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UNCERTAINTIES IN THE PROTON LIFETIME

John Ellis
CERN, Geneva, Switzerland

Mary K. Gaillard
LAPP, Annecy-le-Vieux, France

D.V. Nanopoulos and Serge Rudaz
CERN, Geneva, Switzerland

ABSTRACT

We discuss the masses of the leptoquark bosons m_X and the proton lifetime in Grand Unified Theories based principally on SU(5). It is emphasized that estimates of m_X based on the QCD coupling and the fine structure constant are *probably* more reliable than those using the experimental value of $\sin^2\theta_W$. Uncertainties in the QCD Λ parameter and the correct value of α are discussed. We estimate higher order affects on the evolution of coupling constants in a momentum space renormalization scheme. It is shown that increasing the number of generations of fermions beyond the minimal three increases m_X by almost a factor of 2 per generation. Additional uncertainties exist for each generation of technifermions that may exist. We discuss and discount the possibility that proton decay could be "Cabibbo-rotated" away, and a speculation that Lorentz invariance may be violated in proton decay at a detectable level. We estimate that in the absence of any substantial new physics beyond that in the minimal SU(5) model the proton lifetime is $8 \times 10^{30\pm2}$ years.

1. INTRODUCTION

Grand Unified Theories (GUTs)[1),2),3),4)] are very beautiful, but they do not make very many testable predictions[5)]. Of course they do explain one or two long-standing mysteries, such as why the electromagnetic charges of the proton and electron are equal and opposite, and why leptons and quarks have such qualitative resemblances. GUTs are also able to string together phenomena which have no apparent connection. For example, given any one of the following striking features of our fundamental world, they are able to predict[3),4)] the other two: that the strong interactions are strong on a scale of 1 GeV, that $\sin^2\theta_W \simeq 0.2$ and that the proton is very stable. GUTs also enable one to interrelate some quark and lepton masses[6),4),7)]. What one gets in a specific theory depends of course on its Higgs and fermion representation content, but the original and simplest prediction made for m_b in terms of m_τ before[6)] the discovery of the bottom quark seems experimentally to be reasonably valid. Another very exciting possibility is that of generating the apparent baryon asymmetry of the universe by grand unified particle interactions violating C, CP and baryon number conservation very early in the big bang[8)]. This requires a tremendous extrapolation of our present cosmological knowledge, and recent considerations on grand unified monopole production[9)] and on thermal equilibration[10)] and dissipative phenomena[11)] in the early universe may require us to complicate the simple picture of the first 10^{-35} seconds originally used[8)]. Furthermore, reliable numerical calculations in specific models are quite tricky[12)]. Nevertheless, it seems likely that the qualitative solution to the longstanding astrophysical problem of the origin of matter has now been found. All well and good, but the scale of grand unification is estimated to be of order 10^{15} GeV, which is rather remote from our present "high energy" physics laboratories[*)]. What are the practical and significant low energy tests of GUTs?

The most dramatic testable predictions of GUTs are for the decays of protons and bound neutrons. The very fact of proton decay would be poetic: detailed studies of its decay modes might be our only open window on grand unified interactions. The present limit[13)] on the proton lifetime is somewhat model-dependent, but of order 10^{30} years in a class of popular models. It was realized early on that this longevity betokens very massive leptoquark bosons X to violate baryon number conservation, since

$$\tau_{proton} = C\,\frac{m_X^4}{m_{proton}^5} \qquad (1.1)$$

The first serious estimate of the parameter C was made in Ref. 4), and considerable effort has in the past been devoted[14),15),16),17)] to estimating it more reliably, as well as to estimating the leptoquark mass m_X, on which τ_{proton} (1.1) is strongly dependent.

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*) According to the conventional scaling laws for particle accelerators, an ep colliding ring machine capable of producing leptoquark bosons would need a proton ring of radius ~ 1 light year, an electron ring of order 10^{14} light years in radius, and a luminosity of order 10^{58} cm^{-2} sec^{-1}. One of us (J.E.) thanks A. De Rújula and C.H. Llewellyn Smith for discussions on this point.

In this paper we assess uncertainties in the proton lifetime, mainly but not exclusively concerned with the estimation of m_x.

Two alternative strategies for calculating m_x have been used. One[4),19)] is to use the disparity between the strong and weak coupling constants at present (low) energy scales, which is related to the leptoquark mass m_x by

$$\frac{1}{\alpha_3(Q)} - \frac{1}{\alpha_2(Q)} = -\frac{(11 + N_H/2)}{12\pi} \ln (m_x^2/Q^2) \qquad (1.2)$$

in a *leading* logarithmic approximation[3)]. Notice the insensitivity to the number of fermions in (1.2), and the sensitivity to the number of Higgs fields present at low ($<<m_x$) energies, which can rule out models with very many low energy Higgs representations - see section 3. There are important subleading logarithmic corrections to equation (1.2) which are the meat[18)] of the calculation of m_x in this approach. The alternative approach is to start[19)] from the experimental value $\sin^2\theta_W$, in terms of which m_x is given in a leading logarithmic approximation[3)] by

$$\sin^2\theta_W(Q) = \frac{3}{8} \left[1 - \frac{\alpha_{em}}{4\pi} \left\{\left(\frac{110 - N_H}{9}\right) \ln (m_x^2/Q^2)\right\}\right] \qquad (1.3)$$

where N_H is the number of complex Higgs doublets in the Weinberg-Salam model at "low" energies. The formula (1.3) is also subject to significant but calculable subleading corrections, but we can already see that it will yield a rather approximate method of estimating m_x. Suppose we consider an experimental error $\Delta(\sin^2\theta_W)$ in the determination of $\sin^2\theta_W$. This will correspond approximately to a change ΔL in $\ln(m_x^2/Q^2)$ given according to equation (1.3) by

$$\Delta(\sin^2\theta_W) = \frac{3}{8} \left(-\frac{1}{128}\right) \left(\frac{1}{4\pi}\right) \left(\frac{110 - N_H}{9}\right) \Delta L \qquad (1.4)$$

If we take the present error[20)] in the determination of $\sin^2\theta_W$ to be ± 0.015, then we find from (1.4) that $\Delta L = \pm 5.2$, corresponding to a change in m_x by a factor $(14)^{\pm 1}$! By way of contrast, in the approach starting from the "low" energy strength of the strong interactions and the appropriately sophisticated version of equation (1.2), we find that m_x is almost proportional to Λ, the familiar scale parameter of QCD. As is argued in section 2[21)], this may be known to within a factor 2, and should therefore give a more accurate way of determining m_x. To reduce the error in m_x to a factor $2^{\pm 1}$ using equation (1.4) would require an error of ± 0.004 in $\sin^2\theta_W$, which is not attainable at present. We will therefore use the scale of the strong interactions to set the scale m_x of grand unification.

Our analysis of uncertainties in the proton lifetime then proceeds in four stages. The first of these is the specification of the input parameters in the calculation of m_x. Section 2 contains a discussion of the meaning and value of the QCD Λ parameter, the appropriate starting value of the electromagnetic fine structure constant α[18),19)], and some further remarks about $\sin^2\theta_W$. It also sets out two calculational methods for estimating the renormalization effects in grand unified theories and estimating m_x. The next stage of the analysis concerns the extrapolation from "low" $Q^2 \leq 10^4$ GeV^2 across the conjectured desert to the region of Q^2 just less than m_x^2. Section 3 contains a discussion of the accuracy of calculations[22),23)] in the threshold region for massive vector bosons, relevant when $SU(5) \to SU(3) \times SU(2) \times U(1)$ or when $SU(2) \times U(1) \to U(1)$, and the likely uncertainty due to 3rd order effects in the desert region between thresholds. Also important are effects due to the possible existence of new physics in the domain of extrapolation. For example a new generation of fermions would increase m_x by almost a factor of 2, changing the proton lifetime estimate by a factor of about 12. A further multiplicative uncertainty of similar order could result from each extra generation of technifermions[24)], should they exist. Notice also that if one removes the conventional low energy fundamental Higgs doublet to replace it by technicolour, then one loses the factor of 2 reduction in m_x which was gained from the N_H dependent terms in the evolution equations (1.2, 1.3). Since one requires at least two generations of technifermions, this suggests that m_x is increased by a factor of at least (4 to 10) in technicolour models, with a corresponding (1.1) increase in the nucleon lifetime. Section 4 is concerned with the third stage of analysis around m_x, including uncertainties due to possible superheavy particles with masses $O(m_x)$[25)] and the possibility[26)] of "Cabibbo-rotating away" proton decay, which we argue[27)] to be unlikely in a wide class of models reproducing the naïve $SU(5)$ prediction for the bottom quark mass. We also make an aside about the possibility of observing a breakdown of Lorentz or Poincaré invariance in proton decay, a logical possibility a priori because the basic decay mechanism is such a short distance process ($\sim 10^{-28}$ cm). Sadly, this possibility seems to be unrealizable in the model[28)] of Lorentz invariance breakdown which motivated our interest, though it should still be borne in mind when considering experiments. Finally, in section 5 we assess uncertainties in the estimation[4),14),15),16),17)] of the quantity C in equation (1.1), which reflects the calculation of the matrix elements of the effective baryon-number violating operator. We then quote the values of m_x obtained in the two different calculational procedures introduced earlier, and then quote the nucleon lifetime expected in the minimal $SU(5)$ model, summarize the overall uncertainties in this estimate, and discuss ways in which they might be reduced.

While writing this paper we received a new paper[29)] by Goldman and Ross which also estimates uncertainties in the proton lifetime, and we comment on the relationship to our work wherever appropriate.

2. LOW ENERGY INPUTS

As was mentioned in the introduction, we believe that the most reliable starting-point for estimating m_x is to use[4),18)] the value of the strong and electromagnetic coupling constants at "low energies". We therefore need to know the appropriate input values of the strong interaction scale parameter Λ, and of the fine structure constant α in some renormalization prescription convenient for considerations on unified gauge theories. We study these two problems in the next two sub-sections, and devote a third sub-section to some remarks about $\sin^2\theta_W$.

2.1. The value of Λ

To get the desired precision in the estimation of m_x, we need to go beyond the leading logarithmic approximation of equation (1.2), which entails considering the evolution of coupling constants at the two-loop order. When this is done, one must specify carefully the renormalization scheme being used and the exact definition of the coupling constant, which was not done explicitly in Ref. 18). To different schemes and definitions will correspond different values of Λ. These are interrelated by simple numerical factors, for example

$$\Lambda_{\overline{ms}} = 2.66\ \Lambda_{ms} \ ; \ \Lambda_{MOM} \simeq 3.6\ \Lambda_{\overline{ms}} \qquad (2.1)$$

where Λ_{ms} and $\Lambda_{\overline{ms}}$ refer to the minimal subtraction scheme and the truncation proposed by Bardeen et al.[30], respectively, and Λ_{MOM} refers to a momentum space renormalization of the $q\bar{q}$ gluon vertex evaluated[29] at the symmetric point in the Feynman gauge in a Q^2 range where 6 flavours of quark are operational. Note that, as discussed in Ref. 31), the numerical factor of 3.6 between Λ_{MOM} and $\Lambda_{\overline{ms}}$ in equation (2.1) should be regarded as being uncertain[*] by O(20)% due to corrections of higher order in α_s. Similar uncertainties also exist for all the other numerical ratios of Λ parameters that we quote. There are now many different phenomenological analyses of deep inelastic scattering data which quote widely different values of Λ. But many of these differences arise because they use different renormalization schemes and hence should get different values of Λ. Among the favoured definitions of Λ are those mentioned in (2.1), and parametrizations of the nth moments of deep inelastic structure functions in terms of n dependent Λ parameters. In these schemes the Λ_n are specified in terms of (for example) $\Lambda_{\overline{ms}}$ by numerical coefficients analogous to those in (2.1). For example in the Λ_n scheme of Para and Sachrajda[32]

$$\Lambda_2\ (\text{for } F_2^{ep-en}) = 1.34\ \Lambda_{\overline{ms}} = 1.37\ \ \Lambda_2\ (\text{for } F_1^{\nu N}) \qquad (2.2)$$

It turns out that when one takes account of these different definitions and values of Λ, and expresses them all in terms of the corresponding values of a "standard" definition such as $\Lambda_{\overline{ms}}$, then different sets of data, modes of analysis and analyzers give surprisingly consistent results[21]. Shown in Table 1 are the results of 7 different analyses[33] determining the varieties of Λ parameters defined above (2.1, 2.2). They are all remarkably consistent and suggest that

$$\Lambda_{\overline{ms}} = 0.3 \text{ to } 0.5 \text{ GeV} \qquad (2.3)$$

Some words of caution[34] are in order: most of the analyses cited in Table 1 neglected possible violations of scaling due to higher twist effects. We may include their leading effects on deep inelastic moments in a parametrization

$$M_n(Q^2) = (\ln Q^2/\Lambda_n^2)^{-d_n}\,(1 + \frac{\mu_n}{Q^2}) \qquad (2.4)$$

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[*] Note that the factor of 3.6 that we have chosen corresponds to working with α_s^{-1} as we do subsequently in our analysis of the renormalization group equations (3.4).

