# ESTIMATION AND CONTROL IN HTGR FUEL ROD FABRICATION



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# ESTIMATION AND CONTROL IN HTGR FUEL ROD FABRICATION

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A control algorithm has been derived for a HTGR Fuel Rod Fabrication Process utilizing the method of Box and Jenkins. The estimator is a Kalman filter and is compared with a Least Square estimator and a standard control chart. The effects of system delays are presented

## INTRODUCTION

Quality control of a process as in Figure 1 is a necessary and vital part of good manufacturing. The quality of a product can be monitored by measuring various characteristics which directly affect its usefulness. These measurements can be compared to control limits. If the measurements fall within the control limits, the process is said to be "in control" and manufacturing continues. Once observations fall outside the control limits, the process is declared "out of control" and manufacturing ceases until the problem is rectified and the process is back in control.

Good measurement systems and sophisticated estimation techniques (e.g., least squares, Kalman filtering) can be used in predicting the true level of the process. These estimation techniques can be used to indicate process changes like step changes or drift. Thus, good measurement systems serve three purposes (1) accurate measure of process level (for example, fuel rod length and fuel rod fissile assay), (2) early warning of process change, and (3) indicate process modifications (i.e., control).

Several techniques exist that can be used in estimating process change. Two well-known techniques are least squares and Kalman filtering. It should be pointed out that for a stationary system, a Kalman filter model can be constructed to yield the same results as those given by least squares.

Some questions of interest concerning these techniques are: (1) How soon will the Kalman filter or least squares detect a drift of given size? (2) How soon will they detect a step change? (3) Under what circumstances would one technique be preferred over the other?

Given that one has a technique to estimate the true level of the process, how can this estimate be used to control the process? Quality control charts have been used since the early 1960s. These charts plot the data and give visual cues as to the quality of the product. In many cases, key variables can be identified which directly affect the quality of the product. In the system studied in this paper, the amount of carbon shim particles directly affects the length of fuel rods. If the length of the fuel rod is a characteristic of interest, it can be controlled by adjusting the amount of shim particles used, or if fuel rod fissile control is of interest, it can be controlled by adjustment of the fissile particle volumetric dispenser. If the transfer function relating the input variable to the output variable is known, then a feedback control scheme can be implemented which uses this information to keep the process in control. Figure 1 is a schematic diagram of the control scheme for fuel rod fabrication.

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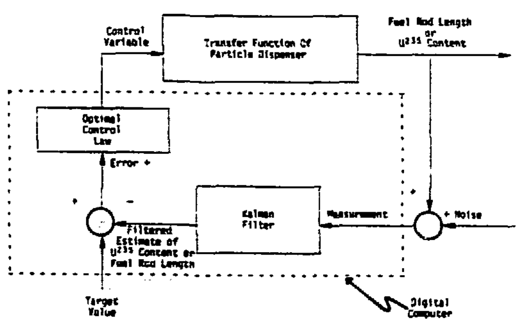


Fig. 1. Schematic Diagram of Control Scheme for Fuel Rod Fabrication

## DETECTING CHANGES IN PROCESS OPERATING LEVEL

Simulations were performed using Kalman Filter, weighted linear least squares, and the Shewhart control chart in order to compare the methods and in order to then utilize the equations such that operating parameters such as speed of fissile assay, source size, and number of rods assayed could be better estimated for equipment in a refabrication facility. The simulation cases studied are given below in Table 1. The simulated general problem of concern is shown in Fig. 2. The picture represents the fissile content of fuel rods on the vertical axis and rod number along the horizontal axis. As we go from left to right, it is assumed that the fissile content is increasing linearly with time. The solid horizontal line extending from the 0.5 gm. mark on the vertical axis is the "target" value, that is, the fissile content we are trying to produce. The dashed lines extending from 0.5(1-x) gms. and 0.5(1+x) gms. are lower and upper control limits, respectively. They are the values such that, if the process level goes below or above them, rods of inferior quality are being produced. The x's indicate a measured fissile control and the u's indicate the true fissile content. The straight line drawn through the points is the least squares line which gives the best estimate of the true fissile content if the process has a linear drift. One question of interest is which of the methods of estimation will detect this trend earliest?

