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ENHANCEMENT OF KALMAN FILTER SINGLE LOSS DETECTION CAPABILITY

G. W. Morrison

D. J. Downing

D. H. Pike

Computer Sciences Division
at Oak Ridge National Laboratory*
Union Carbide Corporation, Nuclear Division
Oak Ridge, Tennessee 37830

MASTER

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ABSTRACT

A new technique to significantly increase the sensitivity of the Kalman filter to detect one time losses for nuclear material accountability and control has been developed. The technique uses the innovations sequence obtained from a Kalman filter analysis of a material balance area. The innovations are distributed as zero mean independent Gaussion random variables with known variance. This property enables an estimator to be formed with enhanced one time loss detection capabilities. Simulation studies of a material balance area indicate the new estimator greatly enhances the one time loss detection capability of the Kalman filter.

It has been shown previously that optimal state estimator techniques (1), (2) employing a Kalman filter and fixed internal smoothing are a sensitive means for detecting small continual losses of material from a material balance area. These techniques have been applied to (1) simulated data from a powder unloading station in a mixed oxide reprocessing plant (3), (2) operational data from the ORNL U^{233} pilot reprocessing plant (4) and (3) simulated data from a chemical reprocessing plant (5). Various loss scenarios of small constant and random losses were analyzed with the Kalman filter and Linear Smoother. In all cases considered, the Kalman filter was a superior estimator of material loss. However, for the one time loss scenario the Kalman filter may be insensitive for loss detection unless the loss is large relative to LEID.

The enhancement of the one time loss detection capability of the Kalman filter is the subject of this research effort.

Utilization of the Kalman filter for material accountability and control requires a system model which specifies the time evolution of the system and a measurement model for the measured quantities. The system model is based on the mass balance equation for the material balance area. The system model is

$$\underline{X}_{t+1} = \underline{A}_t \underline{X}_t + \underline{U}_t + \underline{W}_t \tag{1}$$

where

$\underline{X}$ = n state vector of the system

$\underline{A}$ = n x n state transition matrix

$\underline{U}$ = known input n vector

$\underline{W}$ = zero mean n vector of white noise

The system equations for the inventory and loss are

$$I_{t+1} = I_t + U_t - L_t \tag{2}$$

$$L_{t+1} = L_t \tag{3}$$

where

I_{t+1} = ending inventory for period (t,t+1)

U_t = net transfer during period (t,t+1)

L_t = loss during period (t,t+1)

Note that Eq. (2) is a mass balance for the material balance area while Eq. (3) specifies that the mean of the loss is constant over time. The measurement equation specifies

$$Y_t = I_t + V_t \tag{4}$$

where

V_t = measurement error with $Var(V_t) = R$

or the measured inventory is the true inventory corrupted by additive noise. Utilizing the above system model and measurement model along with initial estimates of the beginning inventory, I_0, and error convariance matrix, $\underline{G}_0$, the Kalman filter can be utilized to sequentially provide optimal inventory and loss estimates at each material balance closing.

The innovations $\tilde{e}_t$, provided by the Kalman filter,

$$\tilde{e}_t = Y_t - \hat{I}_{t|t-1} \tag{5}$$

where

Y_t = measured inventory at time t

$\hat{I}_{t|t-1}$ = predicated inventory at time t given the data up to time t-1

are distributed as normal random variables (6) with mean zero and variance given by

$$Var(\tilde{e}_{t|t-1}) = \underline{H}_t \underline{P}_t \underline{H}_t^T + \underline{R}_t \tag{6}$$

where

$\underline{H}_t$ = measurement matrix

$\underline{P}_t$ = prior state error covariance matrix at time t

$\underline{R}_t$ = inventory measurement covariance matrix

These innovations can be utilized to detect one time losses. The method investigated in this paper is to consider the linear combination of squared innovations

$$L = a_{t-k}\, \tilde{e}^2_{t-k|t-k-1} + a_{t-k+1}\, \tilde{e}^2_{t-k+1|t-k} + \cdots + a_t\, \tilde{e}^2_{t|t-1} \;. \tag{7}$$