One is prejudiced from known higher twist effects (elastic and quasi-elastic form-factors, scattering off diquarks, etc.) and the general falling trends of structure functions at large x to believe that T_n should generally be positive. If so, this means that the values of Λ_n are overestimated by the naïve analyses which neglect higher twist effects, though it is of course logically possible that T_n is in fact negative so that the values of Λ_n are underestimated. If we assume that $T \sim O(\Lambda)$, corresponding to two different measures of the strong interaction scale being comparable, then we find[21] that the preferred values of Λ are reduced by a factor $\lesssim 50\%$. We therefore conclude that it is reasonable to take $\Lambda_{\overline{MS}} = 0.4$ GeV and hope to be correct to within a factor of 2.

We should make a remark about recent experiments[35] from DESY quoting values of $\alpha_s(Q^2)$ and Λ from analyses of hard gluon bremsstrahlung events in $e^+e^- \to q\bar{q}g$ annihilation. Since these experiments are carried out at $Q^2 \approx 1000$ GeV^2, they should be relatively insensitive to higher twist effects. On the other hand, the full QCD radiative corrections to the $e^+e^- \to q\bar{q}g$ process have not yet been calculated. Just as in the case of deep inelastic scattering, this must be done before a reliable value of Λ (whether ms or $\overline{ms}$ or MOM) can be extracted.

There is also a school of thought[36] which holds that Λ is very small ($\lesssim 100$ MeV), largely based on an analysis of resonance phenomena and especially non-perturbative effects. We feel that this type of analysis is open to considerable question. For example, only in one case $[\Gamma(^1S_0 \to \text{hadrons})/\Gamma(^1S_0 \to \gamma\gamma)]$ have the first order QCD radiative corrections to a resonance observable been fully computed[37], and they turned out to be enormous: $(1 + 16\frac{\alpha_s}{\pi})$ times the lowest order result if the $\overline{ms}$ prescription is used. This suggests that using resonance effects to estimate Λ values is a very delicate business.

One point that should perhaps have been emphasized earlier is that with any definition the value of Λ varies with the number of flavours. This is seen clearly from the leading order formula

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2f)\ln(Q^2/\Lambda_f^2)} \tag{2.5}$$

where f is the number of flavours. If we neglect the effects of finite non-zero quark mass in (2.5) and just assume[38] that $\alpha_s(Q^2)$ is continuous at m^2_{f+1}, the $(\text{mass})^2$ of the $(f+1)$th quark, then

$$\Lambda^2_{f+1} = (\Lambda^2_f)^{\frac{33-2f}{31-2f}} \; (m^2_{f+1})^{\frac{-2}{31-2f}} \tag{2.6}$$

This approximation is not exact[4),18),39)], but is sufficient to indicate that the f dependence of Λ is not negligible. If we use the full second-order formula[31] for $\alpha_s(Q^2)$ and make the same assumption of continuity at $Q^2 \approx 2m_q^2$ as required in the $\overline{ms}$ prescription[39] (see also section 3.2), then we find the f dependence of Λ indicated in Table 2. The results quoted in Table 1 generally used 4 flavours. We see that when going through quark thresholds to extrapolate to large momenta Λ must be progressively decreased.

The matching with other coupling constants (in our case we will use α) can be done in either of two ways. One may either

(a) transform α_s from the $\overline{ms}$ to the MOM prescription and then do the rest of the renormalization analysis using momentum space couplings and the analysis by Ross[22] of threshold phenomena in gauge theories, or

(b) keep α_s in the $\overline{ms}$ prescription and transform the other couplings (in our case α) to an $\overline{ms}$ prescription also, and then do the rest of the renormalization analysis using the technique of Weinberg[39] for crossing gauge theory thresholds.

We will use both techniques in this paper so as to get some better impression of the uncertainties inherent to the analysis. In the case of method (a) we will therefore need to transform from $\Lambda_{\overline{MS}}$ to Λ_{MOM} using formula (2.1). There is a problem[*] of principle in using method (a) in that one should use a running Q^2 dependent gauge parameter, and all Ross's[22] calculations of the weak-electromagnetic and grand unified thresholds were calculated using the same (Feynman) gauge. However, this is unlikely[*] to have a significant numerical effect on the results, though if one uses the Feynman gauge in the neighbourhood of m_x, one should in fact use it <u>neither</u> for low energy QCD <u>nor</u> at the $SU(2) \times U(1) \to U(1)$ threshold. We can estimate the effect of this error by computing the Q^2 variation in the covariant gauge parameter ξ ($\xi = 0$ in the Landau gauge, $\xi = 1$ in the Feynman gauge) using the leading order evolution equations

$$Q \frac{\partial}{\partial Q} \xi(g,Q^2) = \begin{pmatrix} \frac{g_3^2}{8\pi^2} \left\{\frac{13}{2} - \frac{3}{2}\xi - \frac{2}{3}f\right\} & \text{for } SU(3) \\ \frac{g_2^2}{8\pi^2} \left\{\frac{13}{3} - \xi - \frac{2}{3}f\right\} & \text{for } SU(2) \end{pmatrix} \tag{2.7}$$

Setting $\xi = 1$ at the grand unification mass $m_x \approx 10^{15}$ GeV we find that at low Q^2

$$\begin{aligned} \left(\frac{5 - 3\xi_3(Q^2)}{2\xi_3(Q^2)}\right) &= \left(\frac{\alpha_3(Q^2)}{\alpha_{GUM}}\right)^{5/14} \quad \text{for } SU(3) \\ \left(\frac{3\xi_2(Q^2) - 1}{2\xi_2(Q^2)}\right) &= \left(\frac{\alpha_2(Q^2)}{\alpha_{GUM}}\right)^{1/10} \quad \text{for } SU(2) \end{aligned} \tag{2.8}$$

Taking representative values of $\alpha_3(Q^2 \approx 2m_t^2)$, $\alpha_2(Q^2 \approx 4m_W^2)$ we find

$$\begin{aligned} \xi_3(Q^2 \approx 2m_t^2) &\approx 0.73 \quad \text{for } SU(3) \\ \xi_2(Q^2 \approx 4m_W^2) &\approx 1.11 \quad \text{for } SU(2) \end{aligned} \tag{2.9}$$

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[*] G.G. Ross and C.H. Llewellyn Smith - private communication.

While they apparently used a different momentum space renormalization prescription than Goldman and Ross[29], the work of Celmaster and Gonzalves[31] may give an idea how the variation in ξ from 1 to 0.73 may influence the ratio between Λ_{MOM} and $\Lambda_{\overline{ms}}$. From their results we deduce

$$\frac{\Lambda_{MOM}(\xi_3 = 0.73)}{\Lambda_{MOM}(\xi_3 = 1)} = 1.05 \tag{2.10}$$

This implies an alteration in scale which is much less than many other uncertainties in the estimation of m_x, and we have not included the factor (2.10) in our subsequent analysis. We neglect the variation (2.9) in ξ_2 since we find it unlikely that the original threshold calculations of Ross[22] would be much affected by changing $\xi = 1$ to $\xi = 1.11$. Any resulting error seems unlikely to be larger than that in (2.10), which is already somewhat derisory.

2.2. The value of α

The value of $\alpha = \frac{e^2}{4\pi}$ in the Thompson limit ($\alpha_T^{-1} = 137.04$) is of course well-established. When following method (a) of section 2.1 the problem is to compute in terms of this definition of α the value at the symmetric renormalization point where contact can be made with the gauge boson threshold analysis of Ross[22]:

$$\alpha^{-1}(-\lambda m_W^2, -\lambda m_W^2, -\lambda m_W^2) - \alpha_T^{-1} = ? \tag{2.11}$$

where $\lambda = 4$ characterizes the effective position of the threshold in a step function approximation; we will see later that our results are relatively insensitive to the exact value of λ. The arguments of α^{-1} in (2.11) are the (off-shell) momenta of the two electrons and the photon, respectively. In this notation $\alpha_T = \alpha(m_e^2, m_e^2, 0)$. When following method (b) of section 2.1 the problem is to compute the finite difference between α in an $\overline{ms}$ prescription and the Thompson limit values:

$$\alpha_{\overline{ms}}^{-1}(\mu) - \alpha_T^{-1} = ? \tag{2.12}$$

We will see later that it is advisable to make the shift from a momentum space renormalization scheme to an $\overline{ms}$ prescription at a scale μ considerably larger than the mass of the heaviest quark, assumed to be the t quark. Therefore we will work with α in momentum space for the time being.

We will work to leading non-trivial order in α so that the usual renormalization group equation

$$\frac{\partial a(m_e^2,m_e^2,-Q^2)}{\partial t} \equiv Q^2\frac{\partial a(m_e^2,m_e^2,-Q^2)}{\partial Q^2} = -\beta_0^{em}\, a^2 + O(a^3) \tag{2.13}$$

becomes

$$\frac{d\alpha^{-1}}{dt}(m_e^2, m_e^2, -Q^2) = \frac{\beta_0^{em}}{4\pi} + O(\alpha) \tag{2.14}$$

In this order β_0^{em} is determined by the conventional vacuum polarization function $\pi(Q^2)$:

$$\beta_0^{em}(-Q^2) = \beta_0^{em} = \frac{d\pi(-Q^2)}{dt} \tag{2.15}$$

and hence

$$\alpha^{-1}(m_e^2, m_e^2, -Q^2) - \alpha^{-1}(m_e^2, m_e^2, 0) = \pi(-Q^2) - \pi(0) \tag{2.16}$$

It is a natural temptation[18),19),29),40)] to use the free fermion form for $\pi(-Q^2)$, in which case[*)]

$$\alpha^{-1}(m_e^2, m_e^2, -Q^2) - \alpha^{-1}(m_e^2, m_e^2, 0)$$

$$= -\frac{2}{\pi} \sum_{\text{fermions } i} \int_0^1 dx\, x(1-x) \ln\left[1 + x(1-x)\frac{Q^2}{m_i^2}\right] Q_i^2 \tag{2.17a}$$

$$= -\frac{2}{\pi} \Sigma \left[\frac{1}{6} \ln\left(\frac{Q^2}{m_i^2}\right) - \frac{5}{18} + \frac{m_i^2}{Q^2} + O\left(\frac{m_i^4}{Q^4}\right)\right] \tag{2.17b}$$

However, this procedure is too naive in the case of quarks, for which strong interaction corrections to the free fermion loops (2.17a) are important. Since we must cross low values of Q^2, we should take into account all orders of QCD perturbation theory, and even non-perturbative phenomena. Rather than insert some fudgy values of the quark masses into (2.17) in an attempt to estimate the integral, we prefer to rewrite it as an integral over the total cross-section for $e^+e^- \to \gamma \to$ hadrons:

$$\alpha^{-1}(m_e^2, m_e^2, -Q^2) - \alpha^{-1}(m_e^2, m_e^2, 0) = -\frac{2}{\pi} \int_0^\infty \frac{ds}{6s} \frac{Q^2}{Q^2+s} R(s) \tag{2.18a}$$

where

$$R(s) = \frac{\sigma(e^+e^- \to \gamma \to \text{hadrons})}{4\pi\alpha^2/3s} \tag{2.18b}$$

We then evaluate the integral in (2.18a) numerically to get the results shown in Table 3 using the following assumptions:

- The ρ is treated as a Breit-Wigner resonance with the parameters listed in Table 3.
- Higher resonances (ω, ϕ, J/ψ, ψ', Υ, ...) are treated in the narrow resonance

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*) Note that the finite part in (2.17b) differs slightly from that quoted in the preprint version of Ref. 29).

approximation, in which case their contributions to α^{-1} are

$$\delta\alpha^{-1} = -\frac{3}{\alpha^2}\sum_V \frac{1}{m_V}\left(\frac{\Gamma_{e^+e^-}\Gamma_{had}}{\Gamma_{tot}}\right)_V \left(\frac{Q^2}{Q^2+m_V^2}\right) \tag{2.19}$$

- Motivated by the latest experimental results[41], we assume that $R = 1$ for $1\ \mathrm{GeV} \leq E_{c.m.} = \sqrt{s} \leq 1.5\ \mathrm{GeV}$, and that $R = 2$ to 2.5 for $1.5\ \mathrm{GeV} \lesssim \sqrt{s} \leq 4\ \mathrm{GeV}$.

