GIANT MULTIPOLE RESONANCES - PERSPECTIVES AFTER TEN YEARS

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Abstract: Nearly ten years ago evidence was published for the first of the so-called giant multipole resonances, the giant quadrupole resonance. During the ensuing years research in this new field has spread to many nuclear physics laboratories throughout the world. A review of the present status of electric giant multipole resonances is provided in this presentation. Other presentations to this conference will describe magnetic giant resonances, giant resonance decay modes, and theoretical implications of the "new" giant resonances.

1. Introduction

It has been nearly ten years since experimental evidence\(^1,2,3\) for the first of the new giant resonances, the giant quadrupole resonance, was published. These initial observations have blossomed into a bona fide subfield of nuclear physics; one that is pursued in nearly every major medium-energy nuclear physics facility throughout the world. An indication of the rapid growth and significant current interest in this field is the fact that five of the invited talks and 80 contributed abstracts to this conference deal entirely or mostly with giant multipole resonances. In addition, giant multipole resonances have found their way into several other areas of nuclear physics.

As this meeting might be considered an anniversary for the giant multipole resonance field it seems appropriate to review the present experimental status of the "new" resonances. In this talk I will concentrate exclusively on the electric giant resonances with special emphasis on the results and interpretations from inelastic hadron scattering. G. Bertsch will tell us about theoretical implications of the new resonances while L. S. Cardman will consider the very interesting experimental work on the decay modes of multipole resonances. S. Gales will review the status of the deep-lying hole states studied via pickup reactions. Finally, a considerable portion of the talk on Saturday by F. Petrovich will deal with the status of magnetic resonances and stretched-configurations studied via the \((p,n)\) and \((e,e')\) reactions.

2. Background

It is important to briefly review a few basic concepts of giant resonances as they apply to this presentation and to the succeeding giant resonance talks. In this short review it is not possible to develop the theory behind the concepts I will describe. It is however, important to understand the nomenclature we will use in our discussions. Much more detailed information may be found in refs. 4 and 5.

Giant resonances are often considered as highly collective modes of nuclear excitation in which an appreciable fraction of the nucleons of a nucleus move together. Indeed the motion is so collective that it is appropriate to think of these modes of excitation in hydrodynamic terms like the oscillation of a liquid drop.

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Figure 1 shows a representation of several of these modes of oscillation, where \( L \) represents the angular momentum quantum number of the modes. The monopole (\( L=0 \)) mode is a spherically symmetric oscillation or compression of the nucleus; the dipole (\( L=1 \)) is pictured as a motion in which the neutrons and protons oscillate in bulk against each other while the quadrupole mode is an oscillation of the spherical nucleus to oblate shape then to prolate shape. Oscillations with \( L > 3 \) are, of course, possible but are not shown. The nuclear fluid has neutron, proton, "spin-up" and "spin-down" components and hence, for each multipolarity (\( L \)) there are four possible combinations of these components as indicated on fig. 1. Modes in which neutrons and protons oscillate in phase are characterized as isoscalar modes (denoted as \( T=0 \) here) while those modes in which the neutrons and protons oscillate out of phase are called isovector (\( T=1 \)). Similarly, spin-up and spin-down nucleons oscillating in phase yield \( S=0 \) modes while the so-called spin-flip modes (\( S=1 \)) are produced by spin-up and spin-down nucleons oscillating out of phase. The \( S=0 \) oscillations are the electric modes while the \( S=1 \) oscillations are the magnetic modes.

The collectivity of these modes (i.e., of the nuclear states that are the observable manifestation of the various modes) can be deduced by studying the transition rates for their electromagnetic excitation (or de-excitation) or by measuring the cross sections for their excitation via direct reactions such as inelastic scattering. Useful "benchmarks" for comparison are the single particle transition rates and sum rules. While the former provide an estimate for a single nucleon promoted from one shell-model level to another, the latter tell us how much total transition strength we can expect for a mode having particular \((L,T,S)\) values. Throughout this talk we will present giant resonance strength in terms of the energy weighted sum rule (EWSR) which is a particularly useful sum rule because it is nearly model independent. The criterion for a transition to be considered collective is that its transition rate be many \((2 \times 10)\) times the single particle value and that it should exhaust an appreciable fraction of the EWSR for the mode in question. As we shall see giant resonances exhaust 20%-90% of their sum rules.