Our purpose is to compare the loss detection capabilities of L for various choices of k and the a_i's. A material balance area was simulated with a loss equal to 70% of the LEID (20 units) at period 60. Figure 1 shows the results of the data when analyzed using the ID/LEID approach, the Kalman filter, the CUSUM approach, and the linear smoother. Note that the loss is smaller in magnitude than the LEID and hence undetected by the ID/LEID approach. The other techniques are designed to detect regularly occuring losses and, therefore, also fail to detect this one time loss. Figure 2 represents the innovation series for the Kalman filter analysis. Figure 3 is a plot of L where k = 2 and $a_{t-1} = a_t = 0.70$, i.e.,

$$L = 0.7\ \tilde{e}^2_{t-1|t-2} + 0.7\ \tilde{e}^2_{t|t-1} \quad . \tag{8}$$

Notice the pronounced spike at time t = 60 indicating a loss. Another spike occurs at time t = 67 which is caused by the overcorrecting feature of the Kalman filter caused by the loss. This can be smoothed by incorporating more innovations in L. Figure 4 shows a plot of L when k = 9 and all the a_i's are unity. This plot clearly indicates a loss at time t = 60 and smoothes the overcorrecting caused by the loss. The optimal choice of k and the a_i's is currently under investigation by Monte Carlo methods.