- We treat heavy quark thresholds as θ functions in $R(s)$. This is motivated by the form of the charm threshold and the expectation that the naïve formula

$$R_0(s) = \sqrt{1 - \frac{4m_q^2}{s}}\left[1 + \frac{2m_q^2}{s}\right] \tag{2.20}$$

is not applicable to heavy quarks because it neglects strong interaction effects close to threshold which tend to fill in the slow free fermion thresold rise exhibited by (2.20). In lowest non-trivial order of strong interaction perturbation theory, (2.20) becomes[42]

$$R_1(s) = R_0(s)\left[1 + \frac{4}{3}\alpha_s f(v)\right] \tag{2.21a}$$

where

$$f(v) = \frac{\pi}{2v} - \frac{3+v}{4}\left[\frac{\pi}{2} - \frac{3}{4\pi}\right] ; \quad v \equiv \sqrt{1 - \frac{4m_q^2}{s}} \tag{2.21b}$$

The singularity in v in (2.21a) exponentiates in higher orders in perturbation theory, and if one sums all these leading singularities one obtains

$$R_\infty(s) = 2\pi\,\alpha_s \tag{2.22}$$

This formula is perhaps reliable when one is a few hundred MeV beyond threshold, above important non-perturbative resonance effects. Putting reasonable values of α_s into (2.22) one finds that even close to threshold $R_\infty(s)$ is essentially the same as its asymptotic value for large s.

On the basis of the results in Table 3 we conclude that

$$\alpha^{-1}(m_e^2,m_e^2,-m_W^2) - \alpha^{-1}(m_e^2,m_e^2,0) = -8.40 \pm 0.42 \tag{2.23a}$$

while

$$\alpha^{-1}(m_e^2,m_e^2,-4m_W^2) - \alpha^{-1}(m_e^2,m_e^2,0) = -9.61 \pm 0.47 \tag{2.23b}$$

and

$$\alpha^{-1}(m_e^2,m_e^2,-7.8m_W^2) - \alpha^{-1}(m_e^2,m_e^2,0) = -10.21 \pm 0.47 \tag{2.23c}$$

We quote the value (2.23) of α^{-1} at several different values of $\lambda = 1,4$ and 7.8 because they are each useful for calculating m_x in different renormalization schemes. Working in momentum space, it will be convenient to replace the slow turn-on of the weak interaction effects on the evolution of α by assuming a θ-function threshold at $Q^2 = \lambda m_W^2$. The appropriate numerical value of λ can in principle be deduced from the threshold calculations of Ross[22]. Unfortunately, he does not give explicit formulae for the Q^2 dependent β-function for electromagnetism in his paper, and his graph of it (his fig. 6) is apparently incorrect[*] because it has the wrong asymptotic limit for the β-function at $Q^2 \gg m_W^2$. From his graph one would have deduced $\lambda = 7.8$: we will take this and the naïve value $\lambda = 4$ as limiting cases of where the appropriate replacement θ-function threshold should be. The uncertainty engendered by varying λ between 4 and 7.8 is not substantial compared with the other uncertainties we encounter. The value of α^{-1} at $\lambda = 1$ is convenient for use in the modified minimal subtraction scheme for calculating m_x which is discussed later.

We may compare, e.g., Eq. (2.23a) with the result we would have obtained from naively using the free fermion form for all quarks as well as leptons:

$$\alpha^{-1}(m_e^2,m_e^2,-m_W^2) - \alpha^{-1}(m_e^2,m_e^2,0) = \begin{cases} -7.23 & \text{for } m_u = m_d = 300 \text{ MeV},\ m_s = 500 \text{ MeV},\ m_t = 15 \text{ GeV} \\ -9.22 & \text{for } m_u = 4 \text{ MeV},\ m_d = 6 \text{ MeV},\ m_s = 125 \text{ MeV},\ m_t = 15 \text{ GeV} \end{cases} \tag{2.24}$$

[*] We may note in passing that the top part of the vertical scale on his fig. 9 seems to be mislabelled, but we do not use this graph in our analysis.

In order to get to the symmetric momentum space point at which Ross[22)] defines his couplings, an additional finite renormalization must be made, which is found[29)] to be

$$\alpha^{-1}(-4m_W^2, -4m_W^2, -4m_W^2) - \alpha^{-1}(m_e^2, m_e^2, -4m_W^2) = 0.60 \qquad (2.25)$$

On the basis of (2.23) and (2.25) we finally conclude that

$$\alpha^{-1}(-4m_W^2, -4m_W^2, -4m_W^2) = 128.03 \pm 0.47 \qquad (2.26a)$$

$$\alpha^{-1}(-7.8m_W^2, -7.8m_W^2, -7.8m_W^2) = 127.43 \pm 0.47 \qquad (2.26b)$$

For orientation purposes, note that an error of ± 0.5 in $\alpha^{-1}(-4m_W^2, -4m_W^2, -4m_W^2)$ corresponds to an error in m_x of the order of about 13 %.

Computation of m_x using method (a) of section 2.2 can now proceed directly using (2.26) and a value for Λ_{MOM}. In order to use method (b) we must instead transform α to an $\overline{ms}$ prescription. We choose to do this at a momentum scale μ obeying the conditions

$$2m_t << \mu << 2m_W \qquad (2.27)$$

so as to be able to use the analysis given previously in this section to take into account strongly interacting quark thresholds which are not trivially accounted for in the $\overline{ms}$ prescription. One must then make the finite renormalization*)

$$\alpha^{-1}(-m_e^2, -m_e^2, -\mu^2) - \alpha_{\overline{ms}}^{-1}(\mu) = 0.83 \qquad (2.28)$$

Using (2.28) and the results of Table 3 we see that

$$\alpha_{\overline{ms}}^{-1}(\mu = m_W) = 127.82 \pm 0.47 \qquad (2.29)$$

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*) P. Binétruy and T. Schücker - to be published in Ref. 39).

which can then be used directly with $\Lambda_{\overline{ms}}$ taken from Tables 1 and 2 in an analysis of renormalization in the $\overline{ms}$ prescription using the analysis of Weinberg[39] to deal with gauge thresholds.

2.3. The evaluation of $\sin^2\theta_W$

We have already mentioned in the introduction that the present experimental determination of $\sin^2\theta_W$ is insufficiently precise to compare with α_s as an input parameter for determining m_X. Conversely, $\sin^2\theta_W$ is in principle quite well determined in a grand unified theory such as SU(5). The issue is however confused by the fact that present experiments measure neutral currents at momentum transfers $Q^2 \ll m_W^2$, the characteristic scale of SU(2) symmetry breaking. There are many different ways of defining $\sin^2\theta_W$ by reference to different low energy phenomena which are equivalent to zeroth order, but differ by radiative corrections of order α. In quoting an SU(5) value for $\sin^2\theta_W$ one should be careful to state one's definition, and experimental comparisons must in principle take into account the radiative corrections relevant to the process measured, relative to the definition of $\sin^2\theta_W$ for which a value is quoted. Not all of these radiative corrections have been calculated: in particular the electromagnetic radiative corrections should be calculated for each individual experiment. It is not obvious that these corrections are negligible at the level of precision which one is now seeking. For example, calculations by Sakakibara[43] have revealed purely weak radiative corrections to the left-handed couplings of u and d quarks - the quantities most precisely determined by inclusive deep inelastic neutral current experiments - which would reduce the value of $\sin^2\theta_W$ deduced from experiment by O(0.01 to 0.02) compared to that obtained from the tree graph approximation conventionally used. With radiative corrections of this order of magnitude, it is not clear that there is a significant discrepancy between the SU(5) value of $\sin^2\theta_W$ recently quoted[44]:

$$\sin^2\theta_W(m_W) = 0.206 \pm 0.006 \qquad (2.30)$$

where the definition is the ratio of the SU(2) coupling $\alpha_2 \equiv \frac{g_2^2}{4\pi}$ and the electromagnetic coupling α at a momentum transfer m_W:

$$\sin^2\theta_W(m_W) \equiv \frac{\alpha(m_W)}{\alpha_2(m_W)} \qquad (2.31)$$

and the world average value of $\sin^2\theta_W$ given by Langacker et al.[45]:

$$\sin^2\theta_W = 0.229 \pm 0.008 \text{ (exp.)} \pm 0.006 \text{ (theor.)}. \qquad (2.32)$$

Another recent analysis[46] of neutral current data gives a value significantly higher than (2.32)

$$\sin^2\theta_W = 0.238 \pm 0.011 \qquad (2.33)$$

The difference between these two values may reflect the uncertainties induced by reasonable differences in assumptions made in theoretical analyses.

Most of these analyses rest heavily on two classes of experiment - the deep inelastic νN neutral current experiments, particularly that of the CDHS collaboration, and the SLAC experiment demonstrating parity violation in eD scattering[47]. We have no comments about the νN experiments, except to recall the necessity of making radiative corrections which was mentioned above. As far as the SLAC experiment[47] is concerned, we merely recall that if the results are interpreted with a two parameter fit of $\sin^2\theta_W$ and the ratio

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \tag{2.34}$$

(the values of $\sin^2\theta_W$ quoted above were for the case $\rho = 1$) then one finds

$$\begin{aligned} \sin^2\theta_W &= 0.293 \begin{array}{l} +\,0.033 \\ -\,0.10 \end{array} \\ \rho &= 1.74 \pm 0.36 \end{aligned} \tag{2.35}$$

The large and correlated errors on these parameters reflect a relatively poor determination of the y dependence of the parity violation effect due to the experiment's limited acceptance in y. The precise value of $\sin^2\theta_W$ usually quoted from this experiment

$$\sin^2\theta_W = 0.224 \pm 0.020 \tag{2.36}$$

is obtained by fixing $\rho = 1$, the value favoured by theory and other experiments. The relatively imprecise results (2.32) indicate that despite the historic achievement of the SLAC experiment[47], there is still information to be gained from a second generation experiment which could make a significant contribution to the determination of neutral current parameters.

3. UNCERTAINTIES IN EXTRAPOLATION

3.1. General remarks

We now turn to the problem of extrapolating from the inputs discussed in the previous section up to the grand unification m_X. The extrapolation has several aspects: threshold regions for $SU(3) \times U(1) \to SU(3) \times SU(2) \times U(1)$ or for $SU(3) \times SU(2) \times U(1) \to SU(5)$, the "desert" region in between the grossly different scales of these two thresholds, and the possible existence of any new physics (technicolour...?) in the "desert". We will discuss individually each of these aspects.

The formulae (1.2) and (1.3) suggest[29),39)] that m_X can be calculated by expanding $t_X \equiv \ln(m_X^2/\mu^2)$ (where μ is some low energy scale), in inverse powers of α starting with α^{-1}:

$$t_X = \frac{A}{\alpha} + B + \ldots \tag{3.1}$$

This is not in fact strictly valid, since there are also $\ln\alpha$ terms. This can be seen by considering the known form for the evolution of coupling constants when the β function is evaluated to $O(g^5)$:

$$\alpha^{-1} = at + b\ln t + c + \ldots \tag{3.2}$$

entailing that actually

$$t = \frac{A}{\alpha} + \tilde{B}\ln\alpha + \tilde{C} + \ldots \tag{3.3}$$

The coefficients A for different strategies for calculating m_x are given in formulae (1.2) and (1.3): the meat of the calculation is now the non-leading logarithms, i.e. $(\tilde{B}, \tilde{C})$ in formula (3.3).

In considering this problem it is useful to consider the evolution of the α_i^{-1} where $\alpha_i \equiv g_i^2/4\pi$: $i = 1, 2, 3$ for U(1), SU(2) and SU(3) respectively. To two-loop order (g^5) the equations of evolution are[18),29),40)]

$$\frac{d\alpha_i^{-1}}{dt} = \frac{\beta_0^i}{4\pi} - \sum_{j=1}^{3} \frac{\beta_1^{ij}\,\alpha_j}{(4\pi)^2} \tag{3.4a}$$

where

$$\beta_0^1 = -\frac{4N_g}{3}, \quad \beta_0^2 = (22 - 4N_g)/3, \quad \beta_0^3 = (33 - 4N_g)/3 \tag{3.4b}$$

and

$$\beta_1^{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_g \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix}. \tag{3.4c}$$

where N_g is the number of fermion generations $[N_g \geq 3$ because of $(u\ d\ e\ \nu_e)$, $(c\ s\ \mu\ \nu_\mu)$, $(t\ b\ \tau\ \nu_\tau)]$. The expressions (3.4b) and (3.4c) are valid in all renormalization schemes in desert regions away from any thresholds. In the neighbourhood of a threshold Q_0 the coefficients β_0^i and β_1^{ij} depend on (Q^2/Q_0^2) in the momentum space subtraction scheme which has previously been used by Ross[22)] and by Goldman and Ross[18),29)], whom we follow here. In the "desert" regions between thresholds we will use the solutions to the lowest order renormalization group equations, $\alpha_i \propto \frac{A_i}{\ln Q^2/\Lambda^2}$, on the right-hand-side of (3.4a). The differences between these forms and the exact ones are of 3 loop order, which is below the level of precision which we seek in this paper.

