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COMPUTER SIMULATIONS OF ANOMALOUS TRANSPORT

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Computer Simulations of Anomalous Transport

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I. Introduction

Numerical plasma simulations have been carried out to study, (1) the turbulent spectrum and anomalous plasma transport associated with a steady state electrostatic drift turbulence, and (2) the anomalous energy transport of electrons due to shear-Alfvén waves in a finite-β plasma. For the simulation of the steady state drift turbulence, it is observed that, in the absence of magnetic shear, the turbulence is quenched to a low level when the rotational transform is a rational number, while the turbulent level remains high for an irrational rotational transform. This result is consistent with recent observations of drift turbulence in the GA octupole, and is interpreted as being the result of the nonlinear excitation of convective cells for a rational transform. For the shear-Alfvén waves in a sheared slab, enhanced thermal fluctuations associated with the damped radial eigenmodes have been observed to nullify the zeroth-order shear near the rational surface through the induced eddy current. Since the eigenmodes have the tearing-type parity of odd ϕ and even $A_{\parallel}$, magnetic islands are formed in the shear-free region which, in turn, cause rapid electron energy transport across the rational surface. The observed eigenmode behavior has been confirmed by the theoretical analysis. We believe this mechanism may be important for anomalous transport in tokamaks.

II. Electrostatic Drift Turbulence

Drift wave instabilities and the associated anomalous diffusion of plasmas have been an important subject in both plasma kinetic theory and laboratory experiments in conjunction with the research of controlled fusion. It is well known that the anomalous diffusion of particles and energy has been observed in a number of laboratory experiments.[1-4] It is generally believed that the presence of low-frequency fluctuations are responsible for the anomalous diffusion observed experimentally.[1-7]

To understand the nonlinear saturation and the anomalous transport associated with drift instabilities, we performed extensive numerical simulations using two-and-a-half and three dimensional electrostatic particle codes in slab and toroidal geometries.[8,9] For those simulations, in which the initial density profile was allowed to diffuse due to the instability induced fluctuations, we found that the profile modification was the dominant stabilization mechanism. Also, diffusion can be coherent when only a few modes unstable or turbulent which leads to the formation of convective cells for both zero β and finite β plasmas.[10,11]

While these numerical simulations are useful in revealing the mechanisms of anomalous transport, particle recycling and plasma heating play a dominant role for establishing a steady state turbulence for laboratory experiments in which the discharge time is much longer than the transport time scale. It is conceivable that under such condition profile modification may no longer be a saturation mechanism.

In order to study steady-state turbulence and the associated plasma transport, three dimensional simulations are carried out which are designed to maintain more or less the initial density profile by recycling the particles reaching the wall. Since the density profile is maintained, particle diffusion coefficient is determined from

$$\nabla\cdot D_\perp\nabla n = S$$

where S is the known source function of particles. It was found that, in the absence of magnetic shear, large anomalous particle diffusion and the high level of turbulence persist with many drift modes unstable for an irrational rotational transform, while the drift instability is quenched to a low fluctuation level for a rational rotational transform. Shown in Figures 1 and 2 are the time development of the radial mode structures for the potential fluctuations for several Fourier modes (m,n) for $q = \sqrt{5}$ (Fig. 1) and $q = 2$ (Fig.2). It is clearly observed that for q irrational $(\sqrt{5})$, potential fluctuations reach higher amplitude while for rational q, drift waves decay in time after reaching the maximum amplitude giving the energy away to the convective cells, $(m,n) = (2,-1)$. For $q = 2$, both drift waves and convective cells saturate roughly at the theoretically predicted value at $e\phi/T_e = (\rho_i/L_n)(\gamma/\omega)$ [9] where ρ_i is the ion gyroradius, L_n is the density scale length, γ and ω are growth rate and the frequency of the unstable drift modes. This observation is consistent with the laboratory experiment,[2] and is interpreted due to the coupling of drift waves to convective cells.

