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# TREATMENT OF MEASUREMENT UNCERTAINTIES

# AT THE POWER BURST FACILITY

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FIGURES

CREDIT LINE -

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## ABSTRACT

The treatment of measurement uncertainty at the Power Burst Facility provides a means of improving data integrity as well as meeting standard practice reporting requirements. This is accomplished by performing the uncertainty analysis in two parts, test independent uncertainty analysis and test dependent uncertainty analysis. The test independent uncertainty analysis is performed on instrumentation used repeatedly from one test to the next, and does not have to be repeated for each test except for improved or new types of instruments. A test dependent uncertainty analysis is performed on each test based on the test independent uncertainties modified as required by test specifications, experiment fixture design, and historical performance of instruments on similar tests. The methodology for performing uncertainty analysis based on the National Bureau of Standards method is reviewed with examples applied to nuclear instrumentation.

## 1. INTRODUCTION

The objective of this paper is to present the treatment of uncertainty of nuclear instrumentation at the Power Burst Facility (PBF). The instrumented experiments at PBF are performed on single fuel rods, or small clusters of fuel rods, and are designed to investigate the behavior of fuel rods under a variety of postulated accident conditions as part of the Thermal Fuels Behavior Program (TFBP). Measurements are obtained from instruments on the fuel rods, the associated test fixture, and plant equipment.

The uncertainties associated with these measurements are determined from computer programs based on the methodology discussed in this paper.

The treatment of uncertainty at PBF was developed and meets the requirements of the Water Reactor Research Directorate (WRRD) Standard Practice #11, "Requirements for Quantifying Measurement Uncertainties of WRRD and LOFT Experimental Data." This approach to uncertainty analysis is compatible with the NRC Reactor Research Division draft policy on reporting uncertainties. If adopted in the present form, the draft NRC policy will be mandatory for all data reported from test facilities such as PBF, which support code assessment, model development, or form a basis for regulatory decision making.

Requirements for reporting uncertainties are covered in Section 2. The approach for testing independent uncertainty is also described in Section 2, and test dependent uncertainty in Section 3. A review of the National Bureau of Standards (NBS) methodology as applied to nuclear instrumentation is presented in Section 4. Conclusions and recommendations are given in Section 5.

## 2. TEST INDEPENDENT UNCERTAINTY ANALYSIS

Measurement uncertainty analysis began at PBF with a test independent evaluation of specific types of instruments and electronic components of the measurement system which are used repeatedly. Typical types of instruments are thermocouples, turbine flowmeters, and pressure transducers. Once the test independent uncertainty values are determined, they can be applied throughout the program and modified as required by test dependent factors peculiar to each test. The test independent analysis results are reported in an internal report for future reference on upcoming tests.

For a Type T (copper-constantan) differential thermocouple, test independent uncertainties were determined to be[1]

Bias error, $B = \pm [0.1\ K + 0.7\%\ (RD-255)]$

random error, $S = \pm 0.4$ K

degrees of freedom, df > 30

uncertainty, $U = \pm$ [0.9 K + 0.7% (RD-255)] .

RD is the temperature in degrees kelvin of the higher of the two thermocouple junctions. These are limit-of-error values including all known sources of error for the instrument and associated cable. These uncertainty values are what would be expected if a new differential thermocouple were used as-received and assumed to meet standard calibration curves for Type T thermocouples without calibrating each unit or correcting for any known in-place errors.

From a detailed test independent analysis of the PBF data acquisition and reduction system (DARS), the most significant uncertainties are those contributed by the analog circuits and analog-to-digital converter (ADC). Once data is in digital form, the various redundancy checks reduce the digital uncertainties to small bit probability errors, due to noise, which is negligible compared with the uncertainties in analog circuits. During the data reduction process uncertainties are found in calibration coefficients, decimation process, and readout devices. Because these DARS errors are based on engineering judgment, they are placed in the bias category. The reason for this is discussed in Section 4.

The DARS is a complex system and no single uncertainty number will suffice for the many combinations of equipment and operating modes. Thus, a set of equations were developed in which the many variables can be represented and an estimated uncertainty obtained for a given channel based on input voltage and selected channel characteristics. This was accomplished by associating the many elemental errors with six groups of equipment which are shown on Figure 1 with the associated equations. The six groups are: (a) low level amplifier, (b) presample filter circuit, (c) remote modulate/demodulate unit, (d) calibration source, (e) computer and data array processor, and (f) readout or plotter device.

The equations are in terms of millivolts and all the bias uncertainty values are referred to the input (RFI) by taking the various system gains into account. The input to the DARS is the reference point for a measurement channel. By referring all error values to the common point of a channel and converting the DARS errors from millivolts to the engineering units for a particular measurement, the total measurement channel uncertainty can be obtained in engineering units.

