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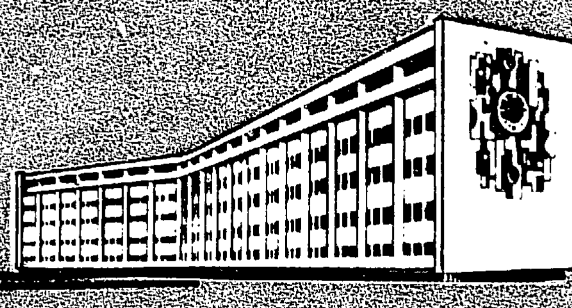
DYNAMIC MASS GENERATION AND RENORMALIZATIONS IN QUANTUM FIELD THEORIES



КИЕВ

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Dynamic Mass Generation and Renormalizations in Quantum Field Theories

V.A.Miransky

Divergences in field theories with scale invariant lagrangians (for example, massless electrodynamics and chromodynamics) lead to a breakdown of scale invariance. In the normal phase of these theories where all bare and physical masses are equal to zero the breakdown is determined by the multiplicative renormalization relations [1]

$$\Gamma^{(n)}(\{p\},\Lambda,g_\Lambda)=Z(\Lambda'/\Lambda,g_\Lambda)\,\Gamma^{(n)}(\{p\},\Lambda',g_{\Lambda'}) \qquad (1)$$

for Green functions $\Gamma^{(n)}$ (here $\{p\}\equiv(p_1,p_2,\dots p_n)$, Λ plays the role of a cut-off parameter, g_Λ is the coupling constant).

It is commonly believed that the dynamic mass generation doesn't destroy these multiplicative relations. The consequence is that the physical masses are renormalization group invariant

$$m(\Lambda,g_\Lambda)=m(\Lambda',g_{\Lambda'}) \qquad (2)$$

and the form of the massive phase Green functions doesn't depend on $\{\Lambda,g_\Lambda\}$.

Is this scale invariance breakdown the most general? Since the renormalization relations (1) are proved only for the normal phase of the theory, the answer to this question is not.

obvious.

In this paper we indicate the dynamic mass generation mechanism which destroys the multiplicative relations (1). In the massive phase divergences of a new type occur. To remove these divergences the value of the bare coupling constant must be fixed. This value coincides with the critical point separating the massless and the massive phases of theories.

A number of dynamic phenomena in quantum field theories have analogues in quantum mechanics (for example, the tunneling process). So, it seems useful to examine the scale invariance breakdown for the massless Dirac equation with the Coulomb potential $-\alpha/r$ $(\alpha = Ze^2/4\pi)$. For the values $\alpha < 1$ $(Z < Z_c \simeq 137)$ the scale invariance of this equation is manifested in the absence of hydrogen-like stationary levels. In this case there is only a continuous spectrum with energy $E \geqslant 0$ and $E < 0$ (Dirac sea). However, when $\alpha > 1$ the situation is drastically changed. For these supercritical values of α the fall into the Coulomb centre (collapse) takes place [2]. In this case it is necessary to complete a definition of the problem by introducing a cutoff $\Lambda \equiv 1/r_0$ at small distances*). The consequence is that the scale invariance becomes broken.

For simplicity, a cutoff of the form

$$V(r) = -\left[\frac{\alpha}{r}\,\theta(r-r_0) + \frac{\alpha}{r_0}\,\theta(r_0-r)\right] \qquad (3)$$

will be used.

Following Refs. [2,3], we shall look for the solutions corresponding to the Breit-Wigner (BW) resonance levels $\mathcal{E}(\mathrm{Im}\,\mathcal{E}<0)$ which define an outgoing positron wave. The physical content of this solution is as follows: the lifetime of a resonance $T \sim 1/|\mathrm{Im}\,\mathcal{E}|$ defines the time of vacuum rearrangement under which the electron-positron pairs are created spontaneously from the vacuum;

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*)The mathematical reason for this is connected with the fact that the Dirac operator for the strong Coulomb field ($\alpha > 1$) has nonzero defect indices, [2].

the positrons go to infinity and the electrons are coupled to the Coulomb centre. As a result, the supercritical charge of the centre is shielded.

