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"THE EFFECTIVE SCHROEDINGER EQUATION OF THE OPTICAL MODEL FOR ELASTIC COLLISIONS BETWEEN COMPOSITE NUCLEI"[+]

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ABSTRACT

An effective hamiltonian for elastic collisions between composite nuclei is obtained from the Schrödinger equation of the complete many-body system and its fully antisymmetric wave functions by means of a projection operator technique. This effective hamiltonian, defined in such a way that it has to reproduce the scattering amplitude in full detail, including exchange effects, is explicitly Galilean invariant. The effective interaction operator is a function of the relative distance between the centers of mass of the colliding nuclei and the constants of the motion of the whole system. The interaction operator of the optical model is obtained next, requiring as usual, that it reproduces the average over the energy of the scattering amplitude and keeping the Galilean invariance. The resulting optical potential operator has some terms identical to those obtained in the Resonating Group Method, and others coming from the elimination of all non-elastic channels and the delayed elastic scattering. This result makes the relation existing among the projection operator method à la Feshbach and the cluster model equations of motion for positive energies (RGM)[14,16] explicit. The additional interaction terms due to the flux loss in the elastic channel are non-local, and non-hermitean operators expressed in terms of the transition amplitudes and the density of states of the compound nucleus in such a way that an approximate evaluation, in a systematic fashion, seems possible. The angular momentum dependence of the optical potential operator is discussed in some detail.

1. INTRODUCTION

In the theoretical treatment of collision experiments between composite nuclei above the Coulomb barrier, the correct treatment of the elastic scattering or entrance channel is of basic importance since it underlies the description of the more complicated inelastic and rearrengement processes that usually occur to a large number of open channels.

The elastic scattering of complex nuclei shows well defined single particle features[1,2,3,4,5], like gross structure in the excitation function, it also has an intermediate structure[4,5] indicating the presence of non-elastic states strongly coupled to the elastic channel, usually collective motion states, and it also has a finite structure,[5,6] that appears as narrow resonances and fluctuations in the cross sections, and it is characteristic of the many-body nature of the system.

The effective interaction operators and optical model potentials for the elastic channel and those inelastic states strongly coupled to it allow the dynamical description of the gross and intermediate structures[7]. A detailed description of the fine-structure requires the knowledge of the solutions of the many-body problem, something which at present is not possible. On the other hand, it seems that at least part of the understanding of the fine structure can be achieved without this detailed knowledge, since it is closely related to the gross features of the elastic channel amplitude through statistical arguments[8]. It follows that a

good knowledge of the effective interaction operators in the elastic channel is of great importance in nuclear reaction theory.

The aim of the present work is the derivation of a formalism appropriate for the description of elastic collisions between complex nuclei. We start from the many-body Schrödinger equation, and in our treatment of this problem we use Green's functions and projection operators, as were used in references (9,10,11), and in this our work is similar to Feshbach's work. This same problem has been studied in the framework of the cluster model by Benöhr and Wildermuth[12], who made a variational analysis of the scattering amplitude using cluster model wave functions[13,14]. Both, these authors are mostly interested in the general structure of nuclear reaction theory and its basic features, and therefore concentrate their attention on the transition amplitudes.

We formulate the effective hamiltonian method in a systematic and straightforward way, taking into account antisymmetrization and Galilean invariance at all stages of the development. We arrive at simple expressions for the effective interaction operators exhibiting explicitly the relations existing between the optical potential model and the resonating group method[14], and making it possible, we hope, their approximate evaluation in a systematic fashion.

In the second section of this paper, we state the problem and fix the notation, in the next section we state the conventions about antisymmetrization to be used later, in section four we make an explicit construction of a projection operator on the elastic channel as an explicitly Galilean invariant integral operator carrying with it the effects of antisymmetrization, in section

five an explicit expression is obtained for the effective interaction operator between two complex nuclei colliding elastically, as well as the effective Schrödinger equation describing the relative motion of the colliding nuclei, the formulation of the theoretical, as opposed to phenomenological, optical model, and its relation to the effective interaction operator is made in the sixth section, in this section we give also formulae expressing the optical potential in terms of the projectivel and target's ground state wave functions, and the density of states of the compound nucleus; in section seven, the orbital angular momentum dependence of the optical potential is examined in some detail, and in section eight we make some final comments.

2. NOTATION

Let us consider two composite nuclei that collide. Before the collision, the system is characterized by specifying the energy and momentum of the relative motion, as well as the mass numbers, the charges and the quantum numbers of the internal states of the target and projectile. Long after the collision has occurred, the system is made of two or more fragments that move away from each other, as for the initial state, the final states are characterized by the energies and momenta of the relative motions and the quantum numbers of the internal motion of the fragments.

$$n_{A_i} = \{ \mathcal{E}_{A_i}, J_{A_i}, M_{A_i}, T_{A_i}, T_{3A_i}, \Pi_{A_i} \} \tag{2.1}$$

The set of internal motion quantum numbers characterizing each possible fragmentation define a channnel. In this work, we shall consider two-body channels only and they will be denoted by a greek letter, i.e.

$$\kappa = \{ n_{A_1}, n_{A_2} \} \tag{2.2}$$

will correspond to the elastic or entrance channel, in this channel, both target and projectile are in their ground state.

The hamiltonian of the complete system, projectile plus target, made of A nucleons is:

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m_i} + \frac{1}{2} \sum_{j=1}^{A} \sum_{\substack{k=1 \\ i \neq k}}^{A} v_{ij}(|r_i - r_j|) \tag{2.3}$$

H is defined in the center of mass coordinate system, in the laboratory frame H becomes

$$H_{Lab.} = H + \frac{P^2}{2M} \tag{2.4}$$

where $\frac{P^2}{2M}$ is the kinetic energy of motion of the center of mass.

The hamiltonian H can be written in many different ways according to the possible partitions of A nucleons. For a two-body channel corresponding to the partition

$$A = A_1 + A_2 \tag{2.5}$$

the hamiltonian is split in the following way

$$H = H_{1\alpha} + H_{2\alpha} + \frac{p_\alpha^2}{2\mu_\alpha} + V_{12} \tag{2.6}$$

where

$$H_{1\alpha} = \sum_{i=1}^{A_1} \frac{p_i^2}{2m} - \frac{P_{1\alpha}^2}{2M_{A_1}} + \frac{1}{2}\sum_{j=1}^{A_1}\sum_{\substack{i=1 \\ i\neq j}}^{A_1} V_{ij}$$

and

$$H_{2\alpha} = \sum_{i=A_1+1}^{A} \frac{p_i^2}{2m} - \frac{P_{2\alpha}^2}{2M_{A_2}} + \sum_{j=A_1+1}^{A}\sum_{\substack{i=A_1+1 \\ i\neq j}}^{A} V_{ij}$$

In these equations, $\frac{P_{1\alpha}^2}{2M_{A_1}}$, $\frac{P_{2\alpha}^2}{2M_{A_2}}$ are the kinetic energies of the center of mass motion of the fragments in channel α , $\frac{P_\alpha^2}{2\mu_\alpha}$ is the kinetic energy of the relative motion of the two fragments and is expressed as

$$\frac{P_\alpha^2}{2\mu_\alpha} = \frac{P_{1\alpha}^2}{2M_{1\alpha}} + \frac{P_{2\alpha}^2}{2M_{2\alpha}} - \frac{P_\alpha^2}{2M_A} = -\frac{\hbar^2}{2\mu_\alpha}\nabla_{r_\alpha}^2 \tag{2.7}$$

where

$$\frac{M_{1\alpha} M_{2\alpha}}{M_{1\alpha} + M_{2\alpha}} = \mu_\alpha$$

is the corresponding reduced mass,

$$\vec{r}_\alpha = \vec{r}_{2\alpha} - \vec{r}_{1\alpha} \tag{2.8}$$

is the relative coordinate between the centers of mass of the two fragments.

The interaction potential energy of the two fragments is

$$V_{12} = \sum_{j\in[A_2]}\sum_{i\in[A_1]} v_{ij}(|r_i - r_j|) = \sum_{j\in[A_2]}\sum_{i\in[A_1]} v_{ij}(X_i - X_j + r_\alpha) \tag{2.9}$$

where the X_i are the internal coordinates of the particles

$$X_i = r_i - r_{i\alpha} \qquad i \in [A_1] \tag{2.10}$$

Since V_{12} is a function of r_α and all the internal coordinates of the individual nucleons, this interaction can produce not only elastic collisions between the projectile and target, but it can also give rise to inelastic collisions and nuclear reactions.

The wave function of the complete system satisfies the Schrödinger equation

$$H\Psi = E\Psi \tag{2.11}$$

In view of the later developments it is convenient to write the complete system's wave function Ψ as a product of the free center of mass motion wave function times the system's wave function in the center of mass frame of reference.

$$\Psi^{(k)}(r_1, r_2, \ldots r_A) = e^{iK\cdot R}\,\Psi^{(0)}(r_1 - R, r_2 - R, \ldots r_A - R) \tag{2.12}$$

in the above expression, R is the position vector of the center of mass of the whole system

$$\vec{R} = \frac{\sum_{i=1}^{A} m_i \vec{r}_i}{\sum_{i=1} m_i}$$

and

$$\vec{K} = \frac{1}{\hbar}\vec{P}$$

is the wave vector corresponding to the whole system's motion with total momentum $\vec{P}$. The wave function $\Psi^{(0)}$ is obtained from the solution Ψ of eq. (1.11) by means of the unitary operator generating finite translation[22]

$$\Psi^{(0)}(r_1 - R, r_2 - R, \ldots\ldots r_A - R) = \int d^3k' \int d^3r' \left[e^{iK'\cdot(r'-R)} e^{iK(R-r')} \Psi(r_i - r') \right] \tag{2.13}$$

It is easily verified that

$$H\Psi^{(0)} = E\Psi^{(0)} \tag{2.14}$$

and

$$\frac{\hbar}{i}\nabla_R \Psi^{(0)} = 0 \tag{2.15}$$

hence, $\Psi^{(0)}$ is not a function of R .