The bias errors are further divided into constant or variable errors. Constant errors are those which are expressed in terms of full-scale values. Variable errors are given as percent of reading. Thus, the operating conditions must be clearly stated before an uncertainty value can be determined. For example, the uncertainty for steady state operation over a specified range can be determined in terms of the maximum error that would occur within this range. The bias uncertainty value is then determined, which includes both constant and variable errors.

As an example, the DARS channel for the Type T differential thermocouple previously discussed, is configured with a low level amplifier gain $G_1$ of 1000, the programmed gain $G_3$ equal to 4, and the presample filter gain $G_2$ equal to one with a 10 Hz filter. These values are applied to the appropriate equations given on Figure 1 and the resulting bias values combined with the differential thermocouple bias values by the root-sum square method. The total uncertainty is calculated by the method discussed in Section 4, which involves adding the total bias plus the random component times the student-T factor. A plot of these test independent uncertainties for the differential thermocouple example shown on Figure 2.

## 3. TEST DEPENDENT UNCERTAINTY ANALYSIS

Uncertainties associated with environmental factors and experiment fixture configuration are considered as test dependent uncertainties. An analysis is performed for each test in three phases which are the (a) pretest uncertainty analysis, (b) performance uncertainty analysis, and (c) test uncertainty analysis. The uncertainties are determined by means of a calculator program which is in the process of being adopted to the DARS as a part of the PBF software package.

## 3.1 Pretest Uncertainty Analysis

The pretest uncertainty analysis is based upon past history of similar instruments, requirement documents, and vendor instrument data. It is completed before instruments are purchased or final test fixture designs are finalized. This analysis considers the uncertainty associated with the instrument, the installation of the instrument, the data acquisition system, data reduction system, and the effect of the instrument on the system response.

From these considerations, it can be determined if the instrumentation accuracy requirements can be satisfied, what instruments must be improved to meet the accuracy requirements, and in some instances, whether the experimental project should be abandoned.

Since some instruments cannot be recalibrated after being in the reactor, this may be the only uncertainty analysis that will be made on them.

## 3.2 Performance Uncertainty

The performance uncertainty analysis is a reevaluation of the pretest uncertainty analysis using laboratory calibrations traceable to the National Bureau of Standards (NBS). Where possible, the analysis should use data from in-place calibrations, also. Static, steady state, and transient calibration should be made when possible. The in-place calibrations should consider temperature, pressure, radiation, vibration or shock, flow regimes, rate of change, the instruments effect on the system, and any other effect the experimentalist considers influential on the measurement uncertainty.

## 3.3 Test Uncertainty

The test uncertainty analysis is a modification of the previous two uncertainty analyses using in-place calibrations and calibration checks just prior to the test. The input for the test uncertainty modification consists of four types. First, the in-place calibrations which are performed before a test series starts. It consists primarily of static calibrations. The in-place calibrations should reveal abnormal shifts in instrument outputs which would require a modification to the uncertainty analysis. Second are the warmup checks. These checks are part of the test procedure and are performed as the facility is being readied for the test. Included in these checks are isothermal checks, frequency, static pressure checks and data system calibrations. Third are the pre and posttest electronics calibration. Fourth, posttest calibration and data interpretation consisting of (a) calibration if abnormal shifts in the instrument output occurred during the test, (b) data system calibration, and (c) instrument effects on the system.

As an example, the differential thermocouple previously discussed would have uncertainties as shown on Figure 3 when the adjustments are applied and the delta temperature set to zero at the end of electrical heatup just prior to starting nuclear operation of the reactor.

# 4. METHODOLOGY

A variety of methods have been used to perform uncertainty analysis, each probably being optimum for some particular situation or application. The method used at PBF is based on the NBS model and is briefly described herein as it applies to nuclear instrumentation. For a detailed discussion of the general methodology, see Reference 2. This methodology is an adaptation from the aerospace industry.

All measurements have errors which are the difference between the measurement and the true value, as shown on Figure 4. The true value, in some cases, may be arbitrarily defined as the value that would be obtained by the NBS. The maximum error that might reasonably be expected is called uncertainty. This is the closeness of the measurement to the true value. Uncertainty has two components; a fixed error which is called bias, and a random error which is called precision.

### 4.1 Bias (Fixed Error)

The bias, B, is the constant or systematic error. Each measurement has the same bias in repeated measurements and the bias cannot be determined unless the measurements are compared with the true value of the quantity measured. However, the true value for most nuclear instrumentation measurements is unknown and the bias is obtained from engineering estimates.

### 4.2 Precision (Random Error)

The variation between repeated measurements is called precision error. A measure of the precision error is the standard deviation. The statistic, S, is calculated to estimate the standard deviation and is called the precision index

$$S = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N-1}}$$

where

$X_i$ = $i^{th}$ measurement

$\bar{X}$ = arithmetic mean

N = number of measurements.