III. Shear-Alfvén Waves

In recent years, the contribution to the anomalous electron energy transport in tokamaks due to magnetic field fluctuations associated with low-frequency microinstabilities has been an area of active research.[12-14] It has been suggested that the magnetic field line reconnection and stochastic field lines could provide the most effective means for the electron energy transport. Various linear stability analysis have been performed to determine the onset conditions for those instabilities.[14-17] However, the theoretical understanding of their nonlinear behavior and the consequent anomalous transport have not been well established. Owing to its intrinsic nature, particle code simulation is the most suitable for studying these types of problems. Here, we will present the results of our investigation of shear-Alfvén waves in a sheared slab using a two and one-half dimensional finite-β particle code.

Let us first describe briefly the present theoretical understanding of the low frequency microinstabilities for a collisionless plasma. With the magnetic field $\underset{\sim}{B}_0 = B_0(\hat{\underset{\sim}{z}} + \hat{\underset{\sim}{y}}\, x/L_s)$ where L_s is the shear scale length, the governing equations for the scalar potential ϕ and the vector potential $A_{\|}$ can be written as

$$d^2\phi/dx^2 = (\sigma/x_A^2)(\psi/x - \phi) + b_s\phi$$
$$d^2\psi/dx^2 = (\sigma/x)(\psi/x - \phi) + b_s\,\psi$$

where the perturbed quantities are assumed to be of the form $\exp(ik_y y - i\omega t)$, $\psi = \omega A_{\|}/k_{\|}'c$, $k_{\|}' = k_y\rho_s/L_s$, $\rho_s = (\tau)^{1/2}\rho_i$, $b_s = k_y^2\rho_s^2$, $\tau = T_e/T_i$, $\rho_i = v_{ti}/\omega_{ci}$, $x_A = (1 + \omega^*/\tau\omega)^{1/2}\omega/k_{\|}'v_A$ with v_A being the Alfvén speed, $\sigma = -x_A^2F$, $F = [aZ'(\xi_e) + \tau Z'(\xi_i)]/[Z'(\xi_i)(x_A^2/x^2) + 2\xi_i Z(\xi_i)]$, $\xi_\alpha = \omega/\sqrt{2}\,k_{\|}'xv_{t\alpha}$, $a = (1 - \omega^*/\omega)/(1 + \omega^*/\tau\omega)$, Z is the plasma dispersion function and α denotes species. The solutions of these equations indicate that drift-Alfvén eigenmodes, which include both the finite-β modified drift branch and the shear-Alfvén branch, do exist in a sheared slab, and they are always stable.[15,16] However, in the presence of electron temperature gradient[14] and parallel electron current,[17] these eigenmodes can become unstable. Calculations have also been carried out to show that, in the absence of density gradient, i.e., $\omega^* = 0$, stable shear-Alfvén eigenmodes of the tearing type (even $A_{\|}$ and odd ϕ) still exist with $\omega \ll |\gamma|$.[18] The eigenmodes are localized near the rational surface due to the finite Larmor radius effects as well as ion Landau damping. The damping rate is found to decrease with plasma β.

The particle simulation model[19] is based on the Darwin formulation of Maxwell's equation for $\omega \ll kc$, in which the transverse displacement current in Ampere's law is neglected. The compression along the external magnetic field has also been ignored in accordance with the usual tokamak approximation for a low (but finite) β plasma. For the particle pushing, the exact dynamics for the ions are kept in the code, whereas the electrons are treated as guiding center particles. With the additional assumptions of $\omega \ll \omega_{pe}$ and $k_{\parallel}/k_{\perp} \ll 1$, the field equations used in the code are

$$\nabla^2\phi = -4\pi\rho$$

$$\nabla^2 A_{\parallel} = -(4\pi/c)J_{\parallel}$$

where ρ is the charge density and $J_{\parallel}$ is the parallel current. The electric field can then be calculated by

$$\underset{\sim}{E} = -\nabla\phi - (1/c)\partial \underset{\sim}{A}_{\parallel}/\partial t$$

and the magnetic field by

$$\underset{\sim}{B} = \nabla \times \underset{\sim}{A}_{\parallel} \quad .$$

The code is considerably simpler than the conventional magnetostatic code.[20] At the same time, it has kept intact all the physical features relevant to the low frequency microinstabilities. Furthermore, we can use rather large time steps ($\Delta t \sim \omega_{pi}^{-1}$) in the code, because the lower-hybrid oscillations are now the highest frequencies in the system. As we can see, this simulation model is most suitable for studying the phenomena associated with drift-Alfvén oscillations.