For large separations of all fragments, the wave function has the following asymptotic behaviour

$$\Psi^{(K)} \sim e^{iK\cdot R}\left\{ \mathcal{A}(\Phi_\alpha^{(0)}\psi_{\alpha k}) + \sum_\beta \mathcal{A}(\Phi_\beta^{(0)}\psi_{\beta k'}) \right\} \tag{2.16}$$

where

$$\Phi_{\alpha}^{(0)} = \Phi_{0_{A_1}}^{(0)}(\vec{r}_1-\vec{R}_1,\ldots,\vec{r}_{A_1}-\vec{R}_1)\,\Phi_{0_{A_2}}^{(0)}(\vec{r}_{A_1+1}-\vec{R}_2\ldots,\vec{r}_A-\vec{R}_2) \qquad (2.17)$$

is the product of the two wave functions describing the internal motion of the nucleons inside the fragments $[A_1]$ and $[A_2]$, and $\psi_{\alpha k}$ is the wave function of the relative motion of the fragments in channel α; the symbol $\mathcal{A}$ stands for the anti-symmetrizer of A objects.

$$\mathcal{A} = \frac{1}{\sqrt{A!}}\sum_{P}(-1)^{I_P}P \qquad (2.18)$$

In what follows, we shall assume that the wave functions $\Phi_{0_{A_1}}^{(0)}$ and $\Phi_{0_{A_2}}^{(0)}$ are completely antisymmetric with respect to the exchange of all internal coordinates in each fragment.

The wave functions describing the relative motion of the fragments in the various channels have the following asymptotic behaviour:

In the entrance (elastic) channel

$$\psi_{\alpha k} \xrightarrow[r_\alpha\to\infty]{} e^{ik\cdot r_\alpha} + f_{\alpha k}^{(+)}(\theta_\alpha)\frac{e^{ik_\alpha\cdot r_\alpha}}{r_\alpha}$$

and for inelastic and reaction channels

$$\psi_{\sigma k'} \xrightarrow[r_\alpha\to\infty]{} f_{\sigma}^{(+)}(\theta_\sigma)\frac{e^{ik'_\sigma\cdot r_\alpha}}{r_\sigma}$$

where

$$k_\alpha \cdot r_\alpha = k_\alpha \cdot r_\alpha - \eta_\alpha \ln 2 k_\alpha r_\alpha$$

and

$$k_\alpha^2 = \frac{2 M_\alpha \mathcal{E}_\alpha}{\hbar}$$

$$\eta_\alpha = \frac{M_\alpha Z_{A_1} Z_{A_2} e^2}{\hbar^2 k_\alpha}$$

The wave functions of the fragments and can be written as products of the corresponding center of mass motion and internal motion wave functions in exactly the same way as was done for the complete system's wave function

$$\Phi^{(k_1)}_{n_1 A_1}(\vec{r}_1 \cdots \vec{r}_{A_1}) = e^{i k_1 \cdot R_1} \Phi^{(0)}_{n_1 A_1}(\vec{r}_1 - \vec{R}_1 \cdots, \vec{r}_{A_1} - \vec{R}_1)$$

$$\Phi^{(k_2)}_{n_2 A_2}(\vec{r}_{A_1+1} \cdots \vec{r}_A) = e^{i k_2 \cdot R_2} \Phi^{(0)}_{n_2 A_2}(r_{A_1+1} - R_2 \cdots r_A - R_2) \qquad (2.19)$$

$\vec{R}_1$ and $\vec{R}_2$ denote the position vectors of the centers of mass of fragments $[A_1]$ and $[A_2]$, k_1 and k_2 are the wave vectors corresponding to the center of mass motion of each fragments. The functions $\Phi^{(0)}_{n_1 A_1}$ and $\Phi^{(0)}_{n_2 A_2}$ describing the internal state of fragments $[A_1]$ and $[A_2]$ satisfy the equations

$$H_{A_1}\Phi^{(0)}_{n_1A_1} = \mathcal{E}_{1n_1}\Phi^{(0)}_{n_1A_1}$$

$$H_{A_2}\Phi^{(0)}_{n_2A_2} = \mathcal{E}_{2n_2}\Phi^{(0)}_{n_2} \qquad (2.20)$$

and

$$\frac{\hbar}{i}\nabla_{R_1}\Phi^{(0)}_{n_1A_1} = 0$$

$$\frac{\hbar}{i}\nabla_{R_2}\Phi^{(0)}_{n_2A_2} = 0 \qquad (2.21)$$

and satisfy the orthogonality and completeness relations

$$\int \Phi^{(0)*}_{n'_{A_i}}(X_j)\,\Phi^{(0)}_{n_{A_i}}(X_j)\,d\zeta_{A_i} = \delta_{n_{A_i}n'_{A_i}} \qquad (2.22)$$

$$\sum \Phi_{n_{A_i}}(X'_1, X'_2 \cdots X_{A_i})\,\Phi_{n_{A_i}}(X_1, X_2 \cdots X_{A_i}) =$$

$$\delta^{(3)}(X'_1 - X_1) \cdots \delta^{(3)}(X_{A_i} - X_i)\,\mathbb{1} \qquad (2.23)$$

we recall that the index n_{A_i} denotes the set of quantum numbers characterizing the dynamical state of fragments $[A_i]$, where $\mathbb{1}$ is the unity operator in the Hilbert space appropriate for the description of the spin of fragment $[A_i]$.

3. ANTISYMMETRIZATION

The complete system's wave function has to be antisymmetric with respect to the exchange of any pair of nucleon indices. This condition introduces some complications in the formalism that can be deal with by means of a slight modification in Feshbach's formalism.[9]

In the first place, in the asymptotic region each two-body channel corresponds to some partition of A nucleons. The definition of the relative coordinate $\vec{r}_{A_1 A_2}$ depends on the way the nucleons are distributed in the two fragments. The numbers of different ways of distributing A nucleons in two fragments, containing A_1 and A_2 nucleons each, is

$$\frac{A!}{A_1!\,A_2!} = \frac{A!}{A_1!(A-A_1)!}$$

hence, there will be an equal number of possible ways of defining the relative coordinate, all of them corresponding to the same physical situation.

In what follows we shall take as the basic partition of indices, the following

$$[A_1]: \quad i \in \{1, 2 \cdots\cdots A_1\}$$

$$[A_2]: \quad i \in \{A_1+1, A_1+2, \cdots\cdots A\}$$

We will always assume that the wave functions describing the internal state of each fragment are completely antisymmetrical, hence, any permutation of nucleons belonging to the same fragment

will produce at most a change in sign. Thus, we shall consider only permutations of nucleons belonging to A_1 with nucleons belonging to A_2, these permutations can be classified according to the number, ℓ, of nucleons exchanged. There are

$\frac{A_1!}{\ell!(A_1-\ell)!}$ ways of choosing ℓ nucleons from A_1, and

$\frac{A_2!}{\ell!(A_2-\ell)!}$ ways of choosing ℓ nucleons from A_2, each set

of ℓ nucleons from A_1 is exchanged by ℓ nucleons from A_2 in all possible ways and the resulting number of ways of exchanging ℓ nucleons between the two fragments is

$$\hat{A}_\ell = \frac{A_1!\ (A-A_1)!}{(\ell!)^2\,(A_1-\ell)!\,(A-A_1-\ell)!}$$

$$\sum_{\ell=0}^{\min(A_1,A_2)} \hat{A}_\ell = \frac{A!}{A_1!\,(A-A_1)!}$$

let us call Ψ_K^d the direct term in channel K,

$$\Psi_K^d(r_{A_1A_2}, X_i, X_j) = \Phi_{n_{A_1}}^{(0)}(X_i)\,\Phi_{n_{A_2}}^{(0)}(X_j)\,\psi_{KK}(r_{A_1A_2}, X_i, X_j) \qquad (3.1)$$

where $K = (n_{A_1}, n_{A_2})$ and $i \in [A_1]$, $j \in [A_2]$. Then the asymptotic form of the completely antisymmetrical total wave function in channel K is:[21]

$$\mathcal{A}'\{\Phi^{(0)}_{n_{A_1}}\Phi^{(0)}_{n_{A_2}}\psi_{kk}\} = \Phi^{(0)}_{n_{A_1}}(x_i)\,\Phi^{(0)}_{n_{A_2}}(x_j)\,\psi^{d}_{kk}(r_{A_1A_2};x_i,x_j) +$$

$$+\sum_{l=1}^{\min(A_1,A_2)}(-1)^{l}\hat{A}_l\,\Phi^{(0)}_{n_{A_1}}(x'_i)\,\Phi^{(0)}_{n_{A_2}}(x'_j)\,\psi_{kk}(r'_{A_1A_2};x'_i x'_j)$$

(3.2)

in the above expression, $\hat{A}_l$ counts the different ways of exchanging l particles. The index l appearing in the wave functions means that the term so labeled has been obtained from the direct term by the exchange of l nucleon indices belonging to the first fragment by l nucleon indices belonging to the second fragment, the symbol $\mathcal{A}'$ is defined by equation (3.2).

4. PROJECTORS

Feshbach and others have shown that an effective Schrödinger equation describing the relative motion of projectile and target in the elastic channel can be obtained by formally decoupling all non-elastic channels from the elastic one, with the help of projection operators and the appropriate Green's function of the complete problem[9,10,11]. In what follows, we define a projection operator P_α, such that $P_\alpha \Psi^{(K)}$ contains all the information about the elastic scattering including the Pauli principle and Galilean invariance constraints. We also require of $P_\alpha \Psi$ to be continuous and regular and have first and second space derivatives for all values of the relative distance $|r_{A_1,A_2}|$. Accordingly, P_α satisfies the following equation

$$\Phi^{(0)*}_{0_{A_1}}(X_1 \cdots X_{A_1})\,\Phi^{(0)*}_{0_{A_2}}(X_{A_1+1} \cdots X_A)\,P_\alpha \Psi^{(0)}(r_1 - R \cdots r_A - R)\,d\zeta_{A_1} d\zeta_{A_2}$$

$$= \int \Phi^{(0)*}_{0_{A_1}}(X_1 \cdots X_{A_1})\,\Phi^{(0)*}_{0_{A_2}}(X_{A_1+1} \cdots X_A)\,\Psi^{(0)}(r_i - R_m + R_m - R)\,d\zeta_{A_1} d\zeta_{A_2}$$

(4.1)

$$m = \begin{cases} 1 & \text{if } i \in [A_1] \\ 2 & \text{if } i \in [A_2] \end{cases}$$

The component $\psi_{\alpha k}$ of $\Psi^{(K)}$ describing the relative motion of target and projectile in the elastic channel is defined by the equation

$$\Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\left[\Psi^{(0)}(\vec{r}_i - R) - \mathcal{A}'\left[\Phi^{(0)}_{0_{A_1}}(X_i)\,\Phi^{(0)}_{0_{A_2}}(X_j)\,\psi_{\alpha k}(r)\right]\right] d\zeta_{A_1}\, d\zeta_{A_2} = 0 \tag{4.2}$$

which, together with equation (4.1), is equivalent to

$$P_\alpha \Psi^{(K)}(r_i - R_m + R_m - R) = e^{iK\cdot R}\,\mathcal{A}'\left[\Phi^{(0)}_{0_{A_1}}(X_i)\,\Phi^{(0)}_{0_{A_2}}(X_j)\,\psi_{\alpha k}(r)\right] \tag{4.3}$$

the difference $R - R_m$ is proportional to the relative coordinate

$$R - R_m = \begin{cases} + \dfrac{M_2}{M}\,\vec{r} & m = 1 \\ - \dfrac{M_1}{M}\,\vec{r} & m = 2 \end{cases}$$

so that after integrating over the internal coordinates in (4.2), only the relative coordinate remains.