In the region $z > z_c$ the solution of the Dirac equation with the potential (3) is expressed through the Whittaker functions [4] $W_{p,ig}(x)\left(p=\frac{1}{2}-i\alpha,\ g=\sqrt{\alpha-1},\ x=-2i\varepsilon z\right)$, and in the region $z < z_0$ through the elementary functions. By equating the logarithmic derivatives of these two solutions at $z = z_0$, we get the energy spectrum for the $nS_{1/2}$ $(n=1,2...)$ BW levels

$$\varepsilon^{(n)} = -b\Lambda(\sin\beta - i\cos\beta)\exp\left[\frac{-\pi n}{\sqrt{\alpha-1}}\right], \qquad b \simeq 4{,}2\ , \quad \beta = -\frac{\pi}{2}\cdot 1{,}01. \tag{4}$$

At small distances ($x \ll 1$) the Whittaker function

$$W_{p,ig} \simeq x^{1/2}\left[\frac{\Gamma(2ig)}{\Gamma(\frac{1}{2}+ig-p)}x^{-ig} + \frac{\Gamma(-2ig)}{\Gamma(\frac{1}{2}-ig-p)}x^{ig}\right] \simeq x^{1/2}\cdot A\cos[g\ln x + a] \tag{5}$$

where A, a are some numerical constants. In the limit $\Lambda \to \infty$, $\alpha > 1$ this function has an infinite number of zeroes - the typical evidence of a collapse (the absence of the ground state) [2]. This agrees with the fact that the energy $\varepsilon^{(n)}$ becomes infinite in this limit.

However, there is another limit: $\Lambda \to \infty$, $\varepsilon^{(1)} = const$. In this limit the value

$$\alpha = 1 + \pi^2 / \ln^2 \frac{b\Lambda}{|\varepsilon^{(1)}|} \longrightarrow 1 \tag{6}$$

is fixed, the wave function

$$W_{p,ig} \to W_{p,0} \underset{x \ll 1}{\simeq} -\frac{x^{1/2}}{\Gamma(i)}\ln x \tag{7}$$

and all BW resonances with $n \geqslant 2$ disappear ($\varepsilon^{(n)} \to 0$, $n \geqslant 2$).

We can regard this limit as some renormalization procedure (compare with (2)):

$$\mathcal{E}^{(1)}(\Lambda, \alpha_\Lambda) = \mathcal{E}^{(1)}(\Lambda', \alpha_{\Lambda'}). \tag{8}$$

However, as can easily verified, relations (1) for the function (5) are not valid. Moreover, in the limit $\Lambda \to \infty$, $\mathcal{E}^{(1)} = const$ the form of this function is drastically changed: oscillations disappear (see (7)). The physical reason for this is clear: removing the cutoff and preserving the energy $\mathcal{E}$ to be finite we get rid of the collapse and, as a consequence, get rid of its manifestation, oscillations. Thus, the violation of the relations (1) is determined by the very essense of the fall into the centre phenomenon.

We emphasize that the quantum-mechanical divergence considered above is not connected with the loop divergences of field theories.

In the massless QED the problem of the Coulomb centre is substituted by the positronium problem and BW resonances by tachyons. In Ref. [5] we examined (in the ladder approximation) the Bethe-Salpeter (BS) equation for the positronium and showed that tachyon levels appear only for the supercritical values of the coupling $\alpha_0 > \alpha_c = \pi/4$ *). The mass spectrum of tachyons has the form

$$m_t^2(n) = -\Lambda^2 \exp\left[\frac{-2\pi n}{\sqrt{K-1}}\right]; \quad K = \frac{4\alpha_0}{\pi}, \quad n = 1, 2 \ldots \tag{9}$$

where Λ is the Lorentz-invariant cutoff.

The occurence of tachyons implies the instability of normal phase vacuum. The vacuum rearrangement is determined by

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*) Just as in the Coulomb centre problem, the dynamics at the small distances is primarily responsible for this tachyon solution. Therefore the coupling α_0 must be identified with the bare coupling constant.

tachyon quantum numbers. In our case the BS wave function of the tachyon solution has the form [5]

$$\chi_{\alpha\beta} = \langle 0 | T(\psi_\alpha \bar{\psi}_\beta) | P \rangle = \chi_S(q^2, P^2, qP)\,\delta_{\alpha\beta} + \\ + (\sigma_{\mu\nu})_{\alpha\beta}\,\chi_T^{\mu\nu}(q, P) \tag{10}$$

where q and P are the relative and the total momentum, respectively ($P^2 = -m_t^2$),

$$\chi_T^{\mu\nu}(q, P) = (P^\mu q^\nu - P^\nu q^\mu)\,\chi_T(P^2, q^2, qP),$$

and the scalar functions

$$\chi_S, \chi_T \sim (q^2/p^2)^{\frac{\nu-1}{2}} F\left(\frac{\nu+\mu}{2}, \frac{\nu-\mu}{2}, 1+\nu; -4q^2/p^2\right) \tag{11}$$

where $\nu = \sqrt{1+K}$, $\mu = \sqrt{1-K}$, F is a hypergeometric function. Since Eq. (10) implies

$$(1 \pm \gamma_5)\chi(1 \mp \gamma_5) = 0,$$

the tachyons have the chiral charge $Q_5 = \pm 2$. Therefore the vacuum rearrangement must lead to the chiral symmetry breakdown (fermion mass generation).

