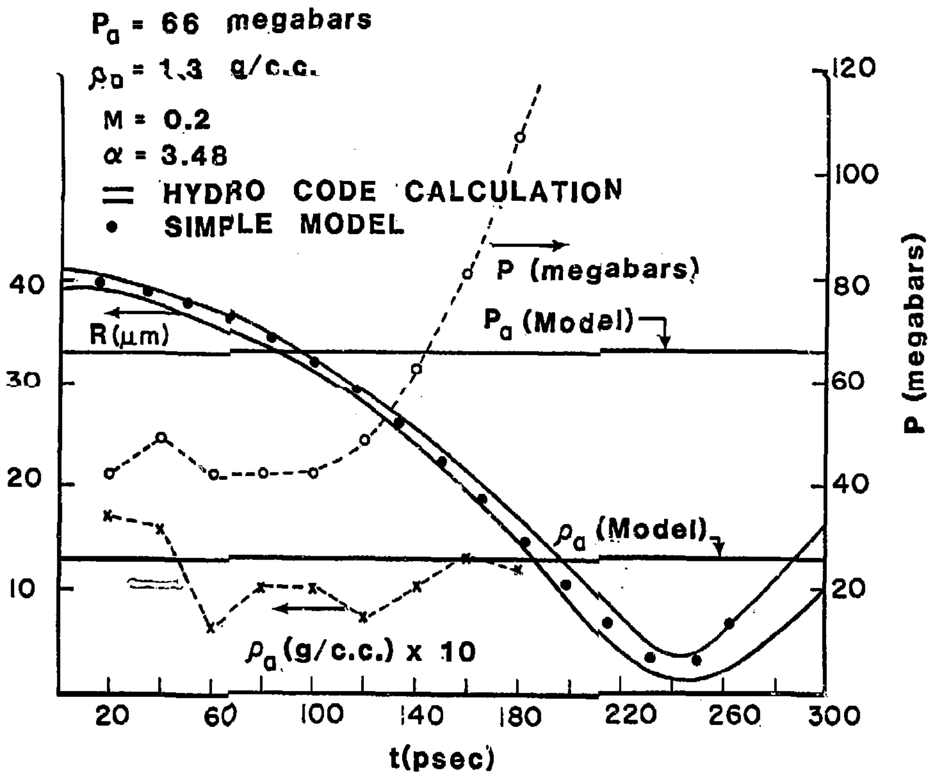
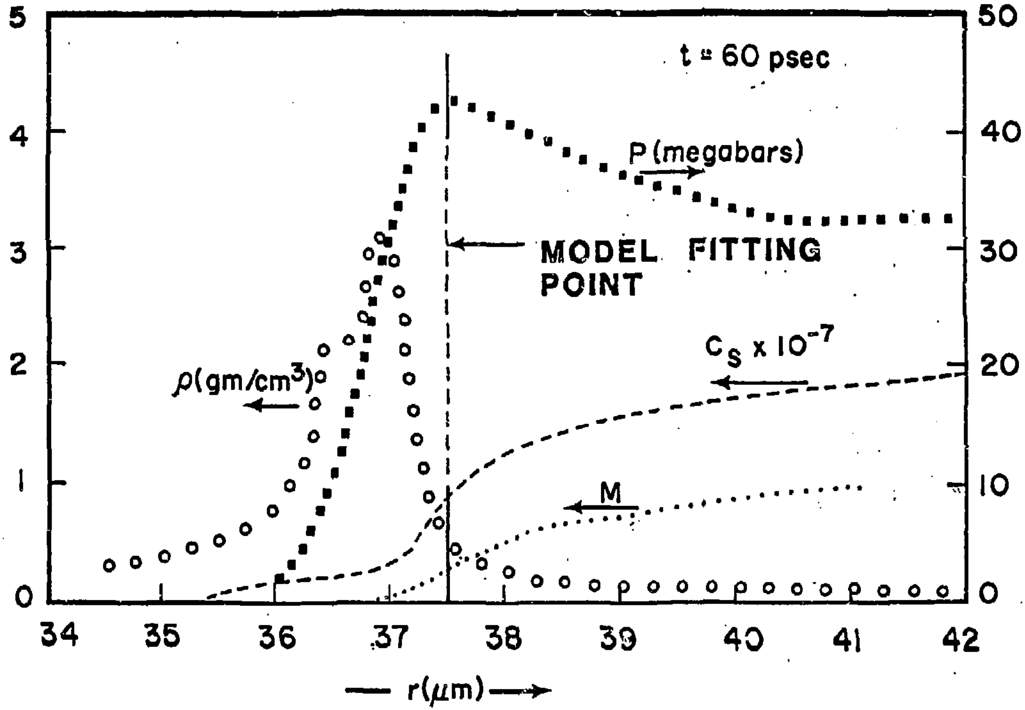


FIGURE 5



Pa = 66 megabars
ρa = 1.3 g/c.c.
M = 0.2
α = 3.48
HYDRO CODE CALCULATION
SIMPLE MODEL
P (megabars)
Pa (Model)
R (μm)
ρa (Model)
ρa (g/c.c.) x 10
40
30
20
10
120
100
80
60
40
20
20
60
100
140
180
220
260
300
t(psec)
P (megabars)