The operator $\mathcal{A}'$ appearing in (4.2) and (4.3) exchanges particles from the target and projectile. Calling $\mathcal{A}_1$ and $\mathcal{A}_2$ the antisymmetrizer for target and projectile we have the following relation

$$\mathcal{A} = \mathcal{A}_1\,\mathcal{A}_2\,\mathcal{A}_{12}$$

therefore

$$\mathcal{A}_{12} = \sqrt{\frac{A_1!A_2!}{A!}} \sum_{P'} (-1)^{I_{P'}} P' \tag{4.6}$$

$$\mathcal{A}_{12} = \sqrt{\frac{A_1!A_2!}{A!}}\ \mathcal{A}'$$

P' denotes all possible permutations of indices in $[A_1]$ with indices in $[A_2]$, including the case when no indices are permuted).

The asymptotic form of $\psi_{\alpha k}$ gives the complete scattering amplitude, with all direct and exchange terms.

Equation (4.2) can be written as

$$U_\alpha(r) = \psi_\alpha(r) + \int K(r-r')\psi_\alpha(r')d^3r' \tag{4.4}$$

where $U_\alpha(r)$ is defined as

$$U_\alpha(r) = \int \Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\,\Psi^{(0)}(r_1-R\cdots r_A-R)\,d\zeta_{A_1}d\zeta_{A_2} \tag{4.4'}$$
$$i\in[A_1]$$
$$j\in[A_2]$$

the kernel $K(r-r')$ is

$$K(r-r') = \Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\Big\{\sum_{l=1}^{\min(A_1,A_2)}(-1)^l\hat{A}_l\,\Phi^{(0)}_{0_{A_1}}(X'_i)\,\Phi^{(0)}_{0_{A_2}}(X'_j)\Big\}d\zeta_{A_1}d\zeta_{A_2} \tag{4.5}$$

From the definition (4.5) it is easy to verity that the kernel $K(r-r')$ is hermitian, bounded and not positive definite due to changes in sign required by antisymmetrizations, furthermore, K decreases exponentially when $r-r'$ goes to infinity. Hence, the eigenvalues of K are real and their absolute value is smaller or eqtal to one

An explicit expression for P_a will be obtained by solving equation (4.4) for ψ_{ak} in terms of $U_a(r)$ and then substituting the resulting expression in equation (4.2)

The eigenfunctions of K satisfy the equation

$$\frac{1}{\lambda_m}\mu_m(r) = \int K(r-r')\mu_m(r')d^3r' \qquad (4.7)$$

and K can be expressed in terms of μ_m as

$$K(r-r') = -\sum_m \frac{\mu_m(r)\mu_m^+(r')}{\lambda_m} \qquad (4.8)$$

and equation (4.4) becomes

$$U_a(r) = \psi_{ak}(r) - \sum_m \frac{1}{\lambda_m}\mu_m(r)\int \mu_m^+(r')\psi_{ak}(r')d^3r'$$

In order to solve for ψ_{ak} in terms of U_a we use the orthonormalityof the set of eigenfunctions μ_m and we obtain the following relations. When $\lambda_m \neq 1$

$$\int \mu_m^+(r')\psi_{ak}(r')d^3r' = \frac{\lambda_m}{\lambda_m - 1}\int \mu_m^+(r')U_a(r')d^3r'$$

When $\lambda_m = 1$

$$\int \mu_1^+(r')U_\alpha(r')d^3r' = \mu_1^+(r)\int \Phi^{(0)*}_{0_{A_1}}(x_i)\Phi^{(0)*}_{0_{A_2}}(x_j)d\zeta_{A_1}d\zeta_{A_2}$$

these relations imply that the component of Ψ in the elastic channel has no contribution coming from terms proportional to the eigenfunction μ_1 of K corresponding to eigenvalues $\lambda_m = 1$ Furthermore, when $\lambda_m = 1$, the eigenvalue equation for K is equivalent to

$$\int \Phi^{(0)*}_{0_{A_1}}(x_i)\Phi^{(0)*}_{0_{A_2}}(x_j)\left[\mathcal{A}'\left[\Phi^{(0)}_{0_{A_1}}(x_i)\Phi^{(0)}_{0_{A_2}}(x_j)\psi_{\alpha k}(r)\right]\right]d\zeta_{A_1}d\zeta_{A_2} = 0$$

which means that these eigenfunctions do not contribute to the second term in equation (4.3), using the above stated results, $\psi_{\alpha k}$ can be expressed in terms of U_α

$$\psi_{\alpha k}(r) = U_\alpha(r) + \sum \frac{1}{\lambda_m - 1}\mu_m(r)\int \mu_m^+(r')U_\alpha(r')d^3r'$$

It is convenient now to substitute for U_α its explicit expression (4.5) then

$$\psi_{\alpha k}(r) = \int \Phi^{(0)*}_{0_{A_1}}(x_i)\Phi^{(0)*}_{0_{A_2}}(x_j)\Psi^{(0)}(r_i - R)d\zeta_{A_1}d\zeta_{A_2} +$$

$$\sum_{\substack{m \\ \lambda_m \neq 1}} \frac{1}{\lambda_m - 1}\mu_m(r)\int \mu_m^+(r')\Phi^{(0)*}_{0_{A_1}}(x_i)\Phi^{(0)*}_{0_{A_2}}(x_j)\Psi^{(0)}(r_i - R)d\zeta_{A_1}d\zeta_{A_2} \qquad (4.9)$$

An explicit expression for the projector operator P_α as an integral operator can now be easily obtained, comparison of the above expression for $\psi_{\alpha k}$ with the definition of $P_\alpha \Psi$ given in equations (4.1) and (4.2) gives

$$P_\alpha \Psi^{(0)} = \int \frac{\mathcal{A}'}{\sqrt{\frac{A!}{A_1!A_2!}}} \left[\Phi^{(0)}_{0_{A_1}}(x_i) \Phi^{(0)}_{0_{A_2}}(x_j) \mathcal{O}\!\!\!/(r-r') \Phi^{(0)*}_{0_{A_1}}(x_i) \Phi^{(0)*}_{0_{A_2}}(x_j) \right] \cdot \frac{\mathcal{A}'}{\sqrt{\frac{A!}{A_1!A_2!}}} \Psi^{(0)}(r_i'-R)\, dz_{A_1} dz_{A_2} d^3r' \tag{4.10}$$

where

$$\mathcal{O}\!\!\!/(r-r') = \delta^{(3)}(r-r') + \sum_{\lambda_m \neq 1} \mu_m(r) \frac{1}{\lambda_m - 1} \mu_m^*(r') \tag{4.11}$$

and

$$\sum_l \hat{A}_l = \frac{A!}{A_1!A_2!} \quad , \qquad \hat{A}_l = \frac{A_1!}{l!(A_1-l)!} \cdot \frac{A_2!}{l!(A_2-l)!}$$

using this notation, relation (4.9) becomes

$$\psi_{\alpha k} = \int \mathcal{O}\!\!\!/(r-r') U_\alpha(r') d^3r' \tag{4.9'}$$

The hermiticity of P_α is obvious from equation (4.10)

$$P_\alpha = P_\alpha^\dagger \tag{4.12}$$

The idempotency of P_α can be verified multiplying both members of equation (4.3) times $\psi_{\alpha k}(r)$, integrating over r, and taking into account that the function in brackets is antisymmetric, in this way we obtain the following relation

$$\mathcal{A}'\{\Phi^{(0)*}_{0_{A_1}}(x_i)\,\Phi^{(0)*}_{0_{A_2}}(x_j)\,\psi_{\alpha k}(r)\cdot$$
$$[\Psi^{(0)}(r_i-R)-\mathcal{A}'\{\Phi^{(0)}_{0_{A_1}}(x_i)\,\Phi^{(0)}_{0_{A_2}}(x_j)\,\psi_{\alpha k}(r)\}]\,d\tau_{A_1}d\tau_{A_2}=0$$

which is equivalent to

$$\int [P_\alpha\Psi]^\dagger[(1-P_\alpha)\Psi]\,d\tau_{A_1}d\tau_{A_2}=0$$

or

$$\int \Psi^\dagger[P_\alpha(1-P_\alpha)]\Psi\,d\tau_{A_1}d\tau_{A_2}=0$$

from here, it follows immediately that

$$P_\alpha(1-P_\alpha)=0$$

or

$$P_\alpha=P_\alpha^2 \qquad (4.13)$$

relations (4.12) and (4.13) show that P_α is a projection operator.

Finally, the condition (4.2) stating that $P_\alpha\Psi$ contains all the elastic scattering information contaimed in Ψ

can be verified when the expression for $\psi_{\alpha k}$ in terms of U_α and $\mathcal{A}$, is inserted in

$$\int \Phi^{(0)*}_{0_{A_1}}(x_i)\,\Phi^{(0)*}_{0_{A_2}}(x_j)\,P_\alpha\,\Psi^{(0)}(r_i - R)\,d\tau_{A_1}\,d\tau_{A_2} =$$

$$\int \Phi^{(0)*}_{0_{A_1}}(x_i)\,\Phi^{(0)*}_{0_{A_2}}(x_j)\,\mathcal{A}'\left[\Phi^{(0)}_{0_{A_1}}(x_i)\,\Phi^{(0)}_{0_{A_2}}(x_j)\,\psi_{\alpha k'}(r')\right]d\tau_{A_1}\,d\tau_{A_2}$$

Furthermore, the relation

$$U_\alpha(r) = \int \Phi^{(0)*}_{0_{A_1}}(x_i)\,\Phi^{(0)*}_{0_{A_2}}(x_j)\,\mathcal{A}'\left[\Phi^{(0)}_{0_{A_1}}(x_i)\,\Phi^{(0)}_{0_{A_2}}(x_j)\,\psi_{\alpha k}(r)\,d\tau_{A_1}\,d\tau_{A_2}\right.$$

is obtained, which means that the asymptotic form of U_α also has the complete elastic scattering amplitude with all direct and exchange terms.