### 4.3 Judgment versus Statistic Estimates

To avoid a complex decision about whether a given elemental error source contributes to bias, precision, or both, the recommendations from

Reference 3 are adopted. That is, depending upon how the error is derived, the uncertainty on the measurement is put into one of two categories. A bias uncertainty is estimated by nonstatistical means and a random uncertainty is derived by a statistical analysis of repeated measurements. This keeps the judgment estimates separate from statistical estimates as long as possible. It also helps identify major sources of error and allows one to make corrections more easily .

## 4.4 Conceptual Measurement Systems

The basic error sources fall into three categories which are: (a) calibration errors, (b) data acquisition errors, and (c) data reduction errors.

Data acquisition errors contribute the most to the total uncertainty. Calibration errors are usually small compared with the data acquisition errors with the NBS being the ultimate reference. Data reduction errors are also usually small. However, all error sources should be listed when performing an uncertainty analysis, no matter how small. This is to ensure that no major error source is overlooked. For illustration purposes, the conceptual design is shown on Figure 5.

## 4.5 Measurement Process

The basis for an uncertainty statement is found in the definition of the measurement process for which the uncertainty values apply. At PBF, the measurement process involves both steady state and transient measurements. In general, the methodology described applies directly to steady state operating conditions. For transient burst or loss-of-coolant accident blowdown reactor operating modes, the dynamic process is less well defined for each parameter and the uncertainty estimates are correspondingly larger. Thus, the present methodology is primarily intended for steady state operation, and extension to transient conditions is handled on an individual parameter basis starting with steady state conditions just prior to the transient.

The uncertainty of the measurement process will also contain errors due to variations between calibrations, test trains, and measurement instruments. The uncertainty analysis for an absolute measurement will be different from a comparative measurement on a single test. Biases can be ignored in comparative testing because the same instrumentation setup is used. Uncertainty will also vary due to test duration.

A reduction in the random uncertainty can usually be obtained if averaging can be used. Averaging is accomplished either with repeated measurements if the measured variable is constant, or with redundant instruments recording simultaneously. Unfortunately, with single burst or blowdown experiments, at PBF it is not always possible to have either redundant instruments or repeat experiments.

### 4.6 Combining Elemental Errors and Degrees of Freedom

The bias and precision elemental errors are combined separately by the root-sum-square (RSS) method, as indicated on Figure 6. Small letters are used for elemental errors and capital letters for the resulting RSS totals in each category which includes calibration errors, data acquisition errors, and data reduction errors.

In a sample measurement, the number of degrees of freedom (df) is the size of the sample. The degrees of freedom associated with the statistic calculated from the sample are reduced by one for every estimated parameter used in calculating the statistic. For example, when the standard error of a curve fit is calculated, the number of estimated coefficients for the curve is equal to the number of degrees of freedom lost. The Welch-Satterthwaite formula is used for calculating the degrees of freedom associated with the precision index. This formula calculates the combined degrees of freedom as a function of the elemental degrees of freedom and magnitudes of each elemental precision index. For example, the degrees of freedom for the data acquisition precision index is

$$df = \left(\sum_{i=1}^{N} S_{i2}^{2}\right)^{2} \quad \left(\sum_{i=1}^{N} \frac{S_{i2}^{4}}{df_{i2}}\right)$$

where

df = degrees of freedom

S = standard deviation.

## 4.7 Uncertainty Model

To obtain a single number for uncertainty, the following recommendation is adopted.[3]

"This method is recognized and recommended by the NBS[2] and has been widely used in the industry.[3] For simplicity of presentation, a single number (some combination of bias and precision) is needed to express a reasonable limit for error. The single number must have a simple interpretation (the largest error reasonably expected) and be useful without complex explanation. It is impossible to define a single rigorous statistic because the bias is an upper limit based on judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). However, the need for a single number to measure error is so great that the adoption of a standard is warranted. The standard most widely used is the bias limit plus a multiple of the precision index."

The equation defining uncertainty is

$$U = \pm (B + t_{95} S)$$

where

U = uncertainty

B = bias

S = precision

$t_{95}$ = 95th percentile point for the two tailed students t - distribution.

The t-value is a function of the number of degrees of freedom (df) used in calculating S, and arbitrarily inflates the limit U to reduce the risk of underestimating S. Since 30 degrees of freedom yields a t-value of 2.04, and an infinite degrees of freedom yields a t-value of 1.96, an arbitrary selection of t = 2 for df values over 30 is used.