Table 1. Simulation Case Studies.

| Simulation | Process Parameters | | | |
|---|---|---|---|---|
| | Slope/Step | Value | Process Error(%) | Measurement Error(%) |
| 1 | Slope | 2.5E-05 | 0.5 | 2.0 |
| 2 | Slope | 5.0E-06 | 0.5 | 2.0 |
| 3 | Slope | 1.0E-06 | 0.5 | 2.0 |
| 4 | Slope | 2.5E-05 | 0.5 | 5.0 |
| 5 | Slope | 5.0E-06 | 0.5 | 5.0 |
| 6 | Slope | 1.0E-06 | 0.5 | 5.0 |
| 7 | Slope | 2.5E-05 | 1.0 | 0.1 |
| 8 | Slope | 5.0E-06 | 1.0 | 0.1 |
| 9 | Slope | 1.0E-06 | 1.0 | 0.1 |
| 10 | Step | 0.01 | 0.5 | 2.0 |
| 11 | Step | 0.01 | 0.5 | 5.0 |
| 12 | Step | 0.01 | 1.0 | 0.1 |

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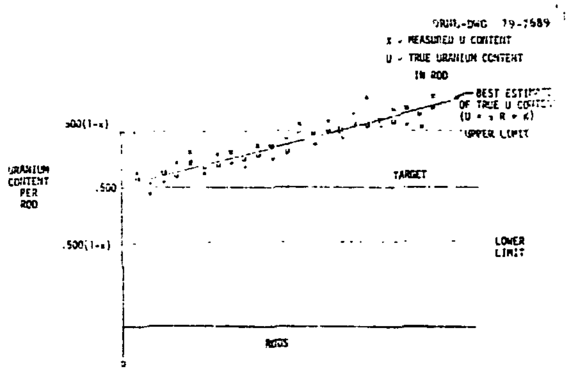


MEASURED URANIUM CONTENT NORMALLY DISTRIBUTED ABOUT TRUE URANIUM CONTENT WITH A STANDARD DEVIATION OF ± 4 %

TRUE URANIUM CONTENT NORMALLY DISTRIBUTED ABOUT MEAN DRIFT LINE WITH A STANDARD DEVIATION OF ± 6 %

Fig. 2. Example of Process Drift in Production of Fuel Rods

Table 2 contains the results of the simulations. The three columns on the left of Table 2 describe the simulation study parameters (i.e., measurement error, process error, and slope/step value).

ORNL-DWG 79-7706

Table 2. Average Number of Rods Before Slope/Step Was Statistically Significant

| Measurement Error[a] | Process Error | Slope/Step | Value | Method of Detection | | | |
|---|---|---|---|---|---|---|---|
| | | | | Least Squares | | Kalman Filter | Control Chart |
| | | | | $\alpha = 0.63$ | $\alpha = 0.9977$ | | |
| 2.0 | 0.5 | Slope | 2.5 E-05 | 28972 | 157 ± 84 | 133 ± 64 | 136 ± 97 (n=5/10) |
| 2.0 | 0.5 | Slope | 5.0 E-06 | 16172 | 386 ± 204 | 294 ± 195 | 167 ± 161 (n=11/10) |
| 2.0 | 0.5 | Slope | 1.0 E-06 | [illegible] | 1971 ± 2014 | 637 ± 566 | 615 ± 421 (n=10/10) |
| 5.0 | 0.5 | Slope | 2.5 E-05 | >30,000 | 230 ± 195 | 203 ± 134 | 273 ± 198 (n=9/10) |
| 5.0 | 0.5 | Slope | 5.0 E-06 | 14081 | 391 ± 687 | 442 ± 354 | 352 ± 323 (n=8/10) |
| 5.0 | 0.5 | Slope | 1.0 E-06 | 3677 | 13032 | 1406 | 1212 |
| 0.1 | 1.0 | Slope | 2.5 E-05 | [illegible] | 91 ± 46 | 80 ± 34 | 69 ± 40 (n=6/10) |
| 0.1 | 1.0 | Slope | 5.0 E-06 | 6763 | 309 ± 179 | 233 ± 147 | 132 ± 71 (n=5/10) |
| 0.1 | 1.0 | Slope | 1.0 E-06 | 16461 | 835 ± 672 | 549 ± 290 | 326 ± 237 (n=8/10) |
| 2.0 | 0.5 | Step | 0.01 | 27 ± 2 (n=3/6)[a] | 38 ± 19 | 34 ± 9 | 35 ± 30 (n=11/11) |
| 5.0 | 0.5 | Step | 0.01 | 2038 (n=1/3)[a] | 71 ± 67 | 74 ± 40 | 207 ± 66 (n=2/10) |
| 0.1 | 1.0 | Step | 0.01 | >10,000 (n=0/3) | 28 ± 2 | 27 ± 9 | — |

[a]Exceeded 10,000 runs without detecting change

Conclusions that can be reached from the simulations are:

1) The Kalman filter detects drift in the process better than weighted least squares or the Shewhart control chart when measurement error dominates.