The orthogonal complement of P_α will be denoted by Q_α.

$$Q_\alpha = 1 - P_\alpha \tag{4.14}$$

5. EFFECTIVE INTERACTION BETWEEN TWO COMPLEX NUCLEI.

Once the projection operator on the elastic channel is obtained, the derivation of the effective Schrödinger equation is straightforward [9].

The Schrödinger equation for the complete system

$$(E - H)\Psi = 0 \tag{5.1}$$

is equivalent to the following two coupled equations

$$(E - P_\alpha H P_\alpha) P_\alpha \Psi = P_\alpha H Q_\alpha (Q_\alpha \Psi) \tag{5.2}$$

$$(E - Q_\alpha H Q_\alpha) Q_\alpha \Psi = Q_\alpha H P_\alpha (P_\alpha \Psi) \tag{5.3}$$

The wave function Ψ described the elastic and inelastic scattering as well as all possible reactions occurring at energy E. Only the elastic component $P_\alpha \Psi$ of the wave function has incoming waves in the asymptotic region. The term $Q_\alpha \Psi$, describing all inelastic processes and the closed channels, has no incoming waves in the asymptotic region. Hence, the formal solution to equation (5.3) has the following form

$$Q_\alpha \Psi = \lim_{\eta \to 0^+} Q_\alpha \frac{1}{E + i\eta - Q_\alpha H Q_\alpha} Q_\alpha H P_\alpha (P_\alpha \Psi) \tag{5.4}$$

where

$$Q_\alpha \frac{1}{E + i\eta - Q_\alpha H Q_\alpha} Q_\alpha$$

is the resolvent in the Q_α subspace, and it can be constructed with the help of solutions of the Schrödinger equation in the Q_α space[24]

$$(Q_\alpha H Q_\alpha - E_i)\chi_i^{(+)} = 0 \tag{5.5}$$

that satisfy the Gamow condition of outgoing waves only in the asymptotic region for all channels in Q space.

The physical meaning of the solutions of the equation (5.5) is transparent if it is recalled that the compound nucleus states have a very long lifetime, compared with the collision times typical of the direct elastic collisions described by the optical model, this means that the contribution to the outgoing prompt current in the elastic channel coming from the decay of the compound nucleus is very small. When the energy is below the first inelastic threshold, all non-elastic states would be stationary states described by wave functions satisfying equation (5.5), when the energy is above the first inelastic threshold there would be outgoing waves in the inelastic channels and the inelastic states would be described by solutions of (5.5) having only outgoing waves in the asymptotic regions and correspondingly having complex eigenvalues of the energy[15]. In this case, the wave functions $\chi^{(+)}$ can be interpreted as compound nucleus

states when it is assumed that the probability for the nucleons in the compound nucleus rearranging themselves so as to form two clusters which, moving coherently, fly apart and appear in the asymptotic region as two fragments identical to projectile and target is equal to zero.

The resolvent operator appearing in (5.4) can be written in terms of the solutions $\chi^{(+)}$ of the homogeneous equation (5.5)

$$Q_\alpha \frac{1}{E^{(+)} - Q_\alpha H Q_\alpha} Q_\alpha = \sum\!\!\!\!\!\!\int \chi^{(+)}(r'-R \cdots r_A - R) \cdot \frac{1}{E^{(+)} - E_i} \chi_i^{(+)\dagger}(r_1 - R, r_2 - R \cdots r_A - R) \tag{5.6}$$

then equation (5.4) takes the form

$$Q_\alpha \Psi^{(0)} = \sum_{E_i}\!\!\!\!\!\!\int \chi_i^{(+)} \frac{1}{E^{(+)} - E_i} \left(\int \chi_i^{(+)\dagger} H P_\alpha \Psi^{(0)} d\zeta_A \right) dE_i \tag{5.7}$$

substituting in (5.2), the many-body effective equation in the elastic channel is obtained

$$P_\alpha H P_\alpha (P_\alpha \Psi^{(0)}) + \sum_{E_i}\!\!\!\!\!\!\int P_\alpha H \chi_i^{(+)} \frac{1}{E^{(+)} - E_i} \left(\int \chi_i^{(+)\dagger} H P_\alpha \Psi^{(0)} d\zeta_A \right) dE_i = E P_\alpha \Psi^{(0)} \tag{5.8}$$

The effective Schrödinger equation describing the relative motion of projectile and target in the elastic channel is obtained from the above equation multiplying both terms by $\Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)$ and integrating over the internal coordinates of the nucleus in the fragments.

From the first term we get

$$\int \Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\,P_\alpha H P_\alpha \Psi^{(K)}\,d\zeta_{A_1}\,d\zeta_{A_2} =$$

$$e^{iK\cdot R}\int\Big[\delta^{(3)}(r-r')\Big(\varepsilon_{A_1}+\varepsilon_{A_2}-\frac{\hbar^2}{2\mu}\nabla^2_{r'}+V_{12}(r')\Big)+$$

$$\mathcal{H}(r-r')\Big]\psi_{\alpha k}(r')\,d^3r' \qquad (5.9)$$

where V_{12} is the direct term

$$V^{D}_{12}(r) = \int V_{A_1A_2}(X_j - X_i + r)\,\Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\,\Phi^{(0)}_{0_{A_2}}(X_j)\,d\zeta_{A_1}\,d\zeta_{A_2}$$

can be written as a direct double folding potential

$$V_{12} = \int V(X_{A_1+1} - X_1 + r)\,\rho_{A_2}(X_{A_1+1})\,\rho_{A_1}(X_1)\,d^3X_1\,d^3X_{A_1+1} \qquad (5.10)$$

$$\int \mathcal{H}(r-r')\,\psi_{\alpha k}(r')\,d^3r' \;=$$

$$\int \Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\,H\left[(\mathcal{A}'-1)\,\Phi^{(0)}_{0_{A_1}}(X_i)\,\Phi^{(0)}_{0_{A_2}}(X_j)\,\psi_{\alpha k}(r)\right] d\zeta_{A_1} d\zeta_{A_2} \qquad (5.11)$$

is the exchange term.

The permuted internal variables occurring in the argument of $[(\mathcal{A}'-1)\,\Phi^{(0)}_{0_{A_1}}(X_i)\,\Phi^{(0)}_{0_{A_2}}(X_j)\,\psi_{\alpha k}(r)]$ can be expressed in terms of the non-permuted internal variables and the difference of the relative coordinates defined before and after the permutation, this gives rise to the $r-r'$ dependence of $\mathcal{H}$.

Let us consider now the second term appearing in the left hand side of equation (5.8), we notice the relation

$$r_i - R = \begin{cases} r_i - R_1 + R_1 - R = X_i - \dfrac{m_2}{M}\vec{r} & i \in [A_1] \\ r_i - R_2 + R_2 - R = X_i + \dfrac{m_1}{M}\vec{r} & i \in [A_2] \end{cases}$$

permitting the integration over the internal variables of the two fragments. It is convenient to introduce the notation for the form factors

$$W_i(r) = \int \Phi^{(0)*}_{0_{A_1}}(X_i)\,\Phi^{(0)*}_{0_{A_2}}(X_j)\,H\chi^{(+)}_i(r_i-R)\,d\zeta_{A_1} d\zeta_{A_2} \qquad (5.12)$$

in this way, we get the following expression

$$\sum_{E_\lambda}\!\!\!\!\!\!\int w_\lambda(r)\frac{1}{E^{(+)}-E}\tilde{w}_\lambda^{*}(r')\mathcal{A}(r'-r'')U(r'')\,d^3r'\,d^3r'' \tag{5.13}$$

where $\mathcal{A}$ and $U_{\alpha k}$ were defined in (4.5) and (4.10). Since the Green's function for the complete system is trnslationally invariant, the function

$$W(r-r') = \sum_{E_\lambda}\!\!\!\!\!\!\int w_\lambda(r)\frac{1}{E^{(+)}-E_\lambda}\tilde{w}_\lambda^{*}(r') \tag{5.14}$$

depends only on the difference of the relative coordinates defined before and after the permutations implicit in $\mathcal{A}'$ are performed.

Recalling the relation

$$\psi_{\alpha k}(r) = \int \mathcal{A}(r-r')U_{\alpha k}(r')\,d^3r' \tag{5.15}$$

where

$$U(r) = \int[\delta^{(3)}(r-r') + K(r-r')]\psi_{\alpha k}\,d^3r'$$

and collecting all terms, we get the effective equation describing the relative motion of the two fragments in the elastic channel

$$-\frac{\hbar^2}{2\mu}\nabla_r^2\psi_{\alpha k}(r) + V_{12}^{D}(r)\psi_{\alpha k}(r) \quad + \int \mathcal{H}(r-r')\psi_{\alpha k}(r')\,d^3r' - E\int K(r-r')\psi_{\alpha k}(r')\,d^3r' + \int W(r-r')\psi_{\alpha k}(r')\,d^3r' = \mathcal{E}_\alpha\psi_{\alpha k}(r) \tag{5.16}$$

The effective interaction operator $\mathcal{U}(r-r')$ can be read of equation (5.16) and is equal to

$$\mathcal{U}(r-r') = V_{12}^{D}(r)\delta^{(3)}(r-r') + \mathcal{H}(r-r') - EK(r-r') + W(r-r') \tag{5.16'}$$

The kernel $[V_{12}^{D}(r)\delta^{(3)}(r-r') + \mathcal{H}(r-r') - EK(r-r')]$ resulting from our calculation has exactly the same functional dependence on the nucleon-nucleon potential and the fragment's wave function as the one appearing in the effective interaction kernel obtained in the resonating group method (RGM) by means of a variational calculation with a cluster model test wave function[14,16], it follows that if we approximate the exact wave functions $\Phi_{0_{A_1}}$ and $\Phi_{0_{A_2}}$ appearing in our kernel by the wave functions used in the cluster model the two effective interaction kernels coincide.

The last term appearing in the left hand side of equation (5.16) gives the contribution to the effective interaction operator coming from the compound nucleus formation and decay, therefore it is a rapidly varying function of the incoming energy as can be seen from (5.14). This rapid energy variation is due to the defining condition put on the effective interaction, i.e. it has to reproduce the elastic cross section in full detail including the sharp compound nucleus resonances. Since the decay of the compound nucleus states doesn't go all through the elastic channel, W is not hermitian and it has a negative imaginary part accounting for the flux loss in that channel. The summation over the energy states in (5.16) can be written in terms

of the density of energy states, so that the imaginary part of can be exhibited.

$$W(r-r') = \Big[\sum_i w_i(r)\,\frac{1}{E-E_i}\,\tilde{w}_i^*(r') + \int_{\epsilon}^{\infty} \rho_c(E',K)\, w_{E',K}(r)\,\frac{1}{E-E'}\,\tilde{w}_{E',K}^*(r')\,dE' - i\pi\,\rho_c(E',K)\, w_{E,K}(r)\,\tilde{w}_{E,K}^*(r')$$

the index i runs over the discrete spectrum of $Q_\alpha H Q_\alpha$, $\rho_c(E',K)$ denotes the density of energy states in the continuum part of the spectrum of $Q_\alpha H Q_\alpha$, and K represents all the additional quantum numbers necessary to characterize the eigenstates of the same hamiltonian.