Figure 2 provides a representation of transitions that might comprise various electric modes. The figure schematically represents single-particle
transitions between shell-model states of a hypothetical nucleus. Collective transitions result from coherent superpositions of many such single-particle transitions. Major shells are denoted as N, N+1, N+2, etc. and are separated by \( \sim 1\hbar \omega \) or \( \sim 41 A^{-1/3} \) MeV. Giant resonances may be considered to result from transitions of nucleons from one major shell to another, under the influence of an interaction that orders these transitions into a coherent motion. The interaction for inelastic scattering can excite a nucleon by at most \( L\hbar \omega \), or, to state it differently, the nucleon can be promoted by at most \( L \) major shells. The number of shells is either odd or even according to the parity. Thus, the isovector giant dipole resonance (GDR) is built up of E1 transitions spanning \( 1\hbar \omega \). The GDR might then be expected to be located at an excitation energy of \( \sim 41 A^{-1/3} \) MeV; however, it is located at \( \sim 77 A^{-1/3} \) MeV. This difference arises from the fact that the spin and isospin dependence of the nucleon-nucleon interaction ensures that the \( S=T=0 \) collective states move down in energy, and that \( S=1 \) or \( T=1 \) states move up from the expected energy.

For E2 excitations two different classes of transitions are allowed. The first of these, with lowest energy, is comprised of transitions within a major shell, the so-called \( 0\hbar \omega \) transitions. A second set is comprised of transitions between shells N and N+2, the \( 2\hbar \omega \) transitions. These transitions would be pushed up or down in energy from \( 2\hbar \omega \) for isovector or isoscalar modes respectively. While the \( 0\hbar \omega \), E2, excitations are identified with the familiar low-lying 2^+ levels, the \( 2\hbar \omega \) class carry most of the EWSR and are associated with the GQR. By similar arguments E3 excitations of \( 1\hbar \omega \) and \( 3\hbar \omega \) and E4 excitations of \( 0\hbar \omega \), \( 2\hbar \omega \) and \( 4\hbar \omega \) are expected.

Fig. 2. Schematic representation of electric multipole transitions between shell-model states of a hypothetical nucleus. Major shells are denoted as N, N+1, N+2, etc. and lie \( \sim 1\hbar \omega \) or \( \sim 41 A^{-1/3} \) MeV apart.
For each class of transitions (E1, E2, E3, etc.) the sum rule should be exhausted by the sum of the strength in all the transitions. For example, the EWSR for T=0, E=2 transitions should be exhausted by the sum of the strength in the 0°ω and 2°ω transitions. Since the low-lying 2° states (0°ω) exhaust only a small part of the EWSR (~30%), most of the T=0 quadrupole strength should in fact be found in the 2°ω transitions. Similar statements can be made about the EWSR strength for E3 and E4 states.

From these brief background comments one derives a complicated picture of possible giant multipole resonances. For each multipolarity there may be four independent modes and for each of these modes there may be more than one class of transitions, e.g., for L=4, S=0, T=0 we could find a 2°ω and 4°ω giant resonance. How then do we sort out this complicated picture? The answer lies in the selectivity of the nuclear reactions we use to search for the resonances. Perhaps the most outstanding example of such a selectivity is for photonuclear reaction studies of the T=1, E1, GDR (L=1, T*1, S=0). An example of the spectrum from the photonuclear reaction on 208Pb is shown on fig. 3. The spectrum contains essentially only GDR cross section since the photonuclear reaction proceeds overwhelmingly by dipole absorption. (Excitation of the GDR is 10-100 times stronger than E2 excitation via photoabsorption). While this selectivity provides an excellent means to study the GDR, measurements of higher multipole resonances are difficult. Photonuclear measurements have been used to study the GDR in many nuclei across the periodic table. While it is not the intention of this presentation to discuss the GDR (the "old" giant resonance) in any detail, consideration of the GDR systematics presented in fig. 4 provide a guide for our search for resonances related to other modes of nuclear excitation.