Why is the normal phase vacuum rearrangement for massless QED so different from the vacuum rearrangement for the supercritical Coulomb centre? Let us imagine a test positron with a supercritical charge $\alpha_0 > \alpha_c$. Like the Coulomb centre, it creates an e^+e^--pair from the vacuum and binds the created electron thus forming a tachyon. But now the fate of the remaining positron is quite different: since its charge is also supercritical, it, in turn, creates an e^+e^--pair, and so on. As a result, the tachyon condensate with a chiral charge Q_5 arises.

The physical reason for vacuum stabilization accompanied by the occurence of a fermion mass is clear: as is known from the problem of the Coulomb centre, the critical value α_c grows with increasing fermion mass [2] . In massless QED the electron acquires a mass but the bare coupling constant remains unchanged. As a result, the vacuum becomes stable.

The value of the physical fermion mass m can be determined by requiring that the tachyon mode disappears and the pseudoscalar Goldstone appears *). As it turns out to be [5]

$$m^2 = - m_t^2/4 .$$

Using asymptotic formulas for the hypergeometric functions we find that for large q^2 (small distances)

$$\chi_{S,T} \sim \left(q^2/p^2\right)^{-3/2} \cos\left(\sqrt{K-1}\, \ln q^2/p^2 + const\right) . \qquad (12)$$

This asymptotic form corresponds to the collapse (comp. with (5)).However, in the limit $\Lambda \to \infty$, $|m_t^2(1)| k \infty$ the coupling constant

$$\alpha_0 = \pi/4 + \frac{\pi^3}{\ln^2 \Lambda^2/p^2} \to \alpha_c = \pi/4 \qquad (13)$$

is fixed, the function

$$\chi_{S,T} \sim \left(q^2/p^2\right)^{-3/2} \ln q^2/p^2 \qquad (14)$$

and all tachyons with $n \geqslant 2$ disappear. In Ref. [5] we identified the critical value α_c with the Gell-Mann-Low fixed point for the bare coupling α_0 . As it is seen, this value is the boundary of the collapse region for α_0 .

Just as the Coulomb wave functions the wave functions (12) don't satisfy the multiplicative relations (1). The formal cause of this violation is connected here with the violation of the relation $\alpha_{\Lambda'} = Z_3 \alpha_\Lambda$ where Z_3 is the renormalization constant cf the photon propagator. Indeed, despite the fact that in the lad-

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*) We note an intriguing parallel between QED and QCD: like the mass of η' -meson, the mass of 0^{-+}-state in QED (parapositronium) is too large for this state to be regarded as an "almost" Goldstone boson ($U(1)$-problem).

der approximation the constant $Z_3 = 1$, the charge renormalization (13) takes place for this solution. Thus, for supercritical $\alpha_0 > \alpha_c$ there is an additional (purely dynamic) renormalization of the charge.

It is useful to compare this dynamic mass generation mechanism with the known Johnson-Baker-Willey(JBW) mechanism[6] . In the approximation with a bare vertex and a free photon propagator[6,7] the ultraviolet asymptotic behaviour of the solution of the Schwinger-Dyson equation for the fermion mass function $B(p^2)$ (propagator $S = \frac{1}{\hat{p}A(p^2) - B(p^2)}$) is

$$B(p^2) \sim (p/m)^{\sqrt{1-K}-1} \tag{15}$$

when the coupling $\alpha_0 < \alpha_c$. This is the JBW solution. However, when the constant α_0 becomes supercritical, the asymptotic behaviour is changed in a drastic way

$$B(p^2) \sim (p/m)^{-1} \cos\left(\sqrt{K-1}\, \ln p/m + const\right). \tag{16}$$

The bare fermion mass [7]

$$m_0 = \left[p^2 \frac{dB}{dp^2} + B(p^2)\right]\Big/_{p^2=\Lambda^2}. \tag{17}$$

Therefore for the JBW solution the bare mass

$$m_0 \sim \Lambda^{\sqrt{1-K}-1} \tag{18}$$

is equal to zero only after the cutoff is removed. Another situation occurs for the solution (16) with the characteristic oscillations: in this case the condition (17) with zero bare mass determines the physical mass spectrum of the form (9).