The uncertainty, U, is not a statistical confidence interval, it is an arbitrary substitute which is probably best interpreted as the largest error expected. Only if the bias limit is negligible does the uncertainty interval become a 95% confidence interval.

The combining equations are summarized on Figure 7 as the uncertainty model equations and the uncertainty interval is shown on Figure 8.

4.8 Propagation of Measurement Uncertainty

When a parameter cannot be measured directly, it is calculated as a function of the measurements such as a venturi meter measurement or mass flow measurement. Error measurement is propagated to the calculated parameter through the function relating the measured parameters to the calculated parameter. The effect of the propagation may be approximated with a Taylor series when there is a mathematical relationship relating the parameter and the measurement. Only bias, precision, and degrees of freedom are propagated; never uncertainty. See Appendix B of Reference 2 for a detailed discussion on propagation of errors by Taylor series.

### 4.9 Reporting Uncertainty

The uncertainty parameter, U, is used for simplicity of reporting with the components bias, precision, and degrees of freedom available in an appendix or in supporting documentation.[4] These three components may be required (a) to substantiate and explain the uncertainty value, (b) to provide a second technical base for improved measurements, and (c) to propagate the error from measured parameters. The reporting format is shown on Figure 9.

## 5. CONCLUSIONS AND RECOMMENDATIONS

In the process of implementing the INEL standard practice at PBF, it has become apparent that a simple standard uncertainty analysis methodology, such as the NBS method, is necessary. Uncertainty analysis can become quite complex and each program may have unique problems. However, by following the methods described in this paper, the end results can be expressed in terms of three components. These components are bias, precision, and degrees of freedom. When they are combined using the NBS model, a single uncertainty value is obtained. This method has gained wide acceptance in other industries and, because it is simple and easily understood, it will provide a consistent method for comparing test results between facilities. At PBF this methodology has been used in the test independent uncertainty analysis and the application is continuing in the test dependent uncertainty studies.

## 6. REFERENCES

1. R. P. Evans, R. D. McCormick, J. E. Byrd, and L. C. Meyer, PBF-TFBP Experimental Measurements Test-Independent Uncertainty Analysis, Draft Internal EG&G Report, 1980.

2. ISAF AEDC Handbook, "Uncertainty in Gas Turbine Measurements," AEDC-TR-73-5 (AD-755356).

3. ASME Draft Standard, "Fluid Flow Measurement Uncertainty," March 28, 1980.

4. R. B. Abernethy, Pratt & Whitney Aircraft, Personal Communications.

| | | |
|---|---|---|
| Calibration Uncertainties | Calibration Source and Electronics | $B\ cal = \pm [U_{DAS} + (0.005\%\ RD)^2 + (0.055)^2]^{1/2}$ mV RTI<br>where $U_{DAS}$ = calibration errors associated with data acquisition system<br>% RD = percent of reading<br>mV = millivolts<br>RTI = referred to input of data system |
| Data Acquisition Systems Uncertainties | Low Level Amplifier | $B_1 = \pm [(10 \div G_1)^2 + (0.0179)^2]^{1/2}$ mV RTI<br>where $G_1$ = gain of low level amplifier |
| | Pre-Sample Filter Circuit | $B_2 = \pm [(0.02\%\ RD)^2 + (K \div G_1 G_2)^2 + (0.0295 \div G_2)^2]^{1/2}$ mV RTI<br>where $G_2$ = gain for pre-sample filter cir.uit<br>K = 2 for 10 Hz and 100 Hz channels<br>= 4.9 for 5 KHz channels<br>= 9.2 for 20 KHz channels |
| | Remote Modulate Demodulate Unit | $B_3 = \pm [(0.065\%\ RD)^2 + (5.7 \div G_1 G_2 G_3)^2]^{1/2}$ mV RTI<br>where $G_3$ = programmable gain in remote modulate demodulater unit |
| Data Reduction Systems Uncertainties | Array Processor and Computer | $B_4 = \pm\ 0.37\%\ RD$ mV RTI |
| | Data Presentation Plotter | $B_5 = \pm\ 6.25 \div G_1 G_2 G_3$ mV RTI |

Figure 1. Bias uncertainty equations associated with the data acquisition and reduction system (DARS).