3.3. *Gauge threshold regions*

The difference between the threshold and asymptotic values of the coefficients β_0^i and β_1^{ij} vanish as powers of Q^2/Q_0^2 above threshold, i.e. as exponentials in

$$|t_{th}| \equiv |\ln(Q^2/Q_0^2)| \;:\; \beta_0^i(\text{asymptotic}) - \beta_0^i(\text{threshold}) \propto e^{-|t_{th}|}\;,\; |t_{th}| \to \infty \tag{3.5}$$

This means that including threshold effects makes a finite $O(\alpha^\circ)$ difference between the coupling constants evaluated in the "desert" region below threshold with and without including threshold effects in the β coefficients:

$$\alpha_i^{-1}(\text{desert})\Big|_{\text{asymptotic } \beta} - \alpha_i^{-1}(\text{desert})\Big|_{\text{threshold } \beta}$$
$$= \frac{1}{4\pi}\int dt \left[\beta_0^i(\text{asymptotic}) - \beta_0^i(\text{threshold})\right] + \frac{1}{4\pi^2}\int dt\left[\beta_0^{ij}(\text{asymptotic}) - \beta_0^{ij}\ (\text{threshold})\right]\alpha_j + O(\alpha^2) \tag{3.6}$$

where the convergence of the t integrals is guaranteed by the $e^{-|t_{th}|}$ factors (3.5). It is clear from (3.3) that an $O(\alpha^\circ)$ change in α_i^{-1} corresponds to a finite change in the coefficient $\tilde{C}$. .

How far does one have to go in computing threshold effects in a momentum space renormalization scheme? It is clear from (3.6) that if one worries about the $O(\alpha_j)$ terms on the right hand side of (3.6) one is getting involved with $O(\alpha^{+1})$ terms on the right-hand side of (3.3), which are smaller than our present concerns.

Another point has recently been made[23)], namely that when using a momentum space renormalization scheme one should take into account the additional forms of invariant coupling which can occur in massive theories, such as arbitrary quadrupole moments of massive vectors denoted by κ. These moments must approach unity in the asymptotic region above a vector threshold, because the gauge theory is renormalizable in the ultraviolet region. Below the threshold they are irrelevant because the massive vector effectively decouples. They are therefore relevant only to the threshold corrections, i.e. to the term B in equation (3.1). Because $\kappa = 1$ in the symmetry limit, it follows that $\kappa - 1 = O(\alpha)$ is finite in perturbation theory for the broken symmetry. Since these couplings will enter in vector loop contributions to vector propagators, the only logarithms proportional to $(\kappa - 1)$ that can develop are $\ln(m_V^2/\mu^2)$, but since $\mu^2 \simeq m_V^2$ in the region where they are relevant, there can be no logarithm large enough to compensate the factor α. Hence they can only make $O(\alpha)$ corrections to B, and are thus of the generic order of terms that we are neglecting.

Gauge threshold effects in the $\overline{\text{MS}}$ prescription can easily be taken into account at the required level of accuracy using the formula of Weinberg[39)]:

$$g_i(\mu) = g\ (\mu) + \frac{g(\mu)^3}{96\pi^2}\ \left\{ \text{Tr}\left[t_{iS}^2\ P\ \ln(m_s/\mu)\right] + 8\ \text{Tr}\left[t_{iF}^2\ \ln(\sqrt{2}m_f/\mu)\right] - 21\ \text{Tr}\left[t_{iV}^2(\ln(m_v/\mu) - \frac{1}{21})\right]\right\} \tag{3.7}$$

The meaning of the symbols in (3.7) is as follows: t_{iS}, t_{iF} and t_{iV} are the representation matrices of the heavy particles of masses m_S, m_F and m_V for the subgroups i of the unified high energy group, while P projects out the Goldstone scalar modes "eaten" by the massive vector bosons. The couplings $g(\mu)$ and $g_i(\mu)$ are respectively the coupling constant of the unified theory and the coupling constant associated with the subgroup i after the massive S, F and V degrees of freedom are integrated out. It is trivially seen[39] from (3.7) that in the $\overline{MS}$ prescription one may take account of an $(f+1)$th new fermion threshold m_F by demanding that the coupling constant for f flavours and that for $(f+1)$ flavours join on continuously at

$$\mu = \sqrt{2} m_F \tag{3.8}$$

as already used in section ' (to construct Table 2. Correspondingly, we see from (3.7) that for a vector threshold the continuity of the $\overline{MS}$ coupling constants occurs at

$$\mu = e^{-1/21} m_V \tag{3.9}$$

which differs by 5% from the naïve prescription $\mu = m_V$ used in the first paper[39] of Binétruy and Schücker. We should note in passing that the numerical ratios (3.8) and (3.9) are subject to uncertainties of $O(\alpha_i)$ due to higher order effects. This uncertainty clearly gives an $O(\alpha_i)$ uncertainty to the estimate of τ_X via equation (3.1), and so is unimportant at our present level of approximation.

3.3. Effect of a new generation threshold

It is well-known that in leading order the estimation of m_X is independent of the number of generations of fermions. This is because in a grand unified theory the contributions of a generation to the leading order β functions β_0^i in (3.4a) have the same contributions $(-4/3)$ from each generation as can be seen from equation (3.4b). Therefore when one computes m_X from $[\alpha_3^{-1}(Q^2) - \alpha_2^{-1}(Q^2)]$ as in equation (1.2), the fermions cancel out and the rate of approach of the couplings depends only on the difference in the "sizes" of the groups as measured by the vector boson contributions to the β_0^i in (3.4b).

It is however clear from the forms of the higher order β_1^{ij} in (3.4c) that this independence from N_g will not persist when one goes beyond leading order in computing m_X. To estimate this sensitivity to N_g we assume that values of $\alpha_3(Q^2)$ and $\alpha_2(Q^2)$ for some Q just above the $SU(3) \times U(1) \to SU(3) \times SU(2) \times U(1)$ threshold region are fixed by low energy phenomenology with 3 generations, and that a mass-degenerate generation of fermions is then found. We compute its effect on the length of the "desert" by observing that the difference between $\alpha_3^{-1}(Q^2)$ and $\alpha_2^{-1}(Q^2)$ is fixed at either end (Q_0^2 and Q_1^2) of the desert - at the upper end Q_1^2 by the integral over the $SU(3) \times SU(2) \times U(1) \to SU(5)$ threshold region of the appropriate mass-dependent lowest order functions β_0^i as discussed in section 3.2. We therefore want the quantity

$$\left[\frac{1}{\alpha_3(Q_0^2)} - \frac{1}{\alpha_2(Q_0^2)}\right] - \left[\frac{1}{\alpha_3(Q_1^2)} - \frac{1}{\alpha_2(Q_1^2)}\right] \tag{3.10}$$

to be unchanged by the addition of an extra generation. The invariance of the quantity (3.10) before and after the addition of a fermion generation is easily seen from equation (3.4) to require:

$$\frac{11}{12\pi}(t_1 - t_0) + \frac{1}{(4\pi)^2}\int_{t_0}^{t_1} dt\left[(102\,\alpha_3 - \frac{136}{3}\alpha_2) - N_g(\frac{64}{3}\alpha_3 - \frac{89}{6}\alpha_2 + \frac{1}{6}\alpha_1)\right]$$

$$= \frac{11}{12\pi}(t_1' - t_0) + \frac{1}{(4\pi)^2}\int_{t_0}^{t_1'} dt\left[(102\,\alpha_3' - \frac{136}{3}\alpha_2') - (N_g + \delta N_g)(\frac{64}{3}\alpha_3' - \frac{89}{6}\alpha_2' + \frac{1}{6}\alpha_1')\right] \qquad (3.11)$$

where $t \equiv \ln Q^2/\mu^2$ for some scale μ. The difference between t_1' and t_1 is the change in the length of the "desert" required to compensate for the new δN_g terms on the right-hand-side of (3.11), and we have written α_i' on the right-hand-side to emphasize that the evolution of the coupling constants changes with the number of generations. It is apparent from the form of (3.11) that the dominant contributions come from the integrals over α_3. We therefore approximate it by

$$\frac{11}{12\pi}\ln(Q_1^2/Q_1'^2) = \frac{38}{(4\pi)^2}\left[\int_{t_0}^{t_1'} dt\,\alpha_3' - \int_{t_0}^{t_1} dt\,\alpha_3\right] - \frac{1}{(4\pi)^2}\frac{64}{3}\left[\int_{t_0}^{t_1'} dt\,\alpha_3'\right] \qquad (3.12)$$

where we have now specialized to the case $N_g = 3$, $\delta N_g = 1$ advertized earlier. Identifying Q_0^2 with the scale of the new heavy generation, we observe that it is convenient to redefine the scale of the logarithm of Q^2 in the expression for α_3' so that (cf. (2.6)) α_3 is continuous at Q_0^2 :

$$\alpha_3(Q^2) = \frac{12\pi}{21t} \rightarrow \alpha_3'(Q^2) = \frac{12\pi}{17\tilde{t}} : \tilde{t} \equiv \ln Q^2/\tilde{\mu}^2 : \tilde{t}_0 = \frac{21}{17}t_0 \qquad (3.13)$$

Equation (3.12) can then be written in the form

$$\frac{11}{3}\ln(Q_1^2/Q_1'^2) = \frac{40}{17}\ln(\tilde{t}_1/\tilde{t}_0) - \frac{104}{21}\ln(t_1/t_0) \qquad (3.14)$$

where $\tilde{t}_1 \equiv \ln Q_1^2/\tilde{\mu}^2$. Putting in reasonable values of the ends (t_0, t_1) of the "desert" before adding a new generation, we compute from (3.14) that

$$\ln(Q_1^2/Q_1'^2) = -1.2 \qquad (3.15)$$

from which we deduce that the estimate of m_X increases by a factor of 1.8 if a fourth generation is added. This corresponds via equation (1.1) to an increase in the proton lifetime by a factor of O(12). This order of magnitude increase is to be expected for each extra generation beyond 3: clearly not many can be tolerated if one wants the proton lifetime to stay observably short. This dramatic increase in τ_p was not noted previously because of arguments restricting the number of generation to 3 or at the most 4 (limits on the Helium generated in the Big Bang[48], the successful calculation of m_b in terms of m_τ[4),6),7)]). However, neither of these arguments may apply to the technicolour models[24)] recently proposed, in which there is a proliferation of fermion generations beyond 3 (these models do not contain extra massless neutrinos which are disfavoured[48]) by the calculations of Helium generation, and they invalidate the previous basis for calculating m_b[4),6),7)]).

It is perhaps worth noting that there is a simple way to understand this increase of O(2) in the estimation of m_X due to the addition of a fermion generation. We recall that in the analysis of Goldman and Ross[18]), the deduced value of m_X decreased by a factor of 4 when 2 loop effects in the β function were included, while the value of Λ was not changed. Looking at equation (3.4c) we see that the most important 2 loop terms – those proportional to α_3 in (3.4b) – approximately cancel in $(\beta_1^{23} - \beta_1^{33})$ if $N_g = 5$. Thus we might expect that the Goldman-Ross[18]) factor of 4 would be completely undone by adding 2 generations, and by $\sqrt{4} = 2$ if just one generation is added, as we have found.