Plotted on fig. 4 are values of the excitation energy (E_x) plotted as E_xA^{1/3} MeV, the width and sum rule depletion for the GDR for many nuclei spanning the periodic table. For nuclei having mass above ~130, the GDR is located at the systematic energy of ~78A^{1/3} MeV and most of the GDR sum rule is accounted for. For lighter nuclei the resonance energy falls steadily from the systematic value and less than 100% of the sum rule is accounted for (at least up to ~30 MeV of excitation). The width of the GDR is narrowest near

![Graph](image)

Fig. 3. Giant dipole resonance in 208Pb as observed in the (γ,n) reaction (ref. 6).
Fig. 4. Systematics of the isovector giant dipole resonance excitation energy, width and sum rule depletion (ref. 6).

shell closures (A=90, 144, 208) and widest for deformed nuclei (e.g., rare earth nuclei).

These systematics yield a few characteristics of giant resonances that may be useful for experimental searches for new resonances.

1) Giant resonances are general properties of nuclei.

2) The excitation energy of a giant resonance varies smoothly with nuclear mass (at least over most of the nuclear mass range).

3) Giant resonances exhaust an appreciable fraction of an appropriate sum rule.
4) Giant resonance strength is generally localized in excitation energy (more an experimental than theoretical necessity).

Since we have said that photonuclear reactions are not especially well suited to study giant resonances other than dipole we must, of course, ask what is an appropriate technique? The answer is through direct reactions, especially inelastic scattering, of medium energy projectiles. It has long been known that the inelastic scattering reaction provides strong excitation of low-lying collective \( T=\bar{T}=0 \) states. Thus, it seems reasonable to assume that high-lying collective states of similar modes should also be excited. Inelastic electron scattering had been used for some time to study the GDR. However, while the electron scattering mechanism (electromagnetic) is well understood the selectivity of the reaction is rather low. The use of hadrons as projectiles provides much more variety for the reactions and thus, more selectivity. This can be seen by considering the effective interaction between a nucleon in the projectile and one in the target nucleus. The interaction is both spin and isospin dependent. For example, the central part of the interaction may be written as in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>Term</th>
<th>Interaction</th>
<th>Label</th>
</tr>
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<tbody>
<tr>
<td>( v_{TS}^{i_{ij}} ) = ( v_{00}^{i_{ij}} + v_{10}^{i_{ij}} ) ( \tau_{i} \cdot \tau_{j} ) + ( v_{01}^{i_{ij}} ) ( \sigma_{i} \cdot \sigma_{j} ) + ( v_{11}^{i_{ij}} ) ( \sigma_{i} \cdot \sigma_{j} ) ( \tau_{i} \cdot \tau_{j} )</td>
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<td>( T=0 )</td>
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<td>GDR</td>
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<td>1st 2^+,3^-</td>
<td>2^-3^+</td>
<td>Gamow-Teller</td>
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<td>IAS</td>
<td>(p,p')</td>
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<td>(a,a')</td>
<td>(p,p')</td>
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<td></td>
<td>(d,d')</td>
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<td>(p,n),(^4He,t)</td>
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<td>(^6Li,^6He)</td>
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The four modes for each multipolarity arise from a term in the effective interaction. For example, the low-lying 2^+ and 3^- nuclear states are produced via the \( T=S=0 \) term, the GDR and the isobaric analog resonance (IAS) are examples of excitations from the isospin term, \( T=1, S=0 \). Isovector electric giant resonances arise from this term. The so-called "spin-flip" states, \( T=0, S=1 \) arise from the \( \sigma \cdot \sigma \) term while the spin-isospin term provides, for example, \( M1 \) states and the Gamow-Teller transitions. The various hadronic probes produce dramatically different strengths for the four components of the interaction. The \( (p,p') \) reaction can excite all four terms, but excitation of the \( T=S=0 \) term would be \( \sim 9 \) times (for cross section) greater than the other three terms. On the other hand alpha particles which have \( T=S=0 \) will produce only \( T=S=0 \) transitions while excluding excitation, to any observable extent, of states arising from the other three terms. Thus, inelastic scattering of medium energy hadrons will dominantly select the \( T=S=0 \) states for excitation. However, in charge
exchange reactions $T=0$ does not contribute to the interaction and it is thus possible to study $T=1$ excitations alone. The $(^6\text{Li},^6\text{He})$ reaction provides further selectivity in that only $S=1, T=1$ modes are excited. Thus, use of a variety of hadronic probes and reactions may help to unravel the expected complicated giant resonance spectra. In this presentation we describe the results from inelastic scattering. F. Petrovich will discuss results from charge exchange reactions in his talk.