The simulation has been carried out in a $L_x \times L_y = 64 \times 32$ grid, in which ϕ and $A_{\parallel}$ are assumed to be periodic in the y direction and zero at $x = 0$, L_x. The shear is initially generated self-consistently by an uniform electron current of $u/v_{te} = 0.05$, and the rational surface ($k_{\parallel} = 0$) is located at the middle of the system in x. The other simulation parameters are: $L_s/\rho_s = 273$, $m_e/m_i = 1/400$, $\beta = 0.4\%$, $b_s = 1.0$ and $\tau = 4.0$. There is no spacial inhomogeneity initially. The simulation results agree very well with the theoretical predictions for the lowest eigenmodes in terms of the measured frequencies and mode structures. Although, they are linearly damped according to the theory, enhanced thermal fluctuations associated with these modes persist for the duration of the simulation. The fluctuations are the result of the balance between Cerenkov emission and the damping as well as the coupling between different eigenmodes. It should be noted here that the presence of a small electron current in the system has negligible effect on the solutions of the original mode equations.[17] In the simulation, we have also observed that the enhanced fluctuations have produced a second-order y-independent eddy current near the rational surface. Its existence can be verified by carrying out the quasilinear analysis of the second-order drift kinetic equation for the electrons.[18] The nonlinear effects come from the $\mathbf{E}\cdot\mathbf{B}$ and $\mathbf{V}_\perp\cdot\nabla\mathbf{V}_\parallel$ terms where $\mathbf{V}$ is the electron fluid velocity. Figure 3 shows the theoretical and simulation results of the second-order eddy current. It can also be verified easily that the presence of the eddy current reduces the shear strength near the rational surface. The saturation takes place when the zero-order current as well as the shear is completely nullified in the region. In the mean time, the oscillations of $A_\parallel$ give rise to the formation of magnetic islands with several ρ_i's in width near the rational surface. *Therefore, when an electron temperature gradient is used in the simulation, rapid electron energy transport across* the rational surface can be observed, as shown in Fig. 4. In the real tokamak geometry, the rational surfaces are closely packed. For the modes with $k_y\rho_i \sim 1$, they are only of the order of ρ_i apart. Therefore, the mechanism described here could be very detrimental to the tokamak confinement.

IV. Conclusions

We have presented in this paper the investigation of the low-frequency microinstabilities using particle code simulations. For the electrostatic drift waves, the results agree well with the GA octupole experiments. The simulation of shear-Alfvén waves has helped us to gain insight into the mechanisms for anomalous transport. We hope that these results would serve to demonstrate the versatility and the importance of the simulation techniques in studying low-frequency phenomena in magnetically confined devices.