We will return later to uncertainties in the estimation of m_X in the presence of technicolour, but first let us comment on the modification to the result (3.15) which is to be expected if the pattern of heavy fermion masses is more similar to that of lighter generations:

$$m_{\nu_4} = 0,\ m_{\ell_4^-} = O(40)\ \mathrm{GeV},\ m_{d_4} = O(2)m_{\ell_4^-},\ m_{u_4} = O(3)m_{d_4} \qquad (3.16)$$

In this case the effect on the value of $\alpha_{em}(m_W)$ is essentially within the errors quoted in section 2.2, and the order of magnitude of the dominant effect can probably be estimated by assuming that there is a range of a factor of O(10) in Q^2 during which the fermion contributions to β_0^2 and β_0^3 fail to cancel. Specifically, we might expect that between the d_4 and u_4 thresholds the fourth generation contributions to β_0^2 and β_0^3 would be

$$\Delta\beta_0^2 = -\frac{1}{3},\quad \Delta\beta_0^3 = -\frac{2}{3} \qquad (3.17)$$

resulting in an additional contribution $-\frac{1}{12\pi}\ln 10$ to the right-hand-side of equation (3.11), and hence $+\frac{1}{3}\ln 10$ to the left-hand-side of equation (3.14). This is somewhat smaller than the $\ln(Q_1^2/Q_1'^2)$ term in (3.14), and so does not affect the qualitative conclusion of an order of magnitude increase in the proton lifetime if a fourth generation exists. We should perhaps add in passing that there is an additional effect of increasing the number of generations, this time in the estimation of the coefficient C of equation (1.1). This comes from an increase in the grand unified coupling constant α_{GUM}. However, this change is negligible by comparison with the change induced by the effect of an extra fermion generation on the value of m_X.

3.4. Effect of a technicolour threshold

It has recently become trendy[24),49)] to replace explicit Higgs fields in the Weinberg-Salam model by dynamical symmetry breaking derived from a new "ultra-strong" unbroken non-Abelian gauge interaction which becomes O(1) and generates a vacuum expectation value for a fermion-antifermion condensate $\langle o|\bar{f}f|o\rangle \neq 0$ on a scale near 1 TeV. These models all introduce several new generations N_{Tg} of fermions – 4 in the case of SU(4) technicolour – and possibly other structures. In this paper we make the simplifying assumptions that at energies below the scale of SU(5) unification the technicolour group commutes with SU(5) and that the SU(5) representation content is

$$(N_{Tg} + 3)\ (\underline{\bar{5}} + \underline{10}) + (\text{any number of } \underline{1}) \tag{3.18}$$

This is what can happen in naïve extended technicolour models in which each technigeneration contains a right-handed neutral lepton field in addition to the usual 15 fermion helicity states conventionally assigned to a reducible $\underline{\bar{5}} + \underline{10}$ representation of SU(5). No phenomenologically satisfactory model of this type exists[49] but we may hope that it mimics appropriately the physics of a physical technicolour scheme, if that is the way of the world.

In this minimal scenario two phenomena occur whose effects we should try to estimate. One is that in a mass region around 1 TeV the technicolour intersections become strong, creating resonances and other as yet uncalculable phenomena. The other is that at energies below O(1) TeV, the spectrum of techniparticles consists of pseudo-Goldstone bosons (PGBs) with masses O(10 to 300) GeV. We can only give an order of magnitude estimate for the first of these effects, whereas the second can be estimated reasonably accurately.

We suppose that there is a region of a factor of 10 in Q^2 during which technicolour interactions are strong. In this region it is reasonable to expect that the fermion parts of the β_o^i are modified by O(1) relative to their naïve "point-like" values computed from simple fermion loops, i.e. there is an uncertainty in the $(-4N_{Tg/3})$ terms in the β_o^i (3.4a) of order 100%. Integrating the evolution equations (3.4a) over this threshold region t_{min} to t_{max} and discarding the higher order β_1^{ij} terms, we have

$$|\Delta(\alpha_3^{-1} - \alpha_2^{-1})| = \frac{1}{4\pi}\left[+\frac{4N_{Tg}}{3}\right](t_{max} - t_{min}) \tag{3.19}$$

as the change in the calculated difference between α_3 and α_2. Since, as argued in section 3.3, the quantity (3.10) must remain unchanged despite the introduction of technicolour, we must compensate for (3.19) by a corresponding change in the integral of the right-hand-side of (3.4a) which we approximate by considering the β_o^i terms alone and altering the value of t_1:

$$\frac{1}{4\pi}\left(\frac{4N_{Tg}}{3}\right)(t_{max} - t_{min}) = \frac{11}{12\pi}\,|t_1 - t_1'| \tag{3.20}$$

Taking $t_{max} - t_{min} = \ln 10$ we find that

$$|t_1 - t_1'| = (0.84)N_{Tg} \tag{3.21}$$

and that <u>the uncertainty in m_X is therefore a factor of $(1.5)^{\pm N_{Tg}}$</u>. This uncertainty factor is multiplicative with the factor of O(2) increase per generation previously found in section 3.3. It is also multiplicative with an increase of a factor of 2 in m_X caused by the disappearance of the doublet of fundamental Higgs fields previously included (1.2, 1.3) in the renormalization group calculations. The overall increase in

m_x due to technicolour is therefore a factor of (4 to 10) if there are just two technicolour generations. This indicates that even a relatively innocuous-seeming admixture of technicolour could increase m_x sufficiently for proton decay to be unobservable.

However, the estimated uncertainty (3.19) to (3.21) may be too pessimistic. This is because ordinary SU(3) colour as well as weak SU(2) is a relatively weak interaction on a scale of 1 TeV, and hence coloured and uncoloured states may be approximately degenerate in mass, so that while their contributions to the individual β_o^i are wildly fluctuating and incalculable, the difference $(\beta_o^2 - \beta_o^3)$ may be relatively stable near 0. This phenomenon is exemplified by the low energy PGBs of technicolour. It is easy to check that the contributions of the multiplets in Tables Ia and Ib of Ref. 24) make equal contributions to β_o^2 and β_o^3. However, the PGB multiplets are not all degenerate, and between $Q = 400$ GeV and 600 GeV

$$\beta_o^3|_{PGBs} = -4/3, \quad \beta_o^2|_{PGBs} = -2 \qquad (3.22)$$

This difference between β_o^2 and β_o^3 over a range of $t = \ln 9/4$ has the effect of decreasing the best value of m_x by 7% which is much smaller than the effect of (3.21). Higher-lying multiplets might make a smaller change in m_x because they are presumably more degenerate ($\delta m/m$ smaller) than the PGBs. However, they might all tend to have the same sign reducing m_x, since coloured states are presumably always heavier than uncoloured states, and most (all?) contributions to β_o^3 and β_o^2 would be negative.

Despite this relatively optimistic afterthought, it seems that a considerable uncertainty (3.21) and probably net increase (3.15) in m_x is likely to arise from technicolour if it exists.

3.5. Higher order effects

In previous sections we mentioned several effects of higher order which would correspond to the dots of order $O(\alpha^{n+1}(\ln\alpha)^n)$ in the formula (3.3). Since the calculation of $O(\alpha^0(\ln\alpha)^{0,1})$ terms in (3.3) has reduced $\ln(m_x^2/m_W^2)$ by $O(10)\%$, we might expect that the higher order terms could affect $\ln(m_x^2/m_W^2)$ by $O(10)\%$ of $O(10)\%$, i.e. by $O(1)\%$. This corresponds to a possible modification of m_x by a factor of $(1.5)^{\pm 1}$.

We have examined explicitly one possible higher order effect in the desert between the two gauge thresholds. In calculating with equation (3.4) we have followed Goldman and Ross[18] in putting the lowest order α_i on the right-hand-side when computing the evolution of the "true" α_i^{-1}. This differs from the consistent 2 loop solution of (3.4) only in higher order (3 loops). As an excercise, we have calculated the change in m_x obtained if one puts the 2 loop formula

$$\alpha_3(Q^2) = \frac{12\pi}{(33-2f)\ln Q^2/\Lambda^2}\left[1 - \frac{102 - \frac{38}{3}f}{(11-\frac{2}{3}f)^2}\;\frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right] \qquad (3.23)$$

on the right-hand-side of equation (3.4). We found that it reduced m_x by only a few %.

Another handle on the higher order effects comes from the remark of Goldman and Ross[29] that the factor of 3.6 between Λ_{MOM} and $\Lambda_{\overline{MS}}$ could easily be a factor of 3 if the higher order effects in the coupling constant are treated in a different way. This would correspond to an uncertainty of a factor of $(1.2)^{\pm 1}$ in estimating m_x. We therefore feel that an estimated error of a factor of $(1.5)^{\pm 1}$ in m_x from higher order effects is reasonable and conservative.

4. UNCERTAINTIES AT THE GRAND UNIFICATION MASS

We are finally approaching the end of our long march to grand unification, and must now discuss uncertainties in the neighbourhood of the grand unification mass itself. We divide these into three categories – the $SU(3) \times SU(2) \times U(1) \to SU(5)$ (?) gauge threshold transition[25], the possibility of "Cabibbo-rotating" away proton decay[26),27),50], and a speculation that since it is such a short distance process, proton decay might be the place to see a breakdown of Lorentz[28] or Poincaré invariance if it ever occurs.

4.1. The grand unification threshold

The effects of the grand unification threshold have been calculated in a momentum space renormalization scheme by Ross[22], in a paper where the m_x^2/Q^2 effects are retained. We can imagine two modifications to the assumptions used in his calculation. One concerns the Higgs sector, which has recently been studied in a momentum space prescription by Cook, Mahanthappa and Sher[25], and the other concerns the fermion sector. As for the Higgs, it was found[25] that putting a $\underline{45}$ of Higgs into the SU(5) model introduces additional particles with superheavy masses $\geq m_x$ which may affect the renormalization group equations near m_x in such a way as to alter m_x by a factor of $2.8 \to 3$. As far as possible superheavy fermions are concerned, it is difficult to quantify a possible effect because there is no clearly preferred model with a specified representation content (see however Ref. 51)). A possible order of magnitude estimate might be to assume a similar uncertainty to that postulated earlier for technicolour (3.20), namely that over a decade in Q^2 there is an uncertainty equivalent to a complete generation of conventional $\underline{\bar{5}} + \underline{10}$ fermions causing a mismatch between the β_o^2 and β_o^3. In this case the uncertainty would be a factor of 1.5 in m_x, but this number could clearly be considerably larger. In the $\overline{MS}$ prescription there is a neat way to calculate the uncertainties in m_x using equation (3.7) due to Weinberg[39]. Let us suppose that extrapolation of the low energy coupling constants $g_i(\mu)$ and $g_j(\mu)$ for two subgroups of SU(5) yields equality at a scale μ_o:

$$g_i(\mu_o) = g_j(\mu_o) \tag{4.1}$$

Writing equation (3.7) in the form

$$g_i(\mu) = g(\mu) + \frac{(g(\mu))^3}{96\pi^2}\{a_i(\ln m_V/\mu - \frac{1}{21}) + b_i \ln(\sqrt{2}m_F/\mu) + c_i \ln(m_S/\mu)\} \tag{4.2}$$

we see that equation (4.1) implies

$$(a_i - a_j)(\ln m_V/\mu_o - \frac{1}{21}) + (b_i - b_j)\ln(\sqrt{2}m_F/\mu_o) + (c_i - c_j)\ln(m_S/\mu_o) = 0 \tag{4.3}$$

which can be used to determine $m_X = m_Y$ in terms of μ_o. If one neglects heavy fermions and scalar particles one just recovers the trivial result (3.9). To estimate the uncertainty in this we consider varying the masses m_F and m_S in different $SU(3) \times SU(2) \times U(1)$ representations R of decomposed SU(5) multiplets so as to maximize the last two terms in (4.3), while keeping m_F and m_S equal to μ_o within an order of magnitude or so. Since the a_i are fixed (we know which SU(5) bosons are heavy) we can then calculate the maximal uncertainty

$$\Delta(\ln m_V/\mu_o) = \frac{1}{|a_i - a_j|}\left[\sum_{R_F} |(b_i - b_j)_{R_F}| + \sum_{R_S} |(c_i - c_j)_{R_S}|\right] \ln 10 \tag{4.4}$$

in an obvious suffix notation for the b_i and c_i. As an example we may consider the special case of the $\underline{\bar{5}} + \underline{5}$ of Higgs in the minimal SU(5) model which has the $SU(3) \times SU(2)$ decomposition

$$\underline{5} + \underline{\bar{5}} \rightarrow (3,1) + (\bar{3},1) + 2(1,2) \tag{4.5}$$