We conclude the background discussion by showing on fig. 5 spectra$^7$ from the $^{120}\text{Sn}(\alpha,\alpha')$ reaction using 152 MeV alpha-particles at 12° and 13°. At both angles a large peak at ~12 MeV rises from an otherwise flat continuum background. (A peak from hydrogen (H) contamination in the target is visible in the 12° spectrum.) Another broad peak is seen at ~7-MeV. The peaks from elastic scattering and inelastic scattering to low-lying levels are shown on a much reduced scale. It is immediately obvious that the experimental spectra we deal with in inelastic scattering measurements of multipole resonances are more complicated or to state it differently less clean than the photonuclear spectra used to deduce the parameters of the GDR. While the $(\alpha,\alpha')$ reaction provides selectivity of $T=S=0$ modes, all multipolarities may be, and indeed, generally are excited. The generally structureless nuclear continuum underlying the resonances is a feature of all direct reaction spectra and is often ascribed to incoherent processes of preequilibrium particle emission. We have not yet found a way to eliminate the continuum and leave just the resonance peaks.

![Fig. 5. $^{120}\text{Sn}(\alpha,\alpha')$ spectra for $E_\alpha = 152$ MeV (ref. 7). A decomposition of the spectrum into giant resonances (established and possible) and a continuum is shown on the 13 degree spectrum.](image-url)
We show on the 13 degree spectrum a decomposition of the spectrum into quadrupole (GQR), and monopole resonances (GMR) and a peak from the so called low-energy octupole resonance (LEOR). These resonances are discussed in some detail below. A possible additional resonance is shown centered at ~ 24 MeV of excitation. This resonance may be the giant octupole resonance (GOR) recently proposed from inelastic alpha, proton and helium-3 scattering. It is to be noted that the shape and magnitude one assumes for the underlying continuum affects the strength of and indeed at times the very existence of proposed giant resonance peaks. The lack of quantitative theoretical understanding of the nuclear continuum is the major contributor to the uncertainty in the extraction of giant multipole resonance parameters.

The remainder of the presentation will be devoted to a summary of the existing information on the electric monopole resonances.

3. Isoscalar giant quadrupole resonance

In the notation of this presentation the isoscalar giant quadrupole resonance has quantum numbers \((L,T,S) = (2, 0, 0)\). The T=0 GQR was the first of the new giant resonances to be discovered and it remains the most well studied and carefully documented. After much disagreement over its existence during the early 1970's we now find that there is in general excellent agreement between all types of measurements that have studied the GQR including, electron and hadron scattering and particle capture reactions.