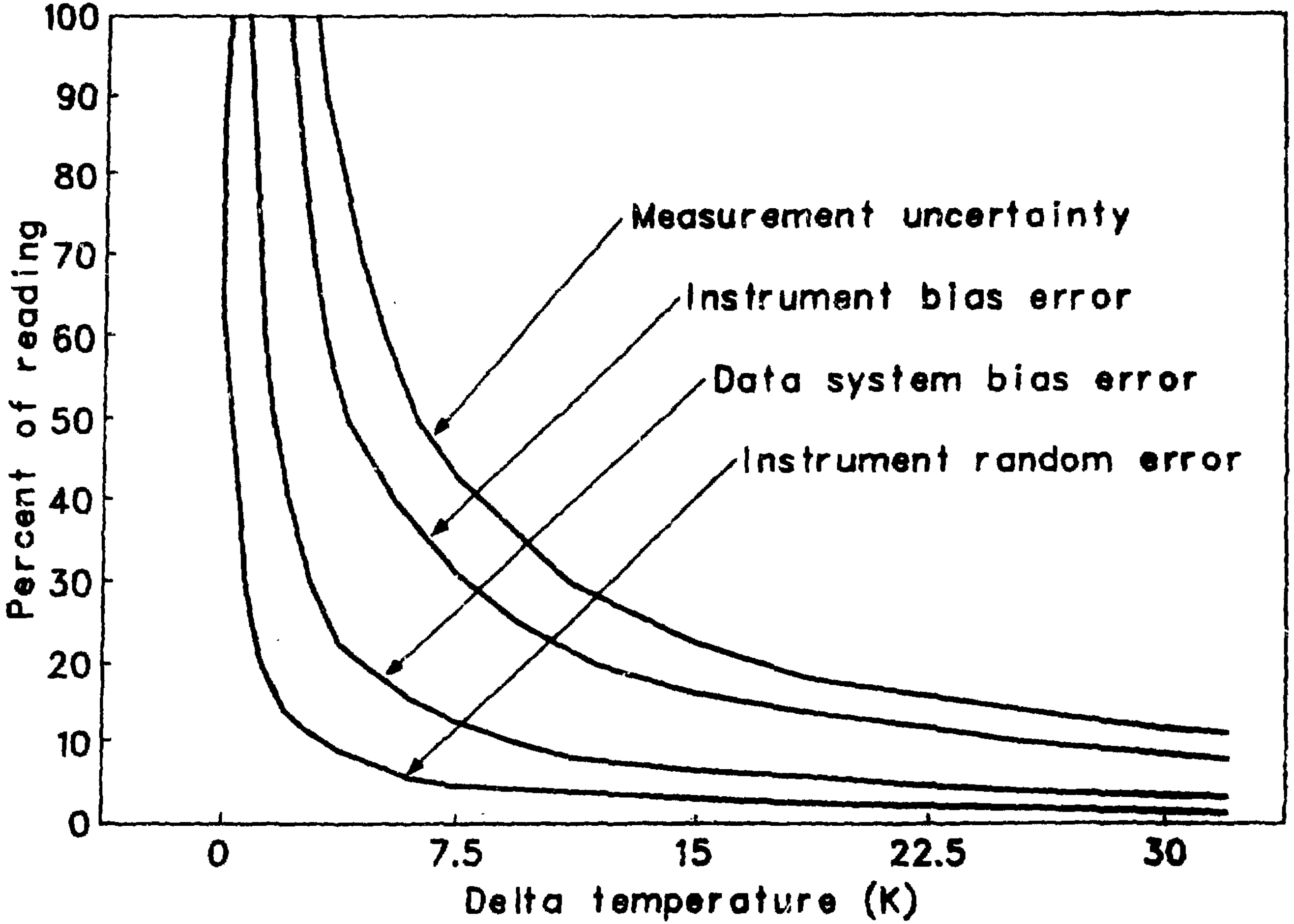


Figure 2. Test independent uncertainty for Type T (copper-constantan) differential thermocouple measurement.