The (1,2) Higgs are of course the conventional low energy Weinberg-Salam Higgs doublet. The uncertainty in $\ln m_V/\mu_o$ then arises from varying the masses of the $(3,1) + (\bar{3},1)$ scalars for which

$$c_3 = 1\ , \quad c_2 = 0 \tag{4.6}$$

Using equation (4.4) we then have

$$\begin{aligned}\Delta(\ln m_V/\mu_o) &= \frac{1}{|a_3 - a_2|} \sum_{R_S} |(c_3 - c_2)_{R_S}| \ln 10 \\ &= \frac{1}{21} \ln 10\end{aligned} \tag{4.7}$$

so that the error in m_X is a factor of $10^{\pm 1/21} \simeq (1.1)^{\pm 1}$. We can extend this to the $\underline{24}$ of Higgs in minimal SU(5), for which the uneaten R_S are

$$\underline{24} \rightarrow (8,1) + (1,3) + (1,1) \tag{4.8}$$

which have respectively

$$\begin{cases} c_3 = 3\ ,\ 0\ ,\ 0 \\ c_2 = 0\ ,\ 2\ ,\ 0 \end{cases} \tag{4.9}$$

In this case there is a restriction on the Higgs masses from the most general form of Higgs potential in the model:

$$m^2_{(8,1)} = \frac{1}{20}\, m^2_{(1,3)} \tag{4.10}$$

which means that the maximal uncertainty (4.4) cannot be attained, and the best one can do is

$$\Delta(\ln m_V/\mu_0) = \frac{1}{21}(3-2)\ln 10 = \frac{1}{21}\ln 10 \qquad (4.11)$$

the same uncertainty as that due to the $\underline{5}+\bar{\underline{5}}$ Higgses. Two other more dramatic cases are shown in Tables 4 and 5. In the case of the $\underline{45}+\overline{\underline{45}}$ of Higgs, we have left out the 2 light (1,2) of fields, and recall that the (3,1) and $(\bar{3},1)$ in the $\underline{45}$ are not self-conjugate and so may have different masses. If we ignored possible restrictions on the ratios of masses analogous to (4.10) and used from Table 4 the fact that

$$\sum_{R_S} |(c_3 - c_2)_{R_S}| = 21 \qquad (4.12)$$

we would deduce from (4.4) an order of magnitude uncertainty in m_V. This however is probably an *overestimate*, precisely because of correlations analogous to (4.10), and we believe the real uncertainty is closer to the factor of 3 quoted earlier in the momentum space prescription. As for the $\underline{5}+\bar{\underline{5}}$ of fermions, their SU(3) × SU(2) decompositions can get arbitrary mass ratios from a combination of a $\underline{24}$ of Higgs and an SU(5) invariant mass term, so we believe that the maximum allowed by (4.4) can be attained. We read off from Table 5 that

$$\sum_{R_F} |(b_3 - b_2)_{R_F}| = 9 \qquad (4.13)$$

so that we get an uncertainty of a factor of about $10^{9/21} \simeq 3$ for each such set of fermions, of which there are three in a minimal E(6) model[51]. We conclude that the estimation of m_X is surprisingly sensitive to the existence of unobservable massive particles with masses close to m_X.

4.2. The grand unified mixing angles

In a recent paper[27] it was demonstrated explicitly that in an SU(5) model with a single $\underline{5}$ of Higgs the grand unified mixing angles were very closely related to those for conventional weak interactions (with extra phases not discussed in Ref. 50)), so that proton decay could not be "Cabibbo-rotated" away[26] in this simple model. In this subsection we extend this argument to versions of SU(5) with more complicated Higgs contents. The representations which can give rise to masses for the conventional generations of fermions are $\underline{5}$ and $\underline{45}$. A $\underline{5}$ gives rise to a symmetric mass matrix for the charge 2/3 quarks, an important property shared by Higgs-fermion couplings in other GUTs[51] based on the groups SO(10) and E(6), while a $\underline{45}$ gives them an antisymmetric mass matrix. We now analyze in turn the situations with different combinations of $\underline{5}$ and $\underline{45}$ representations.

Single 5 H: This was the case studied in Ref. 27). The $f^i_{\bar{5}}\, f^j_{10}\, H$ coupling matrix f_{ij} (i, j generation indices) can be diagonalized in generation space, revealing that for charge $-\frac{1}{3}$ quark and charge -1 lepton mass eigenstates

$$m_{-\frac{1}{3}i} = m_{-1_i}, \quad i = 1 \ldots N_g \qquad (4.14)$$

The symmetric $f_{10}\, f_{10}\, H$ coupling matrix h_{ij} can then be diagonalized by a unitary matrix U (the conventional Kobayashi-Maskawa[52] matrix) with $(N_g - 1)$ independent phases along the diagonal which are observable at high energies but not in conventional low energy weak interactions. In leading order, these phases do not alter the rates for different baryon-number violating processes, which are therefore given by the familiar Cabibbo angles, as conventionally assumed[4),14),15)].

<u>Several $\underline{5}$'s H_a</u>: The situation here is very analogous. We now have a three-index $f_{\bar{5}}^{i}\, f_{10}^{j}\, H_a$ coupling marix f_{ij}^{a} and a three-index $f_{10}^{i}\, f_{10}^{j}\, H_a$ coupling h_{ij}^{a}. The pattern of spontaneous symmetry breaking must be such that the vacuum expectation values of the H_a are all in the same (fifth) direction:

$$\langle 0|H_a|0\rangle = (0,0,0,0,v_a) \tag{4.15}$$

The mass eigenstates are then determined by the matrices

$$\tilde{f}_{ij} \equiv f_{ij}^{a}\, v_a\ , \quad \tilde{h}_{ij} \equiv h_{ij}^{a}\, v_a \tag{4.16}$$

which may be diagonalized in exactly the same way as the f_{ij} and h_{ij} of the single H case, obtaining again the mass relations (4.14). The gauge vector boson couplings are then described by the same Kobayashi-Maskawa[52] matrix and $(N_g - 1)$ additional phases as before, and proton decay still cannot be rotated away. The only difference from the previous case is that there the interactions mediated by Higgs bosons are also determined by the same "Cabibbo-angles", whereas this is no longer the case with several $\underline{5}$'s. However, it is generally presumed that Higgs exchenges are not the dominant mechanism for proton decay[*)].

<u>One or more $\underline{45}$'s $\mathcal{H}_a$</u>: In this case the $f_{\bar{5}}^{i}\, f_{10}^{j}\, \mathcal{H}_a$ coupling matrix $\mathcal{F}_{ij}^{a}$ contracted with the vector of vacuum expectation values $\langle 0|\, \mathcal{H}_a|0\rangle = \mathcal{V}_a$:

$$\tilde{\mathcal{F}}_{ij} \equiv \mathcal{F}_{ij}^{a}\, \mathcal{V}_a \tag{4.17}$$

can be diagonalized in the same way as for the case of several $\underline{5}$'s. However, the resulting mass relations are of course

$$m_{-\frac{1}{3}i} = \frac{1}{3}\, m_{-1_i} \quad (i = 1,\ldots,N_g) \tag{4.18}$$

instead of (4.14). In order for the relations (4.18) to be compatible with the observed mass ratio of the b quark and τ lepton, it is necessary to put 5 or 6 fermion generations into the SU(5) model[54]. This raises problems of comptatibility with the cosmolo-

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*) See Ref. 4). In order for Higgs exchange contributions to be comparable to vector boson exchanges, the Higgs would have to be as light as 10^{10} or 10^{11} GeV (Ellis, Gaillard and Nanopoulos, Ref. 8)), which is presumably unlikely to occur with a realistic Higgs potential for SU(5) when radiative corrections are taken into account - see Ref. 53) and references therein.

gical restriction on the number of associated "massless" neutrinos[48]. Setting these aside, we have seen earlier that having so many generations would increase m_x by O(4 to 8), while a further increase in m_x might be occasioned by the physical superheavy Higgs bosons[25]. However, there is a clear objection to having only $\underline{45}$'s of Higgs: it would give an antisymmetric mass matrix for the charge 2/3 quarks, which is phenomenologically unacceptable. For example, in the case of three generations it would predict

$$m_u = 0\ ,\quad m_c = m_t \tag{4.19}$$

which is experimentally not quite correct!

Combination of 5's and 45's: We can distinguish two subcases of this most general case. One is if the $\underline{10}$ - $\underline{10}$ mass matrix $\tilde{\mathcal{K}}_{ij} \equiv \mathcal{K}^a_{ij}\ \mathcal{V}_a$ due to $\underline{45}$ Higgs representations is comparable in magnitude to that generated by $\underline{5}$ Higgs representations. In this case the charge 2/3 quark mass matrix is not symmetric[a)], the analysis of Ref. 27) does not go through, and the grand unified mixing angles need bear no relation to the Kobayashi-Maskawa[52)] matrix. Proton decay could be rotated away. On the other hand, one can argue that if $\tilde{\mathcal{K}}_{ij}$ is in fact negligible, then proton decay "probably" cannot be rotated away.

Phenomenologically, if there are no more than 3 generations, the relation (4.14) seems to hold for the third generation b and τ masses. This means that we can choose a basis in generation space such that the mass matrices for the charge − 1/3 quarks and charge − 1 leptons take the form

$$m_{-\frac{1}{3}} = \begin{pmatrix} A+\mathcal{A} & B+\mathcal{B} & 0 \\ B+\mathcal{B} & C+\mathcal{C} & 0 \\ 0 & 0 & D \end{pmatrix} \quad m_{-1} = \begin{pmatrix} A-3\mathcal{A} & B-3\mathcal{B} & 0 \\ B-3\mathcal{B} & C-3\mathcal{C} & 0 \\ 0 & 0 & D \end{pmatrix} \tag{4.20}$$

with the $\underline{45}$'s making no significant contribution to the b and τ masses. We can then diagonalize the matrices (4.20) to obtain mass eigenvalues

$$m_{\pm} = \frac{a+c}{2} \pm \frac{1}{2}\sqrt{(a+c)^2 + 4(b^2 - ac)} \tag{4.21}$$

in an obvious notation. Phenomenologically, we know that both for charge − 1/3 quarks and charge − 1 leptons one mass eigenvalue is much larger than the other. We may therefore approximate (4.21) by

$$m_{\pm} \simeq a + c,\ \frac{b^2 - ac}{a + c} \tag{4.22}$$

with the requirements

$$4(b^2 - ac) \ll (a + c)^2 \tag{4.23}$$

We now make the reasonable (?) assumptions that

$$|A + \mathcal{A}| \ll |C + \mathcal{C}|\ ,\quad |A - 3\mathcal{A}| \ll |C - 3\mathcal{C}| \tag{4.24}$$

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a) We reminder the reader that in GUTs[51)] based on groups bigger than SU(5), such as SO(10) or E(6), the charge 2/3 quark mass matrix is also symmetric, so that the argument of Ref. 27) remains valid.

in which case we can make small angle approximations for the angles of rotation between the basis used for the original $M_{-1/3}$ and M_{-1} matrices (4.20), and the basis of eigenvectors with eigenvalues (4.22):

$$\theta_{-1} \approx \frac{B + \mathcal{B}}{(A + \mathcal{A})} \qquad \theta_{-1/3} = \frac{B - 3\mathcal{B}}{(A - 3\mathcal{A})} \tag{4.25}$$

The conventional weak Cabibbo-mixing is a product of the rotation θ_{-1} and of the diagonalization of the charge $-2/3$ mass matrix analogous to that discussed in previous cases. It may therefore be reasonable (?) to expect that

$$\theta_{-1/3} = O(\theta_c) \tag{4.26}$$

in which case it may also be reasonable (?) to expect that

$$\theta_{-1} = O(\theta_c) \tag{4.27}$$

in which case the grand unified mixing angles $\theta_{-1/3} - \theta_{-1}$ relevant to proton decay would all be of order θ_c, and the usual estimates of proton decay rates would go through.

This argument may seem rather weak and full of wishful thinking, but its general principles can be illustrated by a specific example of phenomenological interest[55]. It has been suggested that in order to cure the bad prediction

$$m_d/m_s = m_e/m_\mu \tag{4.28}$$

deducible from (4.14), and to get a better (?) absolute value for m_s, one should impose discrete symmetries on the Higgs couplings in such a way as to get mass matrices of the form

$$M_{-\frac{1}{3}} = \begin{pmatrix} 0 & B & 0 \\ B & \mathcal{C} & 0 \\ 0 & 0 & D \end{pmatrix} , \quad M_{-1} \begin{pmatrix} 0 & B & 0 \\ B & -3\mathcal{C} & 0 \\ 0 & 0 & D \end{pmatrix} \tag{4.29}$$

which is an example of the general form (4.20). In this case

$$\theta_{-\frac{1}{3}} = \frac{-A}{\mathcal{C}} , \quad \theta_{-1} = \frac{-A}{3\mathcal{C}} \tag{4.30}$$

and if the presumption (4.26) is valid, then

$$\theta_{-1} - \theta_{-\frac{1}{3}} = O(-\tfrac{2}{3}\,\theta_c) \tag{4.31}$$

and the grand unified mixing angles are indeed sufficiently small that the proton decay rate remains essentially unaffected.