For the JBW solution the mass parameter m doesn't depend on the cutoff ($\frac{\partial m}{\partial \Lambda} = 0$) and the mass renormalization (18) takes place. This leads to an explicit rather than a spontaneous violation of γ_5-symmetry [7] . In the examined approximation the charge renormalization for this solution is absent ($Z_3 = 1$).

So in this case the renormalization group β-function [1] equals zero, which leads to the power asymptotic (15) for $B(p^2)$.

For the solution (16) the mass parameter m depends on Λ ($\frac{\partial m}{\partial \Lambda} \neq 0$) but $\frac{dm}{d\Lambda} = \frac{\partial m}{\partial \Lambda} + \frac{\partial m}{\partial \alpha_\Lambda}\frac{d\alpha_\Lambda}{d\Lambda} = 0$. The mass renormalization is absent, but the additive charge renormalization (13) takes place. As a consequence, the <u>multiplicative relations</u> (1) <u>are destroyed</u> and the violation of the γ_5-symmetry (in the above approximation when γ_5-anomalies are absent) is spontaneous.

We would like to indicate a soluble model, the massless Thirring model, where fermion mass generation of the considered type occurs. In the normal phase of this model the charge renormalization is absent (β-function equals zero) which leads to the scale covariant (with anomalous dimensions) Green functions [8]. However, as it was recently shown by Mc Coy and Wu [9], in a regularized (by a lattice cutoff) version of this model the massive phase exists for the supercritical coupling constant $g < g_c = -\pi/2$. When cutoff is removed and the fermion mass remains finite ($0 < m < \infty$), the coupling g becomes fixed: $g = g_c = -\pi/2$. The critical value g_c coincides with the boundary of the collapse region established by Coleman [10].

The phase diagram of the Thirring model is as follows. Since the β-function equals zero for the subcritical values of the coupling constant, $g > -\pi/2$, these values form the line of the fixed points. The massive phase ($g < -\pi/2$) has only one fixed point, $g = g_c = -\pi/2$, separating the massless and the massive phases.

In the approximation considered above the phase diagram of massless QED is very similar: $\beta = 0$ for all $\alpha_0 < \alpha_c$, and the fixed point α_c separates massless and massive phases*). In

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*) The connection between phase diagrams of 4-dimension gauge theories and some 2-dimension models was discussed in Refs [11] using a different approach.

the full theory the diagram that we suggest takes the following form: when $\alpha_0 < \alpha_c \sim 1$, the vacuum polarization effects renormalize the charge and lead to the Gaussian infrared stable fixed point $\alpha = 0$ [12]. The consequence is that when cutoff is removed, the free field theory corresponds to all values $\alpha_0 < \alpha_c$ (Landau-Pomeranchuk-Fradkin zero-charge situation [1]). When α_0 becomes larger than α_c, the additional charge renormalization arises. This renormalization works in a direction opposite to that of polarization effects: whence for the former α_Λ decreases with increasing Λ (see Eq. 13), for the latter α_Λ increases as Λ increases $(Z_3 < 1)$ [1]. So one can expect that in QED the non-Gaussian fixed point α_c exists and the nontrivial local ($\Lambda = \infty$) field theory corresponds to this value of α_0.

Another situation occurs in quantum chromodynamics (QCD). As was shown in Refs. [13,14], the Bethe-Salpeter equation of a pure Yang-Mills theory has the colourless tachyon solution

$$m_t^2 = -\Lambda^2 \exp[-\frac{const}{g^2}],$$

i.e. the critical $g_c^2 = 0$ here. A similar situation takes place in QCD with a small enough number of quark flavours [14].

The vacuum rearrangement corresponding to this tachyon can result in the gluons acquiring a mass. The colourless nature of the tachyon gaurantees the equality of the mass for all gluons (the role of gluon mass generation for the quark confinement is discussed in Ref. [15]).

The results of the present paper lead naturally to the following questions. Does an additional dynamic divergence correspond to the occurence of this colourless tachyon? Or is its occurrence (as well as the colour tachyon pole in the gluon propagator) determined by ordinary divergences? These questions will be discussed elsewhere.

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