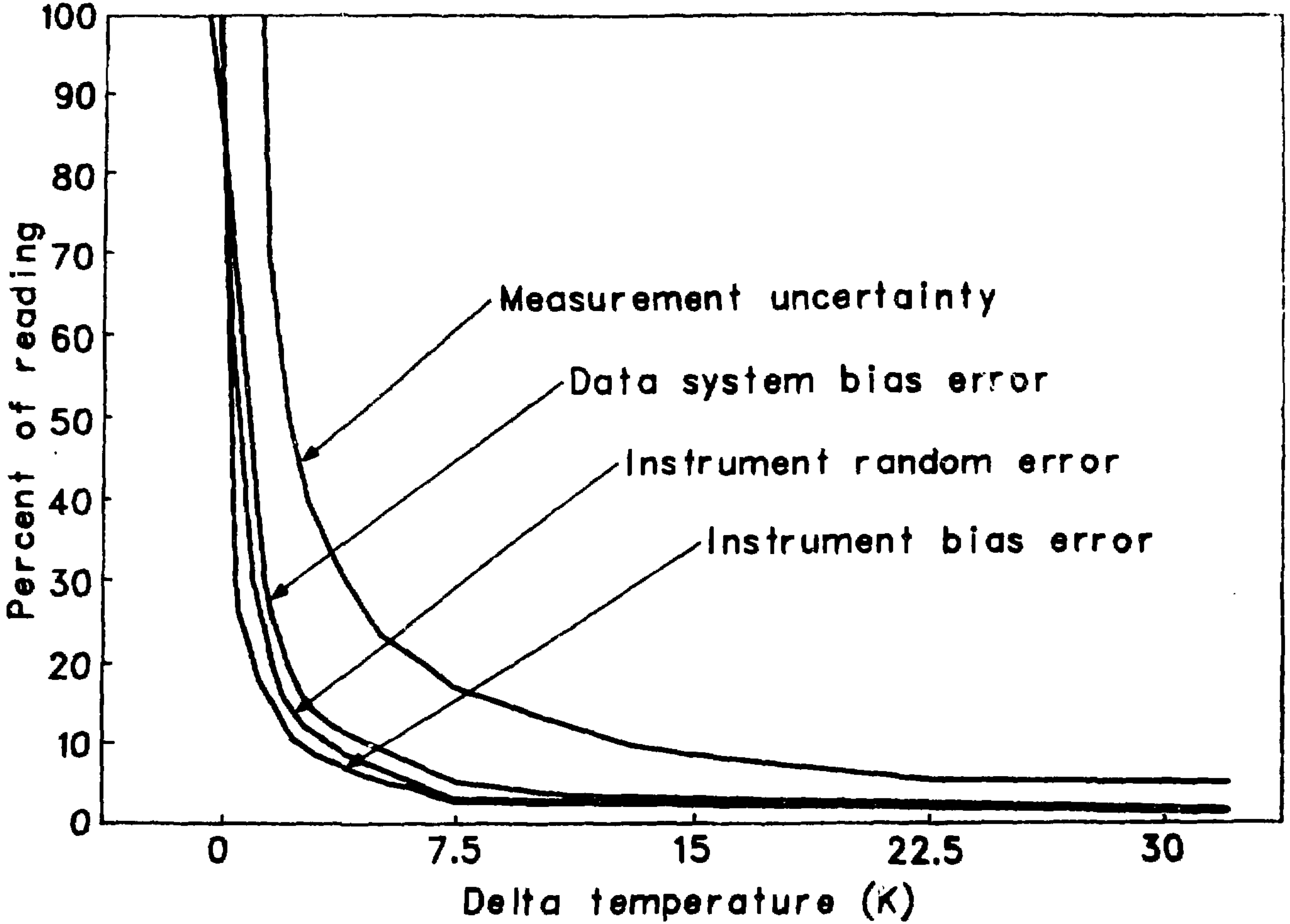


Figure 3. Test dependent uncertainty for Type T (copper-constantan) differential thermocouple measurement.

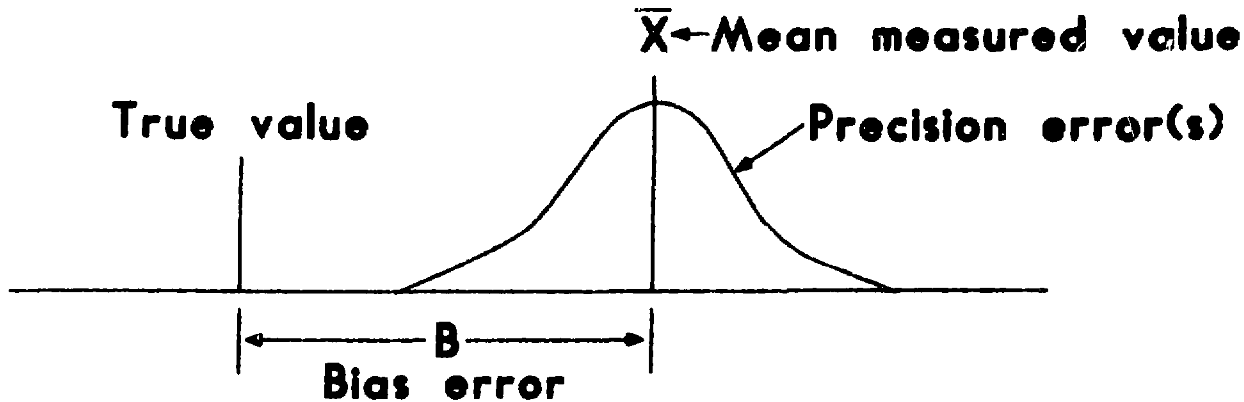


Figure 4. Measurement error.