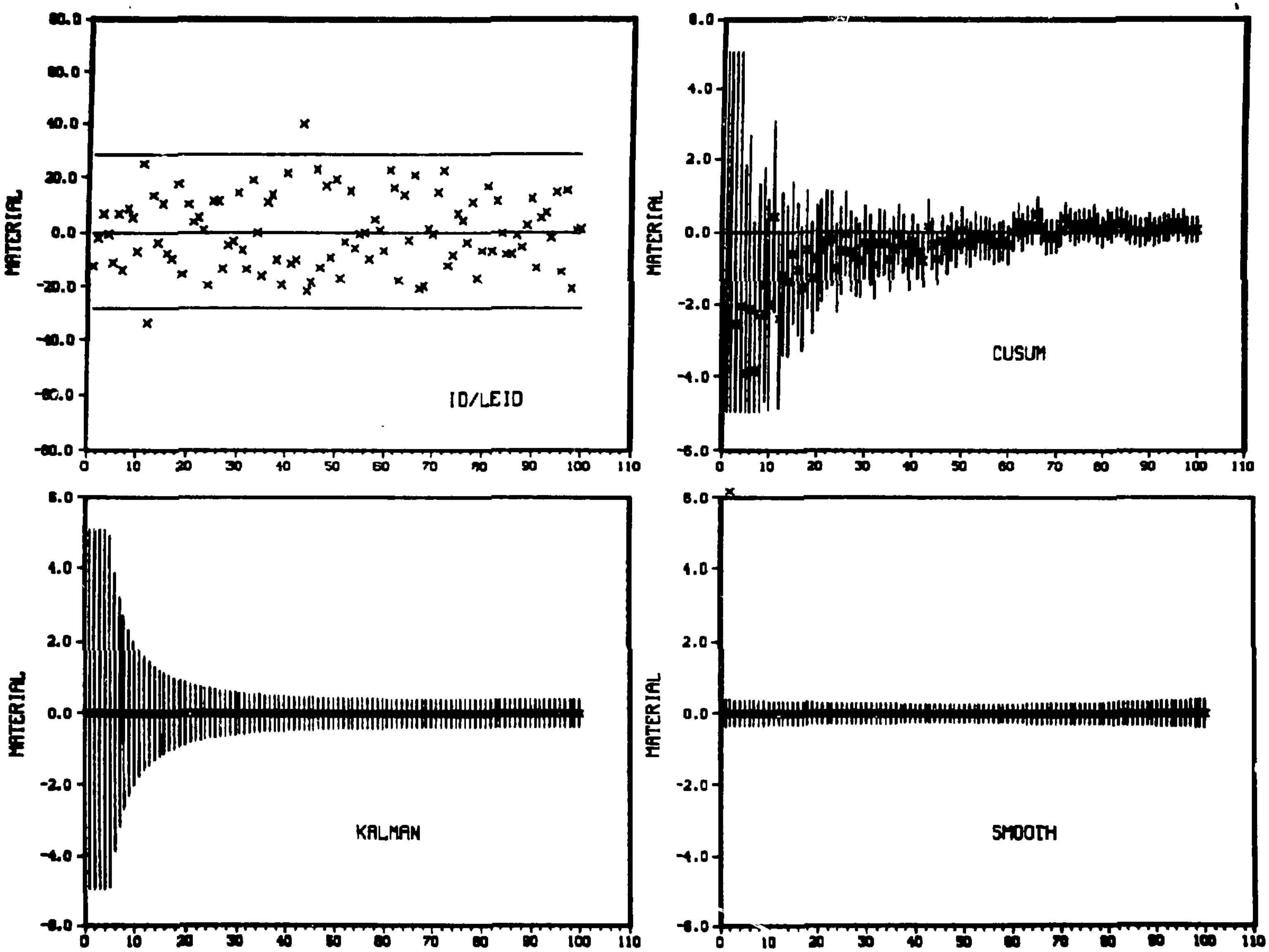

Fig. 1. Comparison of Loss Detection Techniques for single loss of 20 units at time, t = 60.

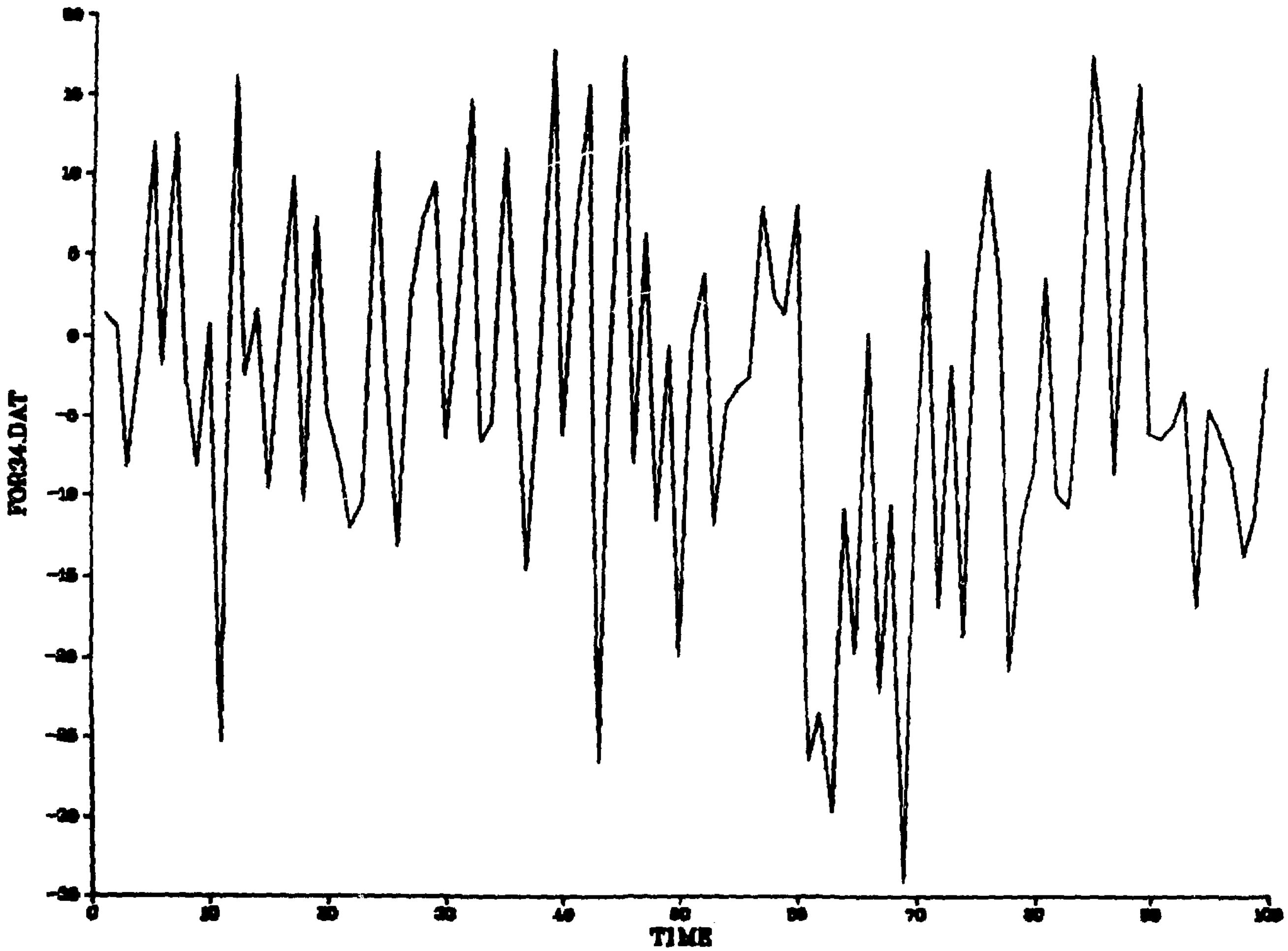

Fig. 2. Innovations Sequence from Kalman filter analysis of Material Balance Area of Figure 1 with single loss of 20 units at time, t = 60.