As was pointed out above, the rapid energy variation appearing in the effective interaction operator we have just found, is such that it has to reproduce the elastic scattering amplitude in full detail[17], and therefore, this effective operator is not equal to the optical model potential operator, since this last one is defined as that potential operator that, when inserted in the Schrödinger equation of the relative motion gives rise to an optical elastic amplitude equal to the energy average of the complete system's elastic amplitude.

6. THE OPTICAL POTENTIAL

The many - body character of the collision of two complex nuclei makes itself apparent, in the occurrence in the cross sections, of the narrow compound nucleus resonances and energy fluctuations. This in turn means that the elastic scattering amplitudes are rapidly varying functions of the energy. Since the effective interaction operator for the elastic channel was constructed in such a way that the solutions of the effective Schrödinger equation reproduce in full detail the elastic scattering amplitude of the complete problem, the effective interaction - operator depends on the energies and wave functions of the compound nucleus states and it is also a rapidly varying function of the energy.

On the other hand, the optical potential, by definition, has to reproduce the elastic amplitude averaged over the energy in such a way that the elastic cross section and polarization calculated with this potential agree with the average over the energy of the measured elastic scattering cross sections and polarization[18].

$$T_{opt.} = \langle T_{el} \rangle \qquad (6.1)$$

The optical potential will be obtained from (6,1) following the general ideas in paper Feshbach's [9]. Firstly, the elastic scattering amplitude is split in two parts, one reproducing the sharp energy variation due to the compound nucleus reso-

nances, and the other varying slowly with energy, the average over the energy of the elastic amplitude is identified with the optical scattering amplitude. Then the relation between $T_{opt.}$ and $V_{opt.}$ is inverted exactly and explicitly. It is shown, in a quite general way, that the imaginary part of the optical potential is negative inthout making any further assumptions. It is also shown, in the same general fashion, that the optical potential operator is a continuous function of the width I of the averaging weight function, that goes to the detailed effective potential operator when I goes to zero, establishing in this way, that in our optical potential operator, the information about the compound elastic part of the scattering amplitude left after taking the average over the energy is correctly represented.

The resonating fine structure in $T_{el.}$ can be exhibited explicitly splitting the effective interaction operator in two terms, one varying rapidly with energy, which is that part of W containing small energy denominators, and the remaining part which varies slowly with energy. The rapidly varying part of the effective interaction is a sum of N terms giving rise to the N compound nucleus resonances comprised in the spectral energy domain $\mathcal{D}$ of widh Δ and centered around E .

$$\mathcal{U} = \mathcal{U}^{(N)}(r,r') + \sum_{\substack{j=1 \\ E_j \in \mathcal{D}}} \frac{w_j(r)\,\tilde{w}_j^*(r')}{E^{(+)} - E_j} \tag{6.2}$$

where $\mathcal{U}$ was defined in (5.16').

The elastic transition amplitude is then written as

$$T_{el.} = \int e^{-ikr} u^{(N)}(r,r') \chi^{(+)}_{\alpha k}(r') dr^3 dr'^3 + \int \chi^{(-)}_{\alpha k}(r) W^{(N)}(r,r') \psi^{(+)}_{\alpha k}(r') dr'^3 dr^3 \tag{6.3}$$

where $\chi^{(+)}_{\alpha k}(r)$ is the solution of the slowly varying part of the effective hamiltonian

$$\frac{\hbar^2}{2\mu} \nabla_r^2 \chi^{(+)}_{\alpha k}(r) + \int u^{(N)}(r,r') \chi^{(+)}_{\alpha k}(r') dr'^3 = \varepsilon_\alpha \chi^{(+)}_{\alpha k}(r) \tag{6.4}$$

with outgoing waves asymptotic conditions.

The rapidly energy varying interaction term $W^{(N)}$ is a separable potential of rank N and, therefore when we assume that $\chi^{(+)}_{\alpha k}$ is known, the solution of the effective equation is reduced to the solution of a secular problem of order N.

Writing.

$$\psi^{(+)}_{\alpha k} = \Omega_+ \chi^{(+)}_{\alpha k} \tag{6.5}$$

the compound nucleus part of the elastic scattering amplitude becomes

$$\iint \chi^{(-)}(r) W^{(N)}(r,r') \psi^{(+)}_{\alpha k}(r') dr^3 dr'^3 = \iint \chi^{(-)}_{\alpha k}(r) W(r,r') \Omega_+ \chi^{(+)}_{\alpha k}(r') dr^3 dr'^3 \tag{6.6}$$

The Møller operator Ω_+ satisfies the Lippmann-Schwinger equation

$$\langle r'|\Omega_+|r\rangle = \delta^3(r'-r) + \sum_{j=1} \int \langle r'| \frac{}{E_a - \mathcal{H}^N} |r''\rangle \cdot$$
$$\int \frac{w_j(r'')\tilde{w}_j^*(r''')}{E^{(+)} - E_j} \langle r'''|\Omega_+|r\rangle d^3r''' d^3r''$$

(6.7)

which is equivalent to

$$\sum_{i=1}^{N} [E\delta_{ij} - |H_{ij}] \frac{1}{E^{(+)} - E_j} v_j^*(r) = \tilde{w}_i^*(r) \qquad (6.7')$$

we notice that $|H_{ij}$ is not a hermitian matrix

$$|H_{ij} = E_i\delta_{ij} + \langle w_i| \frac{P}{E - \mathcal{H}^N} |w_j\rangle - i\pi \langle w_i|\Im_{ak}^{(+)}\rangle\langle\Im_{ak}^{(+)}|w_j\rangle \qquad (6.8)$$

the diagonal terms of $|H$ give the physical resonances in the lowest order approximation

$$E_i^{(1)} \approx \qquad + \Delta_i^{(1)} - i\frac{1}{2}\Gamma_i^{(1)}$$

where

$$\Delta_i^{(1)} = \langle w_i| \frac{1}{E - \mathcal{H}^N} |w_i\rangle$$

and

$$\Gamma_i^{(1)} = 2\pi |\langle w_i|\Im_{ak}^{(+)}\rangle|^2$$

the v_j^* appearing in (6.7') is

$$v_j^*(r) = \int \tilde{w}_j^*(r')\langle r'|\Omega_+|r\rangle d^3r' \tag{6.9}$$

The solution of (6.7) and (6.7') can be easily obtained in terms of the eigenvalues and eigenvectors of H

$$\begin{aligned} H\xi^\lambda &= E_\lambda \xi^\lambda \\ \tilde{\xi}^\lambda H &= E_\lambda \tilde{\xi}^\lambda \end{aligned} \tag{6.11}$$

then, the Møller operator Ω_+ is

$$\langle r'|\Omega_+|r\rangle = \delta^{(3)}(r'-r) + \sum_{j=1}^{N}\Big[\int\langle r'|\frac{1}{E^{(+)}-\mathcal{H}^N}|r\rangle w_j(r'')d^3r'' \times$$

$$\sum_{i=1}\sum_{\lambda} \xi_j^\lambda \frac{1}{E^{(+)}-E_\lambda}\tilde{\xi}_i^\lambda \tilde{w}_i^*(r)\Big\} \tag{6.12}$$

For this expression we get the relative motion wave function $\psi_{\alpha k}^{(+)}(r)$ expressed in terms of $\xi_{\alpha k}^{(+)}$.

$$\psi_{\alpha k}^{(+)}(r) = \xi_{\alpha k}^{(+)}(r) + \sum_{\lambda=1}\frac{\tilde{\gamma}_\lambda}{E^{(+)}-E_\lambda}\sum_{j=1}^{N}\xi_j^\lambda\int\langle r|\frac{1}{E^{(+)}-\mathcal{H}^N}|r''\rangle w_j(r'')d^3r'' \tag{6.13}$$

and the compound elastic transition amplitude

$$\langle\xi_{\alpha k}^{(-)}|W^{(N)}|\psi_{\alpha k}^{(+)}\rangle = \sum_{\lambda}^{N}\frac{\gamma_\lambda\tilde{\gamma}_\lambda}{E^{(+)}-E_\lambda} \tag{6.14}$$

where the reduced width $\tilde{\gamma}_\lambda$ is given by the expression

$$\tilde{\gamma}_\lambda = \sum_{i=1} \tilde{\xi}_i^\lambda \int \tilde{w}_i^*(r)\, \zeta_{ik}^{(+)}(r)\, d^3r \qquad (6.15)$$

finally, the elastic amplitude T_{el} takes the form

$$T_{el} = T^{(N)} + \sum_{\lambda=1}^{N} \frac{\gamma_\lambda \tilde{\gamma}_\lambda}{E^{(+)} - E_\lambda} \qquad (6.16)$$

where

$$T^{(N)} = \iint e^{-ikr} u^{(N)}(\vec{r}-\vec{r}')\, \zeta_{ik}^{(+)}(\vec{r}')\, d^3r\, d^3r'$$

in equation (6.16), the fine resonating structure of T_{el} is evident.

The average over the energy of T_{el} is

$$\langle T_{el} \rangle = T^{(N)} + \sum_{\lambda=1}^{N} \gamma_\lambda C_\lambda \tilde{\gamma}_\lambda \qquad (6.17)$$

where

$$C_\lambda = \int f_I(E-E') \frac{1}{E' - E_\lambda}\, dE' \qquad (6.18)$$

and $f_I(E-E')$ is some normalized weight function of width I, symmetrical around zero and piecewise continuous.