As a final remark on Cabibbo-rotating away proton decay, we just repeat an observation made in Ref. 56). If one wants to cure the bad mass relation (4.28) it is only necessary to introduce into the mass matrices small terms of $O(10\ \text{MeV})$ which do not obey the usual

Clebsch-Gordon relations (4.14) for $\underline{5}$'s of Higgs fields. If we first diagonalize the $\underline{5}$ contributions to the mass matrices along the lines discussed in Ref. 27) and earlier in this section, and then make a final rotation to diagonalize the remaining O(10) MeV terms in the mass matrix, then the angles of this final rotation will be

$$\theta_{10} = O\left(\frac{10\ \mathrm{MeV}}{m_\mu\ \mathrm{or}\ m_s}\right) \lesssim \theta_c \tag{4.32}$$

Once again, proton decay cannot be rotated away in such a scheme. However, the suggestions of small deviations (4.26, 4.27, 4.31, 4.32) from Cabibbo-mixing in baryon-number violating processes are sufficiently interesting to warrant detailed experimental studies of "Cabibbo-suppressed" proton decays, should this ever be feasible.

4.3. Lorentz or Poincaré non-invariance?

So far in this paper we have been rigidly unimaginative, and it seems appropriate at this point to make one or two speculations. Proton decay is a process which originates at a distance scale $\Delta = O(1/m_X) = O(10^{-28})$ cm which is many orders of magnitude shorter than any scale that we can probe in any other foreseeable experiment. It is therefore a natural place to look for novel effects that are unobservable at longer distances. One such possibility is Lorentz invariance breakdown, which Nielsen and Ninomiya[28)] have conjectured may break down at very short distances[*)]. Could a breakdown of Lorentz invariance be observable in proton decay?

To orient our ideas, let us first consider the model of Nielsen and Ninomiya[28)]. They assume the existence of a 4 component non-Abelian gauge field, and assume that its interactions are gauge invariant and renormalizable, but not necessarily Lorentz invariant. The gauge boson interactions then take the form

$$\mathcal{L} = -\frac{1}{4}\eta^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma} \tag{4.33}$$

where $\eta^{\mu\nu\rho\sigma}$ is not in general of the Lorentz-invariant form

$$\eta^{\mu\nu\rho\sigma}_{cov} = \frac{1}{2}(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \tag{4.34}$$

The deviation from internal Lorentz-invariance of the Yang-Mills field can be characterized by a $\delta\eta$ which - since we are dealing with a renormalizable theory - varies logarithmically with the energy scale. The same authors[28)] also introduce a quantity δg which measures the approach of the metric tensor of a fermion to that of the Yang-Mills field. Using the notation

$$\begin{aligned} C_2(G) &: \quad C_2(G)\delta^{ab} = f^{acd} f^{bcd} \\ T(R) &: \quad T(R)\delta^{ab} = \mathrm{Tr}(\sigma^a \sigma^b) \end{aligned} \tag{4.35}$$

*) Another form of Lorentz breakdown at short distance has been proposed by C.H. Wu (private communication via H. Nielsen). This possibility requires the introduction of a dimensional coupling constant, which we disfavour in general[53)].

in terms of the structure constants f^{abc} for a group G and matrices σ^a for a fermion representation R, they find the results

$$\delta\eta(Q) \approx (\ln Q^2/\mu^2)^{\frac{7 C_2(G) + 4 \sum_R T(R)}{11 C_2(G) - 4 \sum_R T(R)}} \qquad (4.36a)$$

$$\delta g(Q) \approx (\ln Q^2/\mu^2)^{\frac{8 C_2(G) + 4 \sum_R T(R)}{11 C_2(G) - 4 \sum_R T(R)}} \qquad (4.36b)$$

where μ is some unknown scale.

Nielsen and Ninomiya[28] estimate that the unobservably short separation in arrival times of differently polarized photons from pulsars imply that $\delta\eta \lesssim 10^{-15}$ for U(1) e.m. This presumably reflects the values of $\delta\eta$ for both the SU(2) and U(1) subgroups of the Weinberg-Salam model on a scale $Q \sim 100$ GeV. Presumably $\delta\eta$ for SU(3) colour is also quite small on a scale of (1 to 100) GeV. For these groups

$$C_2(U(1)) = 0 \ , \quad C_2(SU(2)) = 2 \ , \quad C_2(SU(3)) = 3 \qquad (4.37)$$

while in a grand unified theory such as SU(5)

$$\sum_R T(R) = N_g \qquad (4.38)$$

Taking the coefficients (4.37, 4.38) and substituting them into the evolution formulae (4.36) it is evident that we cannot choose N_g so as to make small simultaneously more than one of the denominators $[11C_2(G) - 4 \sum_R T(R)]$ for G = U(1), SU(2) and SU(3). This means that within this framework the goodness of Lorentz invariance at low energies is sufficient to guarantee that Lorentz invariance is also good at scales $\sim 10^{15}$ GeV (10^{-28} cm). Thus proton decays should be Lorentz-invariant within this approach.

Above the grand unification mass one only has one group G = SU(5) (?) to content with, and it is in principle possible to add superheavy fermions to the theory so that $[11C_2(SU(5)) - 4 \sum_R T(R)] \simeq 0$ and Lorentz-invariance gets broken appreciably at mass-scales above the grand unification mass (the Planck mass?), but this would seem to be rather fortuitous[*]. One could however imagine other ways of violating Lorentz invariance which would be easier to detect. For example, Lorentz violation by non-renormalizable interactions would increase as a power of the momentum scale Q. It may therefore not be entirely a waste of time to look for a violation of Lorentz-invariance in proton decay, though this is clearly wildly speculative. How would one look for such an effect? One possibility that comes to mind is to look for proton decay final states which do not have spin 1/2 : spin 3/2 for example.

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[*] Note however that, amusingly enough, the β-function in the E_6 model of Ref. 51) is almost zero above the grand unification mass, so that this possibility can be realized in that model.

While we are considering the violation of everything else, why not the rest of Poincaré invariance, namely translation invariance in space and time? This could arise from a granularity of space-time on some distance scale between the 10^{-16} cm so far probed[57] and the 10^{-28} cm distance of propagation of an X boson. It would have the signatures of momentum and/or energy non-conservation. Perhaps when a proton decays its decay products will turn out to have a net non-zero momentum? or perhaps their energies will not add up to m_p? These possibilities would be nightmares for the experiments proposed recently[58], which often rely on geometric and/or calorimetric signatures e.g. two particles emerging back-to-back with a total energy $\sim$ 940 MeV.

5. UNCERTAINTIES IN THE PROTON DECAY RATE

Let us suppose we have now established a definite value for m_X: the final problem is to calculate the total decay rate, the parameter C of equation (1.1). Its determination can be split into several steps. First is the derivation of the effective Lagrangian for baryon-nonconserving processes including "Cabibbo" angles, which was given in Ref. 27)[*]:

$$\begin{aligned}
\mathcal{L}_{eff} = e^{i\phi}\frac{G_{GU}}{\sqrt{2}}\Big[\big(\varepsilon_{ijk}\,\overline{u^c_{k_L}}\,\gamma_\mu\,u_{j_L}\big)\Big\{&\big[(1+\cos^2\theta_c)\overline{e^+_L}+\sin\theta_c\cos\theta_c\,\overline{\mu^+_L}\big]\gamma^\mu d_{iL}\\
&+\big[(1+\sin^2\theta_c)\overline{\mu^+_L}+\sin\theta_c\cos\theta_c\,\overline{e^+_L}\big]\gamma^\mu s_{iL}\\
&\cdot\overline{e^+_R}\,\gamma^\mu d_{iR}+\overline{\mu^+_R}\,\gamma^\mu s_{iR}\Big\}\\
+\big[\varepsilon_{ijk}\,\overline{u^c_{k_L}}\,\gamma_\mu(d_{j_L}\cos\theta_c+s_{j_L}\sin\theta_c)\big]&\big[\overline{\nu^c_{eR}}\,\gamma^\mu d_{iR}+\overline{\nu^c_{\mu R}}\,\gamma^\mu s_{iR}\big]\Big]\\
&+\text{H.C.}
\end{aligned} \tag{5.1}$$

where

$$\frac{G_{GU}}{\sqrt{2}} = \frac{g^2}{8m_X^2} = \frac{g^2}{8m_Y^2} \tag{5.2}$$

in the case of the simplest SU(5) model. Note that the form of CP violation in the effective Lagrangian (5.1) through the phase ϕ gives no possibility in leading order for observable CP violating effects of the type recently proposed by Hurlbert and Wilczek[59]. The phase ϕ also does not affect the total decay rate. One must next proceed to the computation of the short distance amplitude enhancement effects due to SU(3), SU(2) and U(1) boson exchanges which are to leading order[4),27),60)]

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[*] Note the change of sign in the e^+_R and μ^+_R terms of (5.1) by comparison with Refs 4) and 27). We thank M.B. Gavela and W. Konetschny for correcting us on this point.

$$A_3 = \left[\frac{\alpha_3(1\ \mathrm{GeV})}{\alpha_{GUM}}\right]^{\frac{2}{11 - \frac{4}{3} N_g}} \qquad A_2 = \left[\frac{\alpha_2(m_W)}{\alpha_{GUM}}\right]^{\frac{27}{86 - 16 N_g}}$$

$$A_1 = \left[\frac{\alpha_1(m_W)}{\alpha_{GUM}}\right]^{-\frac{69}{6 + 80 N_g}} \qquad \text{for the } \overline{e_L^+},\ \overline{\mu_L^+} \text{ operators} \tag{5.3}$$

$$= \left[\frac{\alpha_1(m_W)}{\alpha_{GUM}}\right]^{-\frac{33}{6 + 80 N_g}} \qquad \text{for the } \overline{e_R^+},\ \overline{\mu_R^+},\ \nu \text{ operators}$$

for the different operators appearing in the effective Lagrangian (5.1). To give an idea of the magnitude of these short distance effects for typical values of the different gauge coupling constants one finds

$$A_3^2 \approx 5\ , \quad (A_2\, A_1)^2 \approx \begin{cases} 2.5 & \text{for } \overline{e_L^+},\ \overline{\mu_L^+} \\ 2.2 & \text{for } \overline{e_R^+},\ \overline{\mu_R^+},\ \nu \end{cases} \tag{5.4}$$

The final job is to compute the matrix elements of the effective Lagrangian (5.1) between the proton (or nucleon) initial state and the various meson + lepton final states. So far, two fundamentally different methods of doing this have been used. One[4),14),15),29)] is to treat the decay as if two quarks in the nucleon come together with a probability determined by an SU(6) wave-function $\psi(0)$, annihilating into an antilepton and an antiquark, with the $\bar{q}$ and the spectator quark then combining with probability 1 to form inclusive meson states (the BEGNJYM method). Five calculations of this type have been made: Ref. 14) includes some diagrams which were left out by Ref. 4), but we prefer the assignment of a quark mass of about $m_{N/3}$ to the final state antiquark when computing phase space, as was done in Ref. 15). Recent applications of the method are made in Refs 29 (GR) and 61 (GLOPR). In this method, the decay rate is proportional to $|\psi(0)|^2$, and we now believe on the basis of analyses of hyperon decays[62)] that the value of $0.8 \times 10^{-2}\ \mathrm{GeV}^3$ that we and others originally assumed is probably too large. We now believe that a better value is about $0.7\ m_\pi^3 = 0.2 \times 10^{-2}\ \mathrm{GeV}^3$, which fits Ω and s-wave hyperon decay and agrees with bag model estimates, and we have renormalized the BEGNJYM and GR calculations with this value of $|\psi(0)|^2$. The second class[16),17),63)] of calculations uses a bag model, computes the initial qq overlap in a nucleon bag and then computes the overlap of the resulting $\bar{q}q$ system with different exclusive mesonic final state bags. Three calculations (DGS[16)], D[17)] and G[63)]) of this type have now been made using slightly different techniques: their results differ from each other by factors up to about 12. If we take a representative estimate of $m_x = 5 \times 10^{14}$ GeV, we find the six different proton lifetime estimates shown in Table 6.