2) The Shewhart control chart is best for detecting drift when measurement error is less than process error.

3) The Kalman filter and weighted least squares (with weight $\alpha = 0.9977$) detect step changes equally well.

4) The Shewhart control chart is better for detecting step changes than for trends.

5) The ability of weighted least squares to detect a trend in the process decreases drastically as the weight decreases (and, hence, incorporates fewer past observations).

The above conclusions imply that the Kalman filter may be the best detector for most cases. Two additional reasons to suggest its use are: (1) its modeling capability is superior to the weighted least squares, and (2) it is computationally efficient. As can be seen by the above conclusions, no one method is best for all cases. Thus, it may be necessary to employ more than one technique.

## TECHNIQUES FOR TIME SERIES CONTROL

The transfer function of the system is

$$Y_t = V_0 \, (X_{t-b} - \overline{X}) + N_t \qquad (1)$$

where,

$Y_t$ = deviation from target at time t, (rod length)

$X_t$ = referenced target value at time t, (pinch valve setting)

$\overline{X}$ = mean pinch valve setting

b = delay time between Rod Length measurement and pinch valve change

$V_0$ = system gain.

$N_t$ is the process noise at time t which makes perfect control impossible. By holding the control variable, $X_t$, constant at its steady state value (the value at which the output should be on target), a realization of the process noise, $N_t$, is obtainable. This noise may be modeled by the techniques outlined by Box and Jenkins [1]. To explain the technique consider the noise model to be

$$N_t = \phi N_{t-1} + a_t \qquad (2)$$

where $\phi$ is a constant and $a_t$ is white noise. At time (t-1) we need to estimate $N_t$, and at time t, we need to estimate $N_{t+1}$. Using these forecasts, we can estimate $\Delta X_t$, our change in the control variable at time t. We choose the forecasts of $N_t$ and $N_{t+1}$ so as to minimize the mean square forecast error. Any linear combination of these optimal forecasts will also be optimal in the sense of mean square error. Thus, $\Delta X_t$ is an optimal control. It can be shown that the optimal one-step-ahead forecast of $N_t$ is

$$\hat{N}_{t-1}(1) = \phi a_{t-1} + \phi^2 a_{t-2} + \phi^3 a_{t-3} + \dots = \frac{\phi}{(1 - \phi B)} a_{t-1} \quad . \qquad (3)$$

Similarly, at time t, the optimal one-step-ahead forecast of $N_{t+1}$ is

$$\hat{N}_t(1) = \frac{\phi}{(1 - \phi B)} a_t \quad . \qquad (4)$$

Thus, the change in the control variable at time t is

$$\Delta X_t = -\frac{1}{V_0}\left(\hat{N}_t(1) - \hat{N}_{t-1}(1)\right) . \qquad (5)$$

If one substitutes the optimal one-step-ahead forecast $\hat{N}_{t-1}(1)$ into Eq. (1) and then uses this to estimate the error in the output, we find

$$Y_t = V_0\left(-\frac{1}{V_0}\hat{N}_{t-1}(1)\right) + N_t = N_t - \hat{N}_{t-1}(1) \quad . \qquad (6)$$

Thus, the error in the output at time t is simply the forecast error at lead 1 for the $N_t$ process. That is,

$$N_t - \hat{N}_{t-1}(1) = a_t \quad ,$$

and, hence,

$$Y_t = a_t \quad . \qquad (7)$$

To identify the noise model for the fuel rod lengths, data on three separate campaigns were analyzed, and an appropriate model that describes the stochastic process was chosen for each campaign. These models we used in the simulation study. The simulations were made to compare the process error in a controlled and uncontrolled environment.