Acknowledgement

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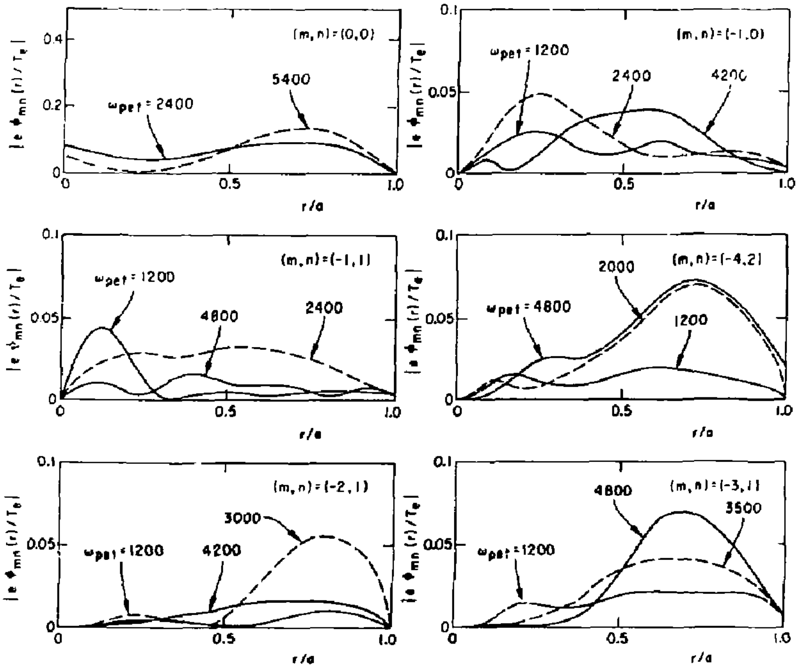


Fig. 1. Radial mode structures for potential fluctuations for $q = \sqrt{5}$ at different times.

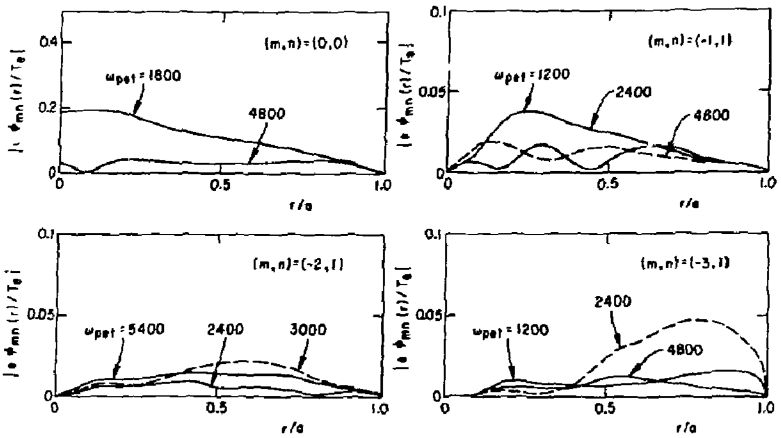
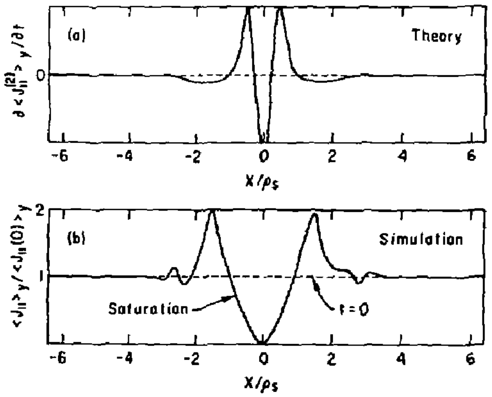


Fig. 2. Radial mode structures for potential fluctuations for $q = 2$ at different times. Note the drift waves are quenched to smaller amplitude compared with $q = \sqrt{5}$.



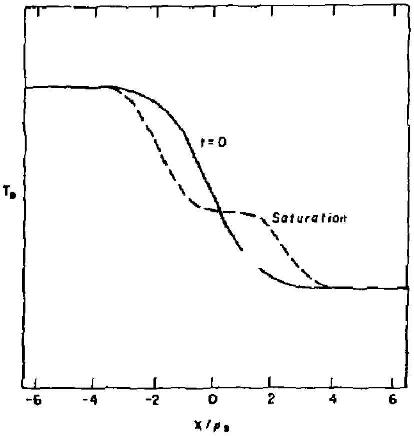


Fig. 3. Theoretical (a) and numerical (b) results for the second-order eddy current. The rational surface is at $x/\rho_s = 0$.

Fig. 4. Anomalous electron energy transport near the rational surface due to shear Alfvén eigenmodes.