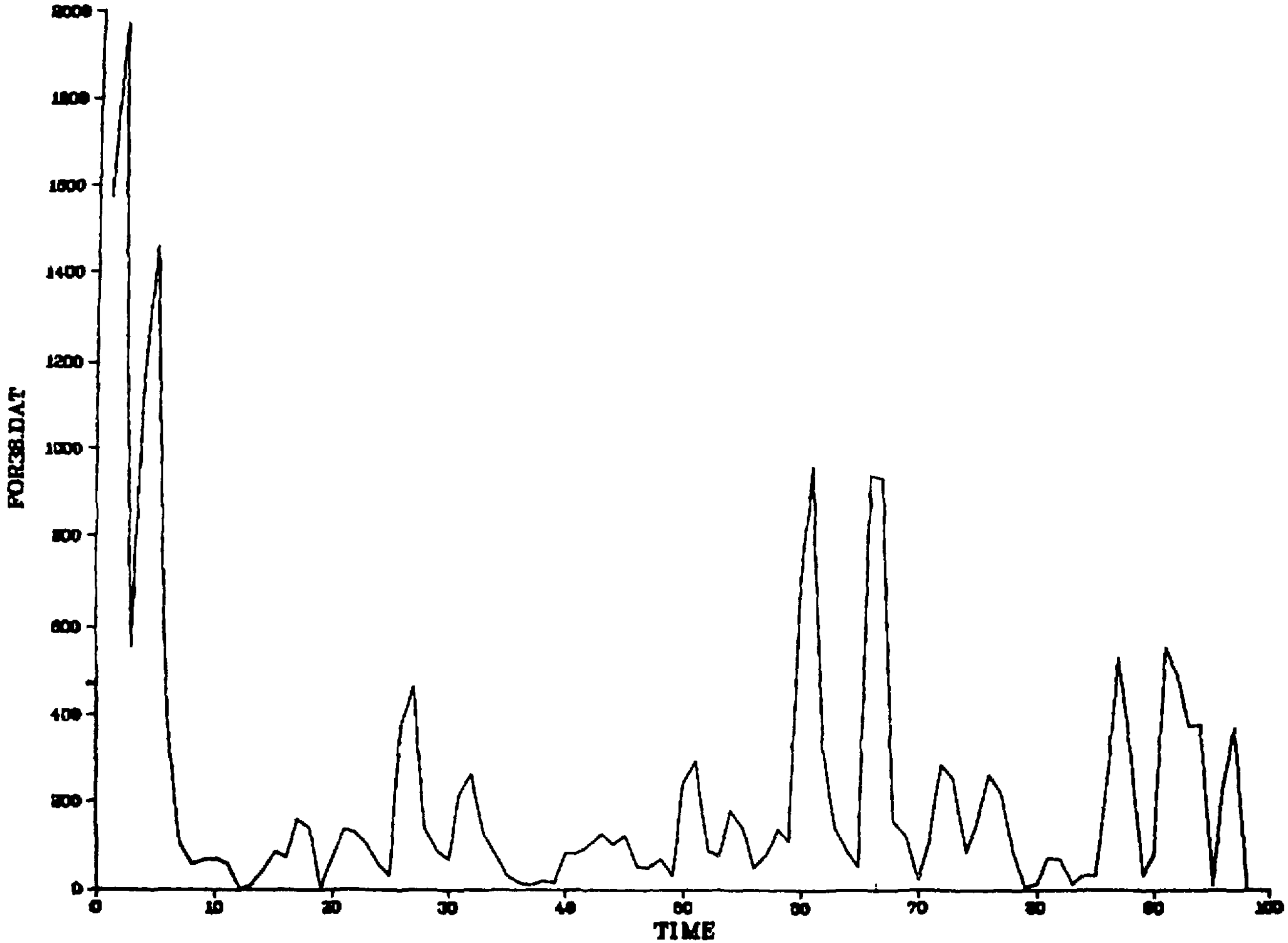

Fig. 3. Loss estimator $L_t = .7e_t^2 + .7e_{t+1}^2$ Innovations Data.