In order to get the optical potential $V_{opt.}$ from (6.1), the optical model Schrödinger equation is written as

$$(\mathcal{H}^{(N)} - \epsilon_\alpha)\varphi_{opt.} = -(V_{opt.} - U^{(N)}) \tag{6.19}$$

then $T_{opt.}$ can be written as

$$T_{opt.} = T^{(N)} + \mathcal{C} \tag{6.20}$$

where $\mathcal{C}$ is defined as

$$\mathcal{C} = \int \mathcal{J}^{(-)*}_{\alpha k}(\vec{r})(V_{opt.} - U^{(N)})\varphi_{opt.}(\vec{r}')\, d^3r\, d^3r' \tag{6.20'}$$

The defining relation (6.1) together with (6.17) and (6.20) give the relation

$$\int \mathcal{J}^{(-)*}_{\alpha k}(\vec{r}')(V_{opt.} - U^{(N)})\varphi_{opt.}(r')d^3r d^3r' = \sum_{\lambda=1}^{N} \gamma_\lambda \mathbb{C}_\lambda \tilde{\gamma}_\lambda \tag{6.21}$$

recalling the definition of γ_λ we get

$$(V_{opt.} - U^{(N)})\varphi_{opt}(r) = \sum_{\lambda=1}^{N}\sum_{i=1}^{N} \xi_i^\lambda W_i(r)\mathbb{C}_\lambda \tilde{\gamma}_\lambda \tag{6.22}$$

When this expression is substituted in (6.19), the resulting equation has the form of a Schrödinger equation only if the right hand side of (6.20) is proportional to $\varphi_{opt.}$. Hence, we write

$$\sum_{\lambda=1}^{N} \xi_i^{\lambda} C_{\lambda} \tilde{\gamma}_{\lambda} \equiv \sum_{j=1}^{N} \alpha_{ij} \int W_j^{*}(r') \varphi_{opt.}(r') d^3r' \qquad (6.23)$$

therefore V_{opt} takes the form

$$V_{opt} = U^{(N)}(\vec{r}, \vec{r}') + \sum_{i=1}^{N} \sum_{j=1}^{N} W_i(r) \alpha_{ij} W_j^{*}(r') \qquad (6.24)$$

where α_{ij} is a $N \times N$ matrix that mo st be determined from the condition (6.21).

In order to solve (6.21) for α we require the explicit dependence of C on α_{ij}, this can be obtained with the help of the Moller operator $\Omega_{+\,opt}$ defined as

$$\varphi_{opt.} = \Omega_{+\,opt} J^{(+)} \qquad (6.25)$$

then $\Omega_{+\,opt.}$ satisfies the Lippmann - Schwinger equation

$$\langle r | \Omega_{+\,opt} | r' \rangle = \delta^{(3)}(\vec{r} - \vec{r}') +$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \left\{ \int \langle r | \frac{}{E^{+} - \mathcal{H}^{(N)}} | r' \rangle W_i(r') d^3r' \times \right.$$

$$\left. \times \alpha_{ij} \int W_j^{*}(r'') \langle r'' | \Omega_{+\,opt} | r' \rangle d^3r'' \right\} \qquad (6.26)$$

which is easily solved to give

$$\langle r|\Omega_{+\,opt.}|r'\rangle = \delta^{(3)}(\hat{r}-\hat{r}') +$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\langle r|\frac{1}{E^{(+)}-\mathcal{H}^{(N)}}|w_i\rangle\left([\alpha^{-1}+\mathcal{E}-H]^{-1}\right)_{ij}\tilde{w}_j^{*}(r')\right\} \qquad (6.26')$$

where

$$\mathcal{E}_{ij} = E_i\,\delta_{ij}$$

and

$$(\mathcal{E}-H)_{ij} = -\langle w_i|\frac{1}{E^{(+)}-\mathcal{H}^{(N)}}|w_j\rangle \qquad (6.8)$$

The transition matrix is obtained from the relation

$$\mathcal{T} = \langle\zeta^{(-)}|\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i\,\alpha_{ij}\,\tilde{w}_j^{*}\right)\Omega_{+\,opt.}|\zeta^{(+)}\rangle$$

which gives

$$\mathcal{T} = \sum_{\lambda=1}^{N}\sum_{\lambda'=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_\lambda\,\tilde{\xi}_i^{\lambda}\left[(\alpha^{-1}+\mathcal{E}-H)^{-1}\right]_{ij}\xi_j^{\lambda'}\,\tilde{\gamma}_{\lambda'} \qquad (6.27)$$

when this expression is compared with the defining condition (6.21) and (6.23) we obtain

$$\mathcal{C}_\lambda\,\delta_{\lambda\lambda'} = \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\xi}_i^{\lambda}\left[(\alpha^{-1}+\mathcal{E}-H)^{-1}\right]_{ij}\xi_j^{\lambda'} \qquad (6.28)$$

or, in matrix notation

$$\mathbb{C} = \left[\alpha^{-1} + \mathcal{E} - \mathbb{H} \right]^{-1} \tag{6.28'}$$

which we solve for α

$$\alpha = \left[\mathbb{C}^{-1} + \mathbb{H} - \mathcal{E} \right]^{-1} \tag{6.29}$$

inserting (6.29) in (6.24) we obtain the optical model potential operator

$$V_{opt.} = U^{(N)} + \sum_{\lambda=1}^{N} \sum_{\lambda'=1}^{N} w_\lambda(r) \left[\left(\mathbb{C}^{-1} + \mathbb{H} - \mathcal{E} \right)^{-1} \right]_{\lambda\lambda'} \tilde{w}^*_{\lambda'}(r') \tag{6.30}$$

where

$$\left.\begin{aligned} w_\lambda(r) &= \sum_{i=1}^{N} \xi_i^\lambda W_i(r) \\ \tilde{w}^*_\lambda(r') &= \sum_{j=1}^{N} \tilde{\xi}_j^\lambda \tilde{W}^*_j(r') \end{aligned}\right\} \tag{6.31}$$

and

$$\left(\mathbb{H} - \mathcal{E} \right)_{\lambda\lambda'} = \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{\xi}_i^\lambda \langle W_i | \frac{1}{E^+ - \mathcal{H}^{(N)}} | W_j \rangle \xi_j^{\lambda'} \tag{6.32}$$

Now, it can be explicitly shown that the optical potential is absorptive, that is $\mathrm{Im}\, V_{opt} < 0$. From (6.29), the antihermitian part of α is

$$\alpha^{\dagger} - \alpha = \alpha\left[\left(\mathbb{C}^{-1}\right)^{\dagger} - \mathbb{C}^{-1} + \mathbb{H}^{\dagger} - \mathbb{H}\right]\alpha^{\dagger} \qquad (6.33)$$

in the λ representation $\mathbb{H}$ and $\mathbb{C}$ are diagonal, therefore we can write

$$\left[\left(\mathbb{C}^{-1}\right)^{\dagger} - \mathbb{C}^{-1} + \mathbb{H}^{\dagger} - \mathbb{H}\right]_{\lambda\lambda'} = \\ = -i\left[i\,\frac{\mathbb{C}_{\lambda} - \mathbb{C}_{\lambda}^{*}}{|\mathbb{C}_{\lambda}|^{2}} - \Gamma_{\lambda}\right]\delta_{\lambda\lambda'} \qquad (6.34)$$

from the definition of $\mathbb{C}$ we have

$$\mathbb{C}_{\lambda} - \mathbb{C}_{\lambda}^{*} = -i\int\Gamma_{\lambda}\frac{1}{\left(E' - \mathrm{Re}\,\mathbb{E}_{\lambda}\right)^{2} + \frac{1}{4}\Gamma_{\lambda}^{2}}\,f_{I}(E - E')\,dE'$$

since Γ_{λ} is a very slowly varying function of E, this relation becomes

$$\mathbb{C}_{\lambda} - \mathbb{C}_{\lambda}^{*} = -i\Gamma_{\lambda}\left\langle\left|\frac{1}{E - \mathbb{E}_{\lambda}}\right|^{2}\right\rangle$$

on the other hand

$$|\mathbb{C}_{\lambda}|^{2} = \left|\left\langle\frac{1}{E - \mathbb{E}_{\lambda}}\right\rangle\right|^{2}$$

then

$$\frac{\mathbb{C}_{\lambda} - \mathbb{C}_{\lambda}^{*}}{|\mathbb{C}_{\lambda}|^{2}} - \Gamma_{\lambda} = i\Gamma_{\lambda}\left[\frac{\left\langle\left|\frac{1}{E - \mathbb{E}_{\lambda}}\right|^{2}\right\rangle}{\left|\left\langle\frac{1}{E - \mathbb{E}_{\lambda}}\right\rangle\right|^{2}} - 1\right] \qquad (6.34')$$

but, from the inequalities of Bessel and Schwarz[27] we have that

$$\left\langle \left| \frac{1}{E-\mathbb{E}_\lambda} \right|^2 \right\rangle \geq \left| \left\langle \frac{1}{E-\mathbb{E}_\lambda} \right\rangle \right|^2$$

which means that the term in square brackets in the right hand side of equation(6.34) is positive definite, that means that $\sum_{i=1}^{N}\sum_{j=1}^{N} W_i(r)\,\alpha_{ij}\,W_j^*(r')$ has a negative definite antihermitian part, and since U^{NLS} is hermitian we finally get

$$\mathrm{Im}\, V_{opt} < 0 \tag{6.35}$$

Next, we shall show that the optical potential operator goes to the effective interaction operator (5.16') when the widht I of the weight function $f_I(E-E')$ goes to zero and this f_I becomes a delta function. From (6.18) we notice that

$$C_\lambda \xrightarrow[I\to 0]{} \frac{1}{E-\mathbb{E}_\lambda}$$

which means that in this limit, α goes to the inverse of the matrix of the unperturbed energy differences

$$\alpha_{\lambda\lambda'} \xrightarrow[I\to 0]{} \sum_{i=1}^{N} \tilde{\xi}_i^{\lambda} \frac{1}{E-E_i} \xi_i^{\lambda'} \tag{6.36}$$

which in turn means that the separable potential term appearing in V_{opt}.

$$\sum_{\lambda=1}^{N} w_\lambda(r)\,\alpha_{\lambda\lambda'}\,w^*_{\lambda'}(r') = \sum_{i,j=1}^{N}\sum_{\lambda,\lambda'=1}^{N} w_i(r)\,\xi^{\lambda}_{i}\,\alpha_{\lambda\lambda'}\,\tilde{\xi}^{\lambda'}_{j}\,w^*_j(r')$$

goes to the corresponding term in the effective interaction operator (5.17)

$$\sum_{\lambda,\lambda'=1}^{N} w_\lambda(r)\,\alpha_{\lambda\lambda'}\,w(r') \xrightarrow[I\to 0]{} \sum_{i=1}^{N} w_i(r)\,\frac{1}{E-E_i}\,w^*_i(r') \tag{6.37}$$

therefore

$$V_{opt} \xrightarrow[I\to 0]{} V_{eff.} \tag{6.38}$$

This result means that the information about the compound elastic scattering left in the scattering amplitude after taking the average over the energy is correctly represented in the theoretical optical potential, and the full amount of information contained in the elastic amplitude is recovered from the optical amplitude when the widht I, appearing in V_{opt} as a parameter, goes to zero.