To a good approximation, these lifetime estimates scale as m_x^4 within the range of uncertainties which we discuss in this paper. From these different estimates of the proton lifetime we therefore have

$$\tau_p \sim (0.\,? \text{ to } 25) \times 10^{28}\left(\frac{m_X}{5 \times 10^{14}\,\text{GeV}}\right)^4 \text{ years} \tag{5.5}$$

We now turn to the actual computation of m_X using the following variations of the input parameters[a)] $\Lambda_{\overline{MS}}$, m_t, α_{em} in the two calculational methods (a) and (b) of section 2.1.

$$\begin{array}{lcccll}
100 \text{ MeV} & \le & \Lambda_{\overline{MS}} & \le & 600 \text{ MeV} & \\
15 \text{ GeV} & \le & m_t & \le & 50 \text{ GeV} & \\
127.56 & \le & \alpha_{em}^{-1} & \le & 128.50 & \text{for } \left\{\begin{array}{l}\lambda = 4 \\ \lambda = 7.8\end{array}\right\} \text{in method (a)} \\
126.96 & \le & \alpha_{em}^{-1} & \le & 127.90 & \\
127.35 & \le & (\alpha_{em}^{-1})_{\overline{MS}} & \le & 128.29 & \text{for method (b)}
\end{array} \tag{5.6}$$

Table 7 shows a representative set of estimates made using method (a). We see that there is very little sensitivity to m_t: this is consistent with the observation of Binétruy and Schücker[39] that the top quark contribution to the relative renormalization of α_{em} and α_s cancels in leading order. The results of Table 7 were calculated using $\lambda = 7.8$ and the central value of α_{em}^{-1} given in (5.6). The effects of using instead $\lambda = 4$ or of taking the extreme values of α_{em} from (5.6) are for $\Lambda_{\overline{MS}} = 400$ MeV and $m_t = 30$ GeV:

$$\begin{array}{l}
\Delta\alpha_{em}^{-1} \text{ for } \lambda = 7.8 \Rightarrow \Delta m_X = \pm\, 0.49 \times 10^{14}\,\text{GeV} \\
\lambda = 7.8 \rightarrow \lambda = 4 \quad \Rightarrow \Delta m_X = +\, 0.56 \times 10^{14}\,\text{GeV}
\end{array} \tag{5.7}$$

We therefore find using method (a) that for $\Lambda_{\overline{MS}} = 400$ MeV

$$m_X = (5.4 \pm 0.8) \times 10^{14}\,\text{GeV} \qquad \text{method (a)} \tag{5.8}$$

while the variation of the central value with $\Lambda_{\overline{MS}}$ can be approximated by

$$m_X \simeq 1.35 \times 10^{15}\left(\Lambda_{\overline{MS}}\right)^{1.01} \tag{5.9}$$

and the significance of the third significant figures in (5.9) is tenuous. Using method (b)

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a) Throughout this discussion, $\Lambda_{\overline{MS}}$ is that found using 4 flavours and the full second order formalism.

we concur with Binétruy and Schücker[39] in finding for $\Lambda_{\overline{ms}} = 400$ MeV and α_{em}^{-1} given by (5.6):

$$m_X = (6.3 \pm 0.7) \times 10^{14}\,\text{GeV} \qquad \text{method (b)} \tag{5.10}$$

Another analysis similar in spirit to our method (b) and Ref. 39) has been made by Hall[64], with the result

$$m_X \approx 5.7 \times 10^{14}\,\text{GeV} \tag{5.11}$$

while the final version of an analysis by Marciano[65] which also uses a modified minimal subtraction technique obtains

$$m_X \approx 6.3 \times 10^{14}\,\text{GeV} \tag{5.12}$$

for $\Lambda_{\overline{ms}} = 400$ MeV. These results all agree within the expected uncertainties of a factor of $(1.5)^{\pm 1}$ coming from higher order effects in the renormalization group equations, such as the uncertainty of 20 % in the $\Lambda_{MOM}/\Lambda_{\overline{ms}}$ ratio of equation (2.1) which was mentioned in section (2.1). We therefore estimate that in minimal SU(5) with $\Lambda_{\overline{ms}} = 400$ MeV

$$m_X = (6 \pm 3) \times 10^{14}\,\text{GeV} \tag{5.13}$$

which should be scaled approximatively by a factor of $(\Lambda_{\overline{ms}}/0.4\ \text{GeV})$ for different values of $\Lambda_{\overline{ms}}$. Assuming that $\Lambda_{\overline{ms}} = 0.4 \times (1.5)^{\pm 1}$GeV we finally conclude[*] from (5.5) and (5.13) that

$$\boxed{\tau_p = 8 \times 10^{30 \pm 2}\ \text{years}} \tag{5.14}$$

in the simple SU(5) model with three generations and no significant new physics before the grand unification point. However, this estimate is subject to all the uncertainties discussed in sections 3 and 4, of which the principal ones are

In m_X

- $\pm$ 13 % due to the uncertainty in renormalization of α_{em},
- 1.8 increase for a new generation of fermions,
- Additional factor of $(1.5)^{\pm 1}$ for each technicolour generation,

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[*] The central value of this estimate is higher than that of Ref. 11) for two reasons. One is that we are now convinced that our earlier[4] value of $|\psi(0)|^2$ was too large, the other is the long proton lifetimes calculated[17),63)] in some bag models.

- Factor of $\leq (1.5)^{\pm 1}$ from higher order effects in the renormalization group calculations,
- Factor of $3^{\pm 1}$ from uncertainties in the superheavy Higgs sector[25], and $(\geq 1.5)^{\pm 1}$ from possible superheavy fermions.

In the effective Lagrangian

- Possible Cabibbo-rotation away of proton decay if the charge 2/3 quark mass matrix contains important antisymmetric pieces.

Experimentalists[58] will be cheered by the estimate (5.14) in the simple SU(5) model[2),3),4] with no significant new physics. They may be dismayed by the uncertainties engendered by new physics. If they do not find decaying protons, they will at least have the first experimental evidence for something more exciting. If they do find decaying protons, then there will be much to be learned, for example from looking at decays which are supposed to be Cabibbo suppressed.

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Table 1: Phenomenological estimates of Λ

Type of Analysis	Type of Λ estimated	Corresponding value of $\Lambda_{\overline{ms}}$ (GeV)
$F_2^{\{\mu(p-n),\, e(p-n)\}}$ moments[a)]	$\Lambda_{n}^{F_2}$, $\Lambda_{\overline{ms}}$	0.53 ± 0.1
$xF_3^{\nu N}$ moments[b)]	$\Lambda_{\overline{ms}}$	0.5
$xF_3^{\nu N}$ moments[c)]	$\Lambda_{n}^{F_3}$	0.45 ± 0.17 (BEBC - GGM)
		0.35 ± 0.27 (CDHS)
$F_2^{eN,\mu N}$ direct[d)]	Λ_{ms}	0.7 ± 0.3
$F_2^{e(p-n)}$ direct[e)]	Λ_{ms}	0.53 ± 0.16
$xF_3^{\nu N}$ direct[f)]	Λ_{ms}	0.46 ± 0.21
$xF_3^{\nu N}$ moments[g)]	Λ_{MOM}	0.41
non singlet moments[h)]	several types	0.3 to 1.0

The original papers cited are listed in Ref. 33).

Table 2: Decrease of Λ with increasing f (m_b = 4.8 GeV)

Number of flavours f	Value of Λ (MeV)		
4	100	400	600
5	62	289	449
6 (m_t = 15 GeV)	29	162	266
6 (m_t = 30 GeV)	27	149	245
6 (m_t = 50 GeV)	25	141	231

Table 3: Contributions to $\delta\alpha^{-1}$: Resonances

Resonances	Parameters	$\delta\alpha^{-1}$
ρ	$m_\rho = 770$ MeV, $\Gamma_{tot} = (155 \pm 25)$ MeV $\frac{\Gamma_{e^+e^-}}{\Gamma_{tot}} = (5.5 \pm 1.2) \times 10^{-5}$	-0.51 ± 0.11 ($\Gamma_\rho = 130$ MeV) to -0.67 ± 0.14 ($\Gamma_\rho = 180$ MeV)
ω	$m_\omega = 783$ MeV, $\Gamma_{tot} = 10$ MeV $\Gamma_{e^+e^-} = 0.77 \times 10^{-6}$ GeV	-0.055
ϕ	$m_\phi = 1.02$ GeV, $\Gamma_{tot} = 4.1$ MeV $\Gamma_{e^+e^-} = 1.3 \times 10^{-6}$ GeV	-0.072
J/ψ	$m_{J/\psi} = 3.1$ GeV, $\Gamma_{tot} = 69$ keV $\Gamma_{e^+e^-} = 4.8 \times 10^{-6}$ GeV	-0.074
ψ'	$m_{\psi'} = 3.69$ GeV, $\Gamma_{tot} = 228$ keV $\Gamma_{e^+e^-} = 2.1 \times 10^{-6}$ GeV	-0.032
ψ''	$m_{\psi''} = 3.77$ GeV, $\Gamma_{tot} = 28$ MeV $\Gamma_{e^+e^-} = 0.36 \times 10^{-6}$ GeV	-0.0054
Υ	$m_\Upsilon = 9.4$ GeV, $\Gamma_{tot} = 25$ keV $\Gamma_{e^+e^-} = 1.3$ KeV	-0.0077
Υ'	$m_{\Upsilon'} = 10.0$ GeV, $\Gamma_{tot} = 25$ keV $\Gamma_{e^+e^-} = 0.4$ keV	-0.0022
	All resonances except ρ	-0.25

This contribution to $\alpha^{-1}(-\lambda m_W^2, -\lambda m_W^2, -\lambda m_W^2)$
is essentially independent of λ for $\lambda \in [1, 7.8]$.

Continuum

Energy range (GeV)	R	$\delta\alpha^{-1}$ $\lambda = 1$	$\lambda = 4$	$\lambda = 7.8$
1 to 1.5	1	− 0.09	− 0.09	− 0.09
1.5 to 4	2	− 0.41	− 0.41	− 0.41
	2.5	− 0.52	− 0.52	− 0.52
4 to 10	4	− 0.77	− 0.78	− 0.78
10 to 30	4	− 0.88	− 0.92	− 0.93
	4.3	− 0.95	− 0.99	− 1.00
30 to ∞	5.5	− 1.22	− 1.97	− 2.36
10 to 100	4	− 1.56	− 1.82	− 1.88
	4.3	− 1.68	− 1.95	− 2.02
100 to ∞	5.5	− 0.29	− 0.74	− 1.05
	All continuum	− 3.12 to − 3.55	− 3.83 to − 4.35	− 4.21 to − 4.75

Leptons

	$\lambda = 1$	$\lambda = 4$	$\lambda = 7.8$
e	− 2.36	− 2.51	− 2.58
μ	− 1.23	− 1.38	− 1.45
τ	− 0.62	− 0.77	− 0.84
Total leptons	− 4.21	− 4.66	− 4.87

	$\lambda = 1$	$\lambda = 4$	$\lambda = 7.8$
Overall Total	− 7.95 to − 8.82	− 9.14 to −10.08	− 9.74 to −10.68

Table 4: $\underline{45} + \overline{\underline{45}}$ of Higgs fields

SU(3) × SU(2) Decomposition R_S	2(8,2)	(3,3)+($\bar{3}$,3)	(3,1)+($\bar{3}$,1)	($\bar{6}$,1)+(6,1)	($\bar{3}$,2)+(3,2)	($\bar{3}$,1)+(3,1)
c_3	12	3	1	5	2	1
c_2	8	12	0	0	3	0

Table 5: $\underline{5} + \bar{\underline{5}}$ of fermions

SU(3) × SU(2) Decomposition R_F	(3,1) + ($\bar{3}$,1)	2(1,2)
b_3	8	0
b_2	0	1

Table 6: Estimates of the proton decay rates

Calculating group	Assumed value of m_x (GeV)	Lifetime quoted (yrs)	Lifetime (years) if $m_x = 5 \times 10$ GeV
BEGNJYM[4),14),15)]	5×10^{14}	3×10^{30}	3×10^{30}
DGS[16)]	5×10^{14}	2×10^{30}	2×10^{30}
D[17)]	3.8×10^{14}	8×10^{30}	24×10^{30}
GR[29)]	4.4×10^{14}	0.9×10^{30}	1.5×10^{30}
GLOPR[61)]	6×10^{14}	1.3×10^{30}	0.6×10^{30}
G[63)]	4.6×10^{14}	1.8×10^{31}	25×10^{30}

Table 7: Estimates of m_X (in units of 10^{14} GeV)

$\Lambda_{\overline{ms}}$ (MeV) \ m_t (GeV)	15	30	50
100	1.28	1.27	1.26
400	5.18	5.16	5.09
600	7.80	7.77	7.67

This table is calculated using
$\lambda = 7.8$ in method (a)

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