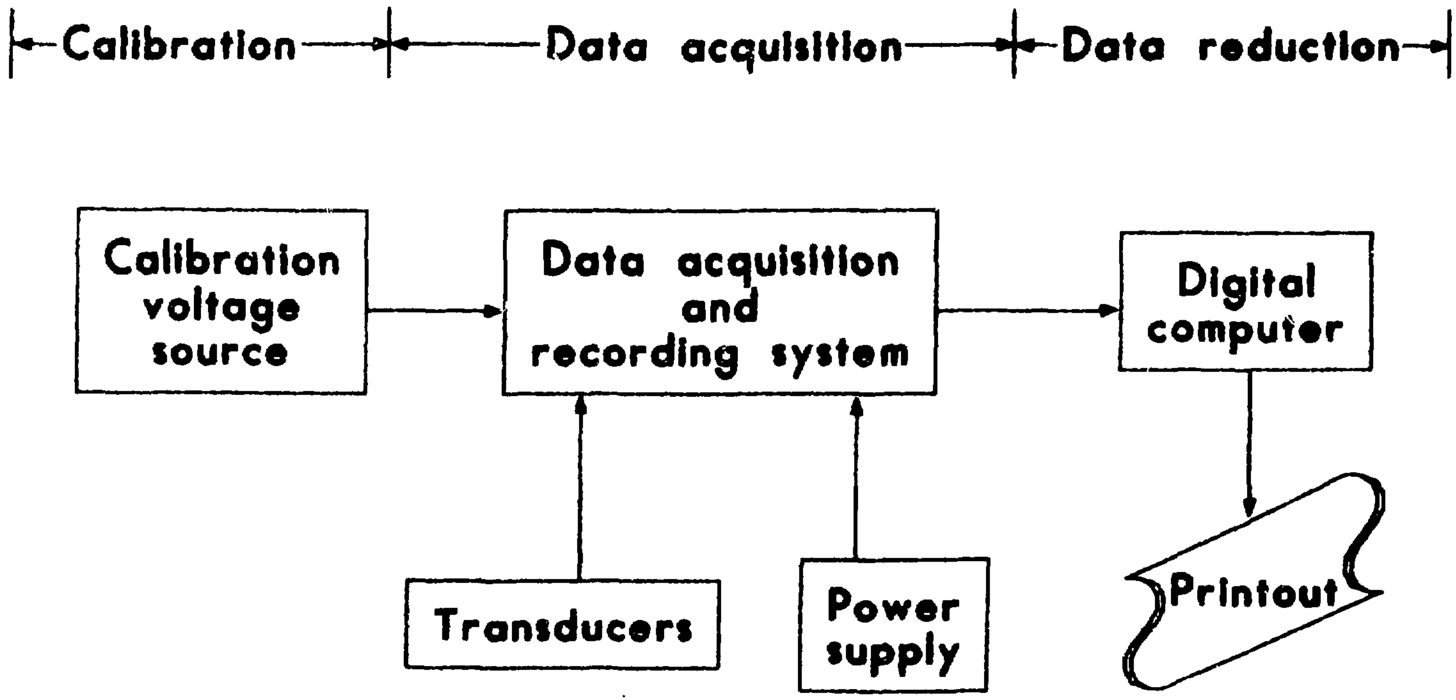


Figure 5. Conceptual design for uncertainty analysis.

| Calibration | Data acquisition | Data reduction |
| --- | --- | --- |
| $b_{11}$ through $b_{11}$ | $b_{12}$ through $b_{12}$ | $b_{13}$ through $b_{13}$ |
| $s_{11}$ through $s_{11}$ | $s_{12}$ through $s_{12}$ | $s_{13}$ through $s_{13}$ |
| $df_{11}$ through $df_{11}$ | $df_{12}$ through $df_{12}$ | $df_{13}$ through $df_{13}$ |

$$B_{cal} = \pm\sqrt{b_{11}^2 + \cdots\cdots b_{11}^2}$$

$$S_{cal} = \pm\sqrt{s_{11}^2 + \cdots\cdots s_{11}^2}$$

$$DF_{cal} = \frac{\left[s_{11}^2 + \cdots\cdots s_{11}^2\right]}{\left[\dfrac{s_{11}^4}{df_{11}} + \cdots\cdots \dfrac{s_{11}^4}{df_{11}}\right]}$$

Figure 6. Method of combining elemental errors and degrees of freedom.

$$B_{Meas} = \pm\sqrt{B_{cal}^2 + B_{\substack{Data\\Acq}}^2 + B_{\substack{Data\\Red}}^2}$$

$$S_{Meas} = \pm\sqrt{S_{cal}^2 + S_{\substack{Data\\Acq}}^2 + S_{\substack{Data\\Red}}^2}$$

$$DF_{Meas} = \frac{\left[S_{cal}^2 + S_{\substack{Data\\Acq}}^2 + S_{\substack{Data\\Red}}^2\right]^2}{\left[\dfrac{S_{cal}^4}{DF_{cal}} + \dfrac{S_{\substack{Data\\Acq}}^4}{DF_{\substack{Data\\Acq}}} + \dfrac{S_{\substack{Data\\Red}}^4}{DF_{\substack{Data\\Red}}}\right]}$$

$$U_{Meas} = \pm\left(B_{Meas} + t_{95}\, S_{Meas}\right)$$

WHERE $t_{95}$ IS THE 95$^{th}$ PERCENTILE POINT FOR THE TWO-TAILED STUDENTS "t" DISTRIBUTION.