In each of these campaigns, the pinch valve setting was kept at a constant level, thought to produce rods of length 1.94 inches. Keeping the pinch valve at a constant level allows one to investigate the noise process, $N_t$. We assume that the noise process $N_t$ will follow an autoregressive-integrated-moving average (ARIMA) model. These models are amply described [1]. Let $N_t$ denote the $t^{th}$ observed rod length minus the target value 1.94, $a_t$ an unobservable white noise random variable, and B the backward shift operator (equivalent to $Z^{-1}$ in sampled-data control theory). Then the ARIMA (p,d,q) model is written as:

$$1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)\nabla^d N_t^* = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)a_t \quad . \qquad (8)$$

where $\nabla = (1 - B)$ and $N_t^* = (N_t - \mu)$ if d = 0; otherwise $N_t^* = N_t$. For example, an ARIMA (1,1,1) model could be written as

$$(1 - \phi_1 B)(1 - B)N_t = (1 - \phi_1 B)a_t \quad . \qquad (9)$$

Expanding the above, we have

$$[1 - (1 + \phi_1)B + \phi_1 B^2]N_t = (1 - \theta_1 B)a_t \quad , \qquad (10)$$

or

$$N_t - (1 + \phi_1)N_{t-1} + \phi_1 N_{t-2} = a_t - \theta_1 a_{t-1} \quad . \qquad (11)$$

Table 3 gives the three best campaigns where campaign 2 has an ARIMA(4,0,1B) model, campaign 3 has an ARIMA(17,0,0) model and campaign 4 is an ARIMA(2,0,0) model.

ORNL-DWG 79-7688

Table 3. Descriptive Statistics for Campaigns Two, Three, and Four.

| | Campaign Two | Campaign Three | Campaign Four |
|---|---|---|---|
| Sample Size | 176 | 300 | 187 |
| Mean | 1.958 | 1.950 | 1.968 |
| Standard Deviation | 0.00774 | 0.01023 | 0.01015 |
| Coefficient of Variation | 0.40% | 0.52% | 0.52% |
| Autocorrelations | | | |
| lag 1 | 0.16 | 0.38 | 0.31 |
| lag 2 | 0.21 | 0.30 | 0.29 |
| lag 3 | 0.09 | [illegible] | 0.24 |
| lag 4 | 0.23 | 0.34 | 0.20 |
| lag 5 | 0.12 | 0.31 | 0.21 |
| lag 6 | 0.18 | 0.30 | 0.18 |
| lag 7 | 0.05 | 0.33 | 0.12 |
| lag 8 | 0.19 | 0.34 | 0.17 |
| lag 9 | 0.03 | 0.34 | 0.06 |
| lag 10 | 0.20 | 0.34 | 0.13 |

The simulation results are summarized in Table 4.

Table 4. Comparison of Mean Square Errors for Controlled and Uncontrolled Processes.

| Simulation | Delay Value | Uncontrolled Process MSE* | Controlled Process MSE* | Percent** Reduction In MSE |
|---|---|---|---|---|
| Campaign 2 | 1 | 1.11 | 1.04 | 6.3 |
| Campaign 2 | 20 | 1.11 | 1.13 | -1.8 |
| Campaign 3 | 1 | 1.07 | 0.92 | 14.0 |
| Campaign 3 | 20 | 1.07 | 1.07 | 0.0 |
| Campaign 4 | 1 | 1.17 | 0.92 | 21.4 |
| Campaign 4 | 20 | 1.17 | 1.17 | 0.0 |
| Nonstationary | 1 | 437.93 | 0.92 | 99.8 |
| Nonstationary | 20 | 437.93 | 165.78 | 62.1 |

*MSE - Mean square error.

**Percent Reduction = (MSE Uncontrolled - MSE Controlled)/MSE Uncontrolled.

In summary, the following conclusions can be made:

1) The campaigns can be analyzed by the methods of Box and Jenkins.

2) To have an effective control system using the methods of Box and Jenkins, the models describing the process cannot change from campaign to campaign as much as witnessed in these trials.

3) The campaigns analyzed are stable and require very little control. This implies that not every item needs to be measured to insure good quality since the process does not change radically in a short time span.

4) The amount of control decreases rapidly as the delay value (the amount of time between a change in the input is observed in the output) increases.

## REFERENCES

1. G.E.P. Box and G. M. Jenkins, "Time Series Analysis Forecasting and Control", Holden-Day, 1970.