FIGURE 6



t = 60 psec
P (megabars)
MODEL FITTING POINT
$c_S \times 10^{-7}$
ρ(gm/cm³)
M
5
4
3
2
1
0
50
40
30
20
10
0
34
35
36
37
38
39
40
41
42
r(μm)

FIGURE 7

READY

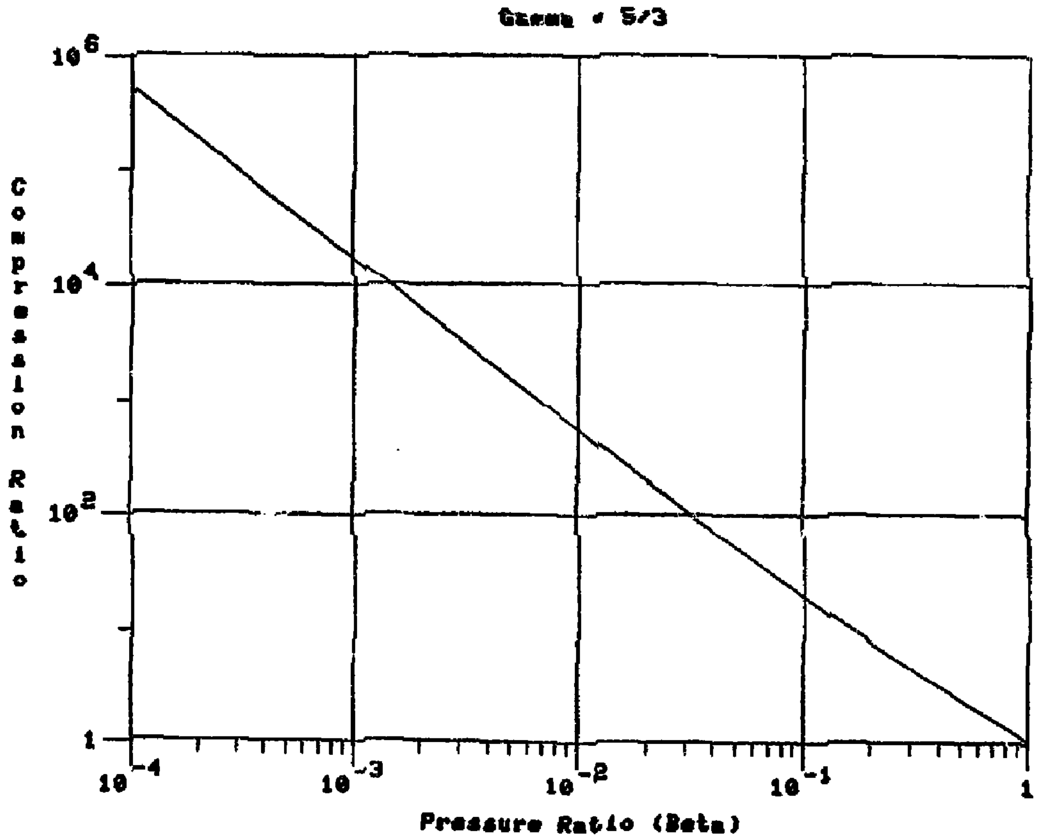


FIGURE 8

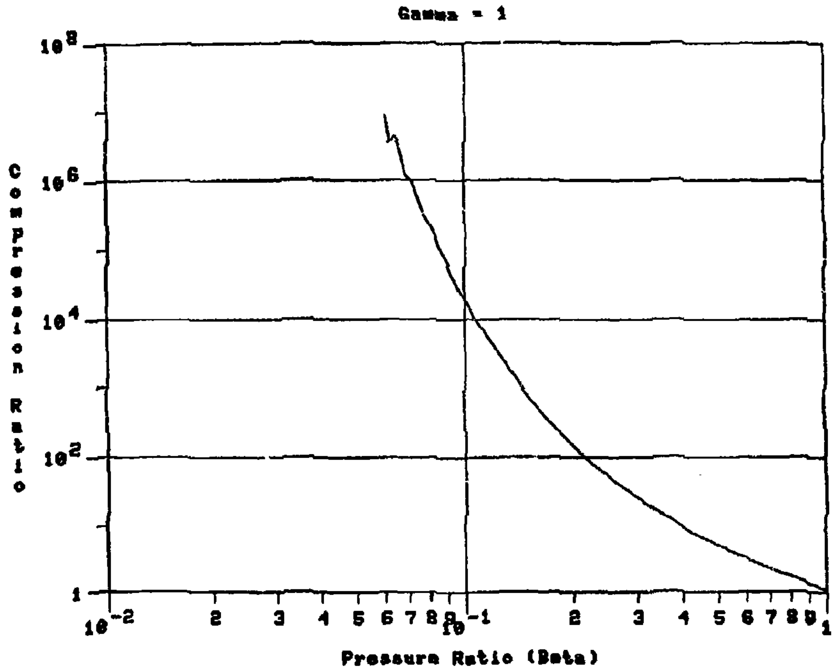


FIGURE 9