To first order in the interaction $w_i(r) = \langle \Phi_{oA_1}\Phi_{oA_2}|V|\chi_i\rangle$ the energies of the physical resonances are just the diagonal terms in H_{ij}

$$E_i^{(1)} = E_i + \Delta_i^{(1)} - i\,\frac{1}{2}\,\Gamma_i^{(1)}$$

$$\Delta_i^{(1)} = \langle w_i|\frac{P}{E-E_i}|w_i\rangle\,,\quad \Gamma_i^{(1)} = 2\pi\left|\langle w_i|\zeta^{(+)}_{\alpha k}\rangle\right|^2$$

Then $\alpha^{(1)}_{ij}$ is diagonal, calling A_i and B_{ij} the real and imaginary parts of C_i, we get

$$\alpha^{(1)}_{ij} = \frac{\dfrac{A_i}{|C_i|^2} + \Delta^{(1)}_i - i\frac{1}{2}\left(\dfrac{2B_i}{|C_i|^2} - \Gamma^{(1)}_i\right)}{\left(\dfrac{A_i}{|C_i|^2} + \Delta^{(1)}_i\right)^2 + \dfrac{1}{4}\left(\dfrac{2B_i}{|C_i|^2} - \Gamma^{(1)}_i\right)^2}\,\delta_{ij} \tag{6.39}$$

we have already shown that

$$\frac{2B_i}{|C_i|^2} \geq \Gamma_i$$

hence, $\mathrm{Im}\,\alpha^{(1)} < 0$.

In order to have a clearer idea of the meaning and magnitude of the different terms appearing in (6.39), it is convenient to evaluate C_i for some particular $f_I(E-E')$. When the $f_I(E-E')$ is the Lorentzian function

$$f_I(E-E') = \frac{1}{2\pi}\,\frac{I}{(E-E')^2 + \frac{1}{4}I^2} \tag{6.40}$$

we obtain

$$C_\lambda = \frac{1}{E - \mathrm{Re}\,E_\lambda + i\frac{1}{2}(\Gamma_\lambda + I)} \tag{6.41}$$

and

$$\frac{A_\lambda}{|C_\lambda|^2} = E - \mathrm{Re}\,E_\lambda$$

to first order this expression becomes

$$\frac{A_i}{|C_i|^2} = E - E_i - \Delta_i^{(1)} \tag{6.42'}$$

similarly

$$\frac{2B_i}{|C_i|^2} = \Gamma_i^{(1)} + I \tag{6.43}$$

Then,

$$\alpha_{ij}^{(1)} = \frac{E - E_i - i\frac{1}{2} I}{(E-E_i) + \frac{1}{4} I^2} \delta_{ij} \tag{6.44}$$

putting this value in (6.30), we get

$$\begin{aligned} \mathrm{Re}\, V_{opt} = {}^{(d)}V_{1,2}(r)\delta^{(3)}(\vec{r}-\vec{r}') + \mathcal{H}(|\vec{r}-\vec{r}'|) - E\,K(|\vec{r}-\vec{r}'|) + \\ + \sum_{\substack{i=1 \\ E_i \in \mathcal{D}}}^{N} W_i(r) \frac{E-E_i}{(E-E_i)^2 + \frac{1}{4}(\Gamma_i + I)^2} W_i^*(r') + \\ + \left(\sum_{E_j \notin \mathcal{D}} W_j(r) \frac{1}{E-E_j} W_j^*(r') + \right. \\ \left. + \int \rho_c(\varepsilon) W_\varepsilon(r) \frac{1}{E-\varepsilon} W_\varepsilon^*(r') \right) \end{aligned} \tag{6.45}$$

and

$$Im\, U_{opt} = -\pi \left[\sum_{i=1}^{N} W_i(r) \frac{1}{\pi} \frac{\frac{1}{2}\,\Gamma}{(E-E_i)^2 + \frac{1}{4}\Gamma^2} W_i^*(r') + \right.$$

$$\left. + \rho_c(r) W_E(r) W_E^*(r') \right] \tag{6.46}$$

The real part of the optical potential is a sum of various terms of different nature. The first three terms appearing in the right hand side of equation (6.45) have exactly the same form as the effective interaction obtained by the resonating group method (RGM)[13,14,16]; the two exchange terms of this part of the hamiltonian can also be expressed as

$$\mathcal{H}(|\vec{r}-\vec{r}'|) - E\,K(|\vec{r}-\vec{r}'|) = V_{1,2}^{exch.}(|r-r'|) -$$

$$- \left(\frac{\hbar^2}{2\mu} \nabla_r^2 K(|\vec{r}-\vec{r}'|) + \epsilon_\alpha K(|\vec{r}-\vec{r}'|) \right) \tag{6.47}$$

in this expression, the exchange term $V_{1,2}^{exch.}$ is the exchange term appearing in the double folding potential

$$V_{1,2}^{exch.}(|\vec{r}-\vec{r}'|) = \iint \Phi^*_{A_1}(x_i)\, \Phi^*_{A_2}(x_j) \left[\sum_{\substack{i\in[A_1] \\ j\in[A_2]}} v_{ij}(|r_i-r_j|) \times \right.$$

$$\left. \times \left\{ (\mathcal{A}'-1)\, \Phi_{A_1}(x_i)\, \Phi_{A_2}(x_j) \right\} \right] d\tau_{A_1}\, d\tau_{A_2} \tag{6.48}$$

In many semiphenomenological calculations[26] only $V^{d}_{1,2}$ and $V^{exch.}_{1,2}$ are taken into account.

There are two correction terms to the RGM part of $Re\ V_{opt}$. The first one represents the effect produced in the direct elastic scattering by the coherent contribution of those compound nucleus resonances contained in the energy interval of width I, centered at E; the contribution of this term to the direct elastic scattering is very small because the energy denominator are large and the energy numerators change sign, when the average is performed with a box weight function[28], and the width I is large compared to the resonance width, this term is usually put equal to zero. The second correction to the RGM part of V_{opt} represents the effect produced in the direct elastic scattering by the coherent contribution of the far away resonances and all inelastic processes, the large energy denominators and the great incoherence between contributions coming from different compound nucleus states and different channels make this contribution negligible.

The imaginary part of the optical potential appears as a sum of non-local separable potentials each term corresponding to the flux loss in the direct elastic scattering due to each one of the compound nucleus states in the energy interval I and the eliminated non-elastic channels.

When the summation over compound nucleus states is made with the help of corresponding smoothed out density of energy states, and the small terms are neglected, we get simplified formulae for $Re\ V_{opt}$ and $Im\ V_{opt}$

$$\mathrm{Re}\, V_{opt.} \cong \left[V^{(d)}_{1,2}(r)\, \delta^{(3)}(\vec{r}-\vec{r}') + \mathcal{H}(|\vec{r}-\vec{r}'|) - E\, K(|\vec{r}-\vec{r}'|) \right] + \\ + \int_{\epsilon_1} \rho_c(\mathcal{E})\, \mathcal{W}_{\mathcal{E}}(r) \frac{1}{E-\mathcal{E}} \mathcal{W}^{*}_{\mathcal{E}}(r')\, d\mathcal{E} \qquad (6.49)$$

$$\mathrm{Im}\, V_{opt.} \cong -\pi\, \overline{\rho}\,(E)\, \mathcal{W}_E(r)\, \mathcal{W}_E(r') \qquad (6.50)$$

where

$$\overline{\rho} = \rho_c + \overline{\rho_D} \qquad (6.51)$$

and ρ_D is the density of energy states in the discrete part of the spectrum

$$\overline{\rho_D} = \int \frac{1}{2\pi} \frac{I}{(E-E')^2 + \frac{1}{4} I^2} \rho_D(E')\, dE' . \qquad (6.52)$$

7. ANGULAR MOMENTUM DEPENDENCE OF THE OPTICAL POTENTIAL

In order to make evident the angular momentum dependence of $V_{opt.}$ we shall consider in more detail the simplest case of a composite target and projectile both with zero spin.

We notice first that all the interaction terms appearing in the effective equation (5.15) are invariant with respect to translations, since the projector P_α , equation (4.10), the Green's functions, and nuclear potentials are all invariant with respect to translations. Making a partial wave expansion of the effective wave function

$$\psi_{eff.} = \sum_{l=0}^{\infty} \frac{u_l(r)}{r} Y_{l0}(\cos\theta) \tag{7.1}$$

we get the following expression for the <u>effective</u> radial equation (not the optical radial equation.)

$$\frac{d^2 u_l}{dr^2} + \left(\frac{2\mu E}{\hbar^2} - \frac{l(l+1)}{r^2}\right) u_l - \frac{2\mu}{\hbar^2} \int \mathcal{U}_l(r,r') u_l(r')\, dr' = 0 \tag{7.2}$$

where $\mathcal{U}_l(r,r')$ stands for all interaction terms.

$$\mathcal{U}_l(r,r') = V_{12}^{0}(r)\delta(r-r') + h_l(r,r') - E\,k_l(r,r') + R_e W_l(r,r') - i\, I_m W_l(r,r') \tag{7.3}$$

$h_l(r,r')$, $k_l(r,r')$ and $W_l(r,r')$ are the radial parts of the

interaction terms

$$h_\ell(r,r') = \frac{4\pi\mu}{\hbar^2} rr' \int_{-1}^{+1} P_\ell(\cos\alpha)\, \mathcal{H}(|r-r'|)\, d(\cos\alpha) \qquad (7.4)$$

$$\mathcal{H}(|r-r'|) = \frac{\hbar^2}{8\pi\mu} \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\alpha)\, \frac{h_\ell(r,r')}{rr'} \qquad (7.4')$$

in these expressions α stands for the angle made by $\vec{r}$ and $\vec{r}'$. Similar expressions relate $h_\ell(r,r')$ to $K(|r-r'|)$ and $W_\ell(r,r')$ to $W(|r-r'|)$.