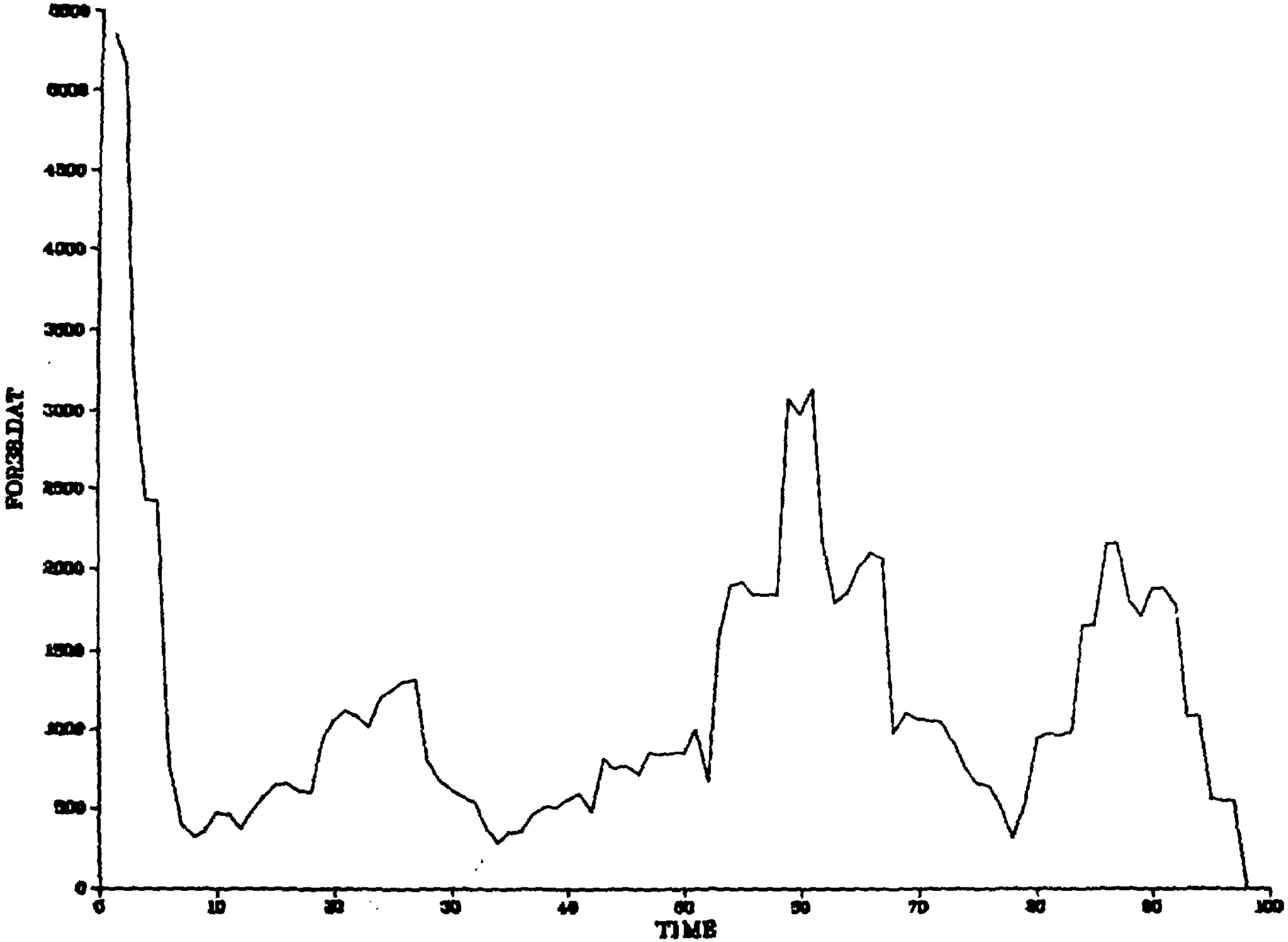

Fig. 4. Loss estimator $L_t = \sum_{k=1}^{10} e^2_{t+k-1}$ for Innovations Data.

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