Figure 7. Uncertainty model equation.

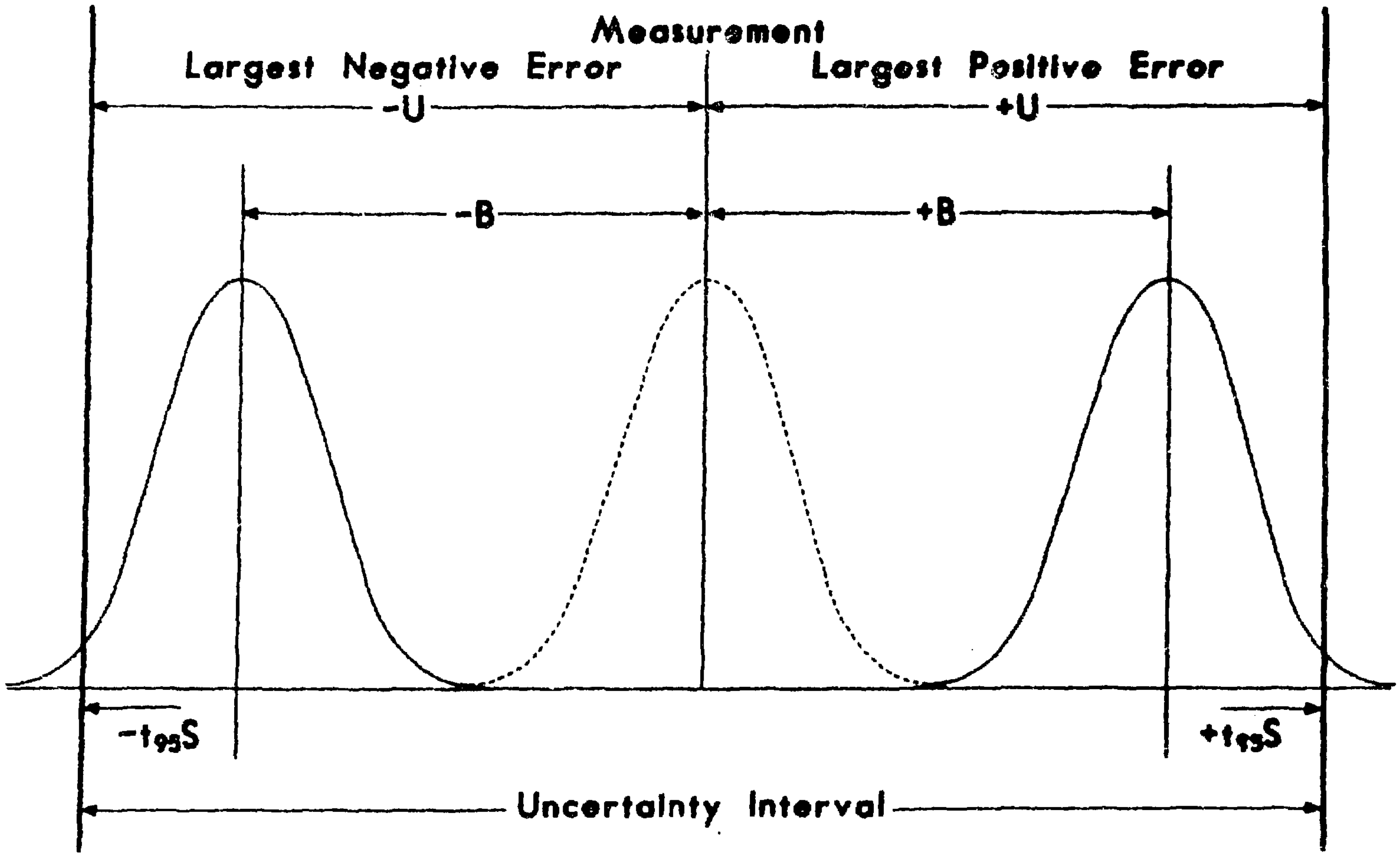


Figure 8. The uncertainty interval.

1. Bias error $B = \sqrt{\sum B_i^2}$

2. Precision error $S = \sqrt{\sum S_i^2}$

3. Degrees of freedom
   (From Welch Satterthwaite equation)

4. Uncertainty $U = B + t_{95}S$

Figure 9. Standard reporting format.