Now, we expand $W(|r-r'|)$ in resonant terms as in equation (5.16') and recalling that each resonant terms has a well defined total angular momentum J_i we obtain

$$W_\ell(r,r') = \sum_{E_i} \sum_{J_i(E_i)} \omega^{J_i}_{E_i,\ell}(r)\, \frac{1}{E^{(+)} - E_i}\, \omega^{*}_{E,\ell}(r') + \int d\mathcal{E}\, \rho_c(\mathcal{E},\ell)\, \omega_{\mathcal{E},\ell}(r)\, \frac{1}{E-\mathcal{E}}\, \tilde{\omega}^{*}_{\mathcal{E},\ell}(r') - i\pi\rho_c(\mathcal{E},\ell)\,\omega_{\mathcal{E},\ell}(r)\,\omega^{*}_{\mathcal{E},\ell}(r') \qquad (7.5)$$

where

$$\omega^{J_i}_{E_i,\ell}(r) = \frac{\sqrt{4\pi\mu}}{\hbar} \int Y_{\ell 0}(\hat{\Omega})\, w^{J_i}_{E_i}(r)\, d\Omega \qquad (7.6)$$

and a similar expression for $\omega^{*}_{\epsilon,\ell}(r)$.

In order to obtain the optical potential in § 5, we split $W(|r-r'|)$ in two parts, one containing the small denominators and the other, $W^{(N)}$, varying smoothly with energy, and we took an average over the energy that changed the analytic form of the second part only. Since the separation of the term $W(|r-r'|)$ made in § 5 in near resonances and far away resonances can also be made for each radial component W_ℓ of W with well defined ℓ, and the ensuing arguments carry through without changing the ℓ dependence, the interaction terms appearing in the optical model radial equation have an ℓ dependence similar to the one found above for the interaction terms in the effective radial equation. Making a partial wave expansion of the optical wave function

$$\varphi_{opt.} = \sum_{\ell=0} \frac{u_\ell(r)}{r} Y_{\ell 0}(\theta,\varphi) \tag{7.7}$$

we get the optical radial wave equation

$$\frac{d^2 u_\ell}{dr^2} + \left(\frac{2\mu\epsilon}{\hbar^2} - \frac{\ell(\ell+1)}{r^2}\right) u_\ell(r) - \frac{2\mu}{\hbar^2}\int V_\ell^{opt}(r,r') u_\ell(r')\,dr' = 0 \tag{7.8}$$

and

$$V_\ell^{opt}(r,r') = V_{12}^{0}(r)\delta(r-r') + h_\ell(r,r') - E k_\ell(r,r') + W_\ell^{(s)}(r,r')$$
$$\Delta R_e V_\ell^{opt} - i\,\mathrm{Im}\, V_\ell^{opt} \tag{7.9}$$

where $W_\ell^{(s)}$ is

$$W_\ell^{(s)}(r,r') = \sum_{E_i \notin D} \omega_{E_i,\ell}(r) \frac{1}{E - E_i} \omega^*_{E_i,\ell}(r') + \int_0^\infty d\varepsilon\, \rho_c(\varepsilon,\ell)\omega_{\varepsilon,\ell}(r) \frac{1}{E-\varepsilon} \omega^*_{\varepsilon,\ell}(r') \qquad (7.10)$$

and

$$\Delta \mathrm{Re}\, V_\ell^{opt}(r,r') = \sum_{i=1} \omega_{i,\ell}(r) \frac{(E - E_i)}{(E-E_i)^2 + \frac{1}{4} I^2} \omega^*_{i,\ell}(r') \qquad (7.11)$$

The imaginary part of the interaction term in the optical radial equation is

$$\mathrm{Im}\, V_\ell^{opt}(r,r') = -\pi \omega_\ell(r) \bar{\rho}_\ell(E,\ell) \omega^*_\ell(r') \qquad (7.12)$$

where

$$\bar{\rho}(E) = \rho_c + \bar{\rho}_D$$

The terms $h_\ell(r,r') - E k_\ell(r,r')$, appearing in (7.9), can be written in such a way as to make evident the relation of the radial component of the optical potential (7.9) with the more frequently used double folding potentials.

$$h_\ell(r,r') - E k_\ell(r,r') = V_\ell^{exch}(r,r') + \eta_\ell(r,r') \qquad (7.13)$$

in (7.13), $V_{12}^{D}(r)$ is the direct part of the double folding potential and $U_{\ell}^{exch}(r,r')$ is given by

$$U_{\ell}^{exch}(|r-r'|) = \frac{4\pi\mu}{\hbar^2} rr' \int_{-1}^{+1} P_{\ell}(\cos\alpha) V_{12}^{exch.}(|r-r'|) d(\cos\alpha) \tag{7.14}$$

where $V_{12}^{exch}(|r-r'|)$ is the exchange part of the double folding potential. $\eta_{\ell}(r,r')$ has the simple expression

$$\eta_{\ell}(r,r') = \frac{\hbar^2}{2\mu}\left(\frac{d k_{\ell}(r,r')}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right) k_{\ell}(r,r') + \frac{2\mu\epsilon}{\hbar^2} k_{\ell}(r,r') \tag{7.15}$$

It can be seen that, besides the usual direct and exchange double folding potentials and the terms coming from the compound nucleus contribution, we obtain a new term $\eta_{\ell}(r,r')$.

A rough estimation of the ℓ dependence of $\mathrm{Im}\, U_{\ell}^{opt}(r,r')$ coming from the density of compound nucleus states can be made using some analytical approximation for the J dependence of and recalling that this is practically the same for the various nuclear models[19].

$$\bar{\rho}(E,J) \simeq \rho(E,0)\,\frac{2J+1}{\sigma^2}\, e^{-\frac{(J+1/2)^2}{2\sigma^2}} \tag{7.16}$$

where σ is the spin dispersion parameter. Now, we recall that for each fixed value E of the compound nucleus energy there is a maximum value of its total spin $J_{max}(E)$ corresponding to the Yrast state, that is, to the situation in which the compound nucleus has all its energy in the state of maximum rotational energy

that it can sustain

$$E \simeq \frac{J_{max}(J_{max}+1)}{2\mathcal{J}}\hbar^2 \tag{7.17}$$

and none of it as internal excitation energy, in (7.16), $\mathcal{J}$ is the moment of inertia. For that energy value there is no compound nucleus state of larger spin, hence the density of states goes to zero, in the rough estimation, we are making we put this condition as

$$e^{-\frac{J_{max}(J_{max}+1)}{2\sigma^2}} = e^{-1} \tag{7.18}$$

which, together with (7.17) gives

$$\sigma^2 \simeq \frac{E\mathcal{J}}{\hbar^2} \tag{7.19}$$

on the Yrast line. Since we are considering the simplest case in which the spins of target and projectile are zero, conservation of total angular momentum implies that the compound nucleus total spin is equal to the orbital angular momentum of the partial wave that excites that compound nucleus state. In this way we obtain

$$Im\, V_\ell^{opt}(r,r') \simeq -\pi \bar{\rho}(E,0)\frac{2\ell+1}{2E\mathcal{J}}\hbar^2 e^{-\frac{(\ell+\frac{1}{2})^2\hbar^2}{2\mathcal{J}E}} \omega_{E,\ell}(r)\omega^*_{E,\ell}(r') \tag{7.20}$$

From (7.20), it follows that, if at some fixed energy E some of the partial waves participating in the nuclear collision

have orbital angular momentum larger than the Yrast spin value of the compound nucleus for that energy, then the absorption is zero for those partial waves, i.e. the optical potential is transparent for those values of l .

8. CONCLUDING REMARKS

In this work the attention is focused on the theory of the optical model for elastic collisions between composite nuclei. Starting from the many-body Schrödinger equation we obtained first an effective hamiltonian describing in full detail the relative motion of target and projectile in the elastic channel. Next, from the effective equation and the usual requirement of the equality of the average over the energy of the elastic scattering amplitude and the optical scattering amplitude we obtained an optical model hamiltonian and a theoretical expression for the optical potential. Finally, we exhibited the optical potential dependence on the orbital angular momentum of the relative motion.

We used projection operators and Green's functions as were used in references (9) and (10), and we took into account the Pauli principle requirements (antisymmetrization), in this respect our formal treatment of the problem is similar to that of H. Feshbach and others[9,21,29]. However, we took into account explicitly the Galilean invariance of the problem (recoil effects), and we chose the effective wave function in such a way as to get a hermitian operator for the effective hamiltonian when the energy is lower than the first inelastic threshold. We found simple and physically transparent formulae for $\mathcal{R}_e\, \mathcal{V}_{opt.}$ and $\mathcal{I}_m\, \mathcal{V}_{opt.}$. We were able to show that, quite explicitly, that the imaginary part of the optical potential is always negative without making any ad hoc assumptions. We showed also that, the information about the compound elastic scattering left in the energy averaged scattering amplitude is correctly represented in the theoretical optical potential, and

the full amount of information contained in the elastic amplitude is recovered from the optical amplitude when the width I of the weight function used to take the energy average, which appears in $U_{opt.}$ as a parameter, goes to zero. In this respect our treatment differs from Feshbach's.

The detailed effective elastic wave function and the optical model wave function are simply related to the ones used by Feshbach and others, in a simple way, by means of an integral transform whose kernel is a delta function plus Wheeler's kernel.[23]

The detailed elastic effective hamiltonian, found in this work, has a term, slowly varying with energy, with the same functional dependence on the nucleon-nucleon interaction and the heavy-ion internal wave functions as the effective hamiltonian of the "cluster model" obtained by the resonating group method (RGM). It has another term, varying rapidly with energy, that produces the compound nucleus resonance effects in the elastic transition amplitude. Therefore, when the "true" internal wave functions of target and projectile are replaced by the same approximate internal wave functions used in the RGM[14], the first terms of the effective hamiltonian and the real part of the optical model potential coincide exactly with the hamiltonian appearing in the cluster model of nuclear structure and collisions[16]. In addition to the "cluster model" hamiltonian, the real part of our optical potential operator has a hermitian correction term coming from the prompt contribution of compound nucleus states to elastic scattering.

The imaginary part of our optical potential is also a hermitian operator, non-local, explicitly dependent on the energy, angular momentum and all the other constants of the motion. It is

proportional to the density of compound nucleus states and this might provide an explanation to the recently experimentally confirmed transparency of the nuclear potential's surface observed in some heavy-ion elastic collisions.

The "cluster model" term in the real part of the optical potential can be rewritten as the sum of a double folding potential, having direct and exchange terms, plus an additional term. This additional term is the sum of a second derivative of Wheeler's kernel plus the product of the elastic channel's relative motion energy times Wheeler's kernel. Since Wheeler's kernel produces the exchange of any number of particles between target and projectile, this term might be related to the observed elastic exchange effects[2], not usually-taken into account by the phenomenological optical potentials.

In this way, we have shown that, when proper care is taken of Galilean invariance and the Pauli principle, the optical model formalism for elastic collisions between composite nuclei contains the usual "cluster model" approximation plus the appropriate compound nucleus corrections and absorption. The relation of this formalism with the frequently used double folding potentials was also exhibited.

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