Figure 6 shows inelastic spectra from five targets spanning a wide nuclear mass range, bombarded by 152-MeV alpha particles. In each spectrum a broad peak is observed at an excitation energy that varies smoothly with target mass. Prior to approximately three years ago the entire peak in each nucleus was attributed to excitation of the GQR. However, we now know, as will be discussed below, that the peak contains both monopole and quadrupole resonances, at least for nuclei heavier than about \(A=50\). The identification of the resonance, the portion marked \(E_2\), as an \(L=2\) excitation is provided by comparison of the measured angular distributions with those calculated using the Distorted Wave Born Approximation (DWBA). The strength of the resonance in terms of percentage depletion of the EWSR is determined by the normalization of the calculated cross sections to those measured. Figure 7 shows such a comparison for the five giant resonance spectra shown on fig. 6. The data are described very well by the DWBA calculation assuming the state to be quadrupole (\(L=2\)). For all five nuclei most of the T=0, L=2, EWSR is depleted in the resonance peak.

For nuclei lighter than mass forty the character of the GQR changes dramatically as is seen in fig. 8. These data were taken with a magnetic spectrograph which provided energy resolution considerably less than 100 keV (FWHM). For \(^{40}\)Ca one observes a broad peak located at ~ 63 \(A^{-1/3}\) MeV (position of arrow) similar to those in heavier nuclei. However, no such broad structure is found at 63 \(A^{-1/3}\) MeV in the lighter nuclei. Rather, the excitation energy region between 12 and 20 MeV is fragmented into a large number of individual states. Most of these peaks are attributable to \(L=2\) excitations. The sum of the \(L=2\), T=0, EWSR depleted in the individual quadrupole states is found to be a significant fraction of the total EWSR. Systematics for the energy, width and strength (sum rule) of the T=0 GQR are presented in fig. 9. The data were taken from ref. 4 and ref. 5. Where more than one measurement has been made on a given nucleus the results were averaged. In general the excitation energy for the GQR for \(A < 100\) falls at the systematic energy of ~ 65 \(A^{-1/3}\) MeV. For nuclei lighter than mass 100 there is a clear tendency for the GQR peak to fall below the systematic energy. For nuclei lighter than \(A=40\) the energy of the strength centroid of the individual fragments has been plotted. The dashed line at 64.7 \(A^{1/3}\) MeV is taken from a recent calculation).
The width of the GQR increases smoothly with decreasing nuclear mass. However, there are apparent shell effects in the data. The narrowest resonance widths occur for A=40, 90, 142 and 208. The resonance is widest in the region of the rare earth deformed nuclei. This behavior is similar to that observed for the GDR although the increase in the GQR width in deformed nuclei is not nearly as great as for the GDR. The A^{-2/3} dependence of the width predicted in ref. 11 provides a good description of the data. The value of the multiplicative constant (90) has been increased from that suggested (58) in ref. 11 in order to fit the data.

The percentage of the T=0, E2 EWSR strength depleted in the GQR is shown at the bottom of fig. 9. It is to be noted that these values do not include contributions to the EWSR from low-lying (0h\(\omega\)) quadrupole excitations. A trend to larger sum-rule depletion with increasing nuclear mass is clearly evident. For nuclei having A \(\geq\) 100 essentially 100\% of the EWSR is found to be depleted in the GQR peak (2\(\hbar\omega\) transitions), while for lighter nuclei 30-50\% of the EWSR is typically found in the high-lying quadrupole states. However, considerably more EWSR strength is located in the low-lying 2^+ states of light nuclei than of heavy nuclei, so that the sum of the GQR (2\(\hbar\omega\)) and low-lying quadrupole (0\(\hbar\omega\)) strengths exhaust ~ 100\% of the T=0, E2 EWSR in light as well as heavy nuclei.

Fig. 6. Spectra from inelastic scattering of 152-MeV alpha-particles on \(^{208}\text{Pb}\), \(^{120}\text{Sn}\), \(^{90}\text{Zr}\), \(^{58}\text{Ni}\) and \(^{46}\text{Ti}\). The giant resonance structure located near the excitation energy 63 x A^{-1/3} MeV has been decomposed into contributions from the giant quadrupole and giant monopole resonances. The peak located at higher excitation energy in the \(^{208}\text{Pb}\) and \(^{120}\text{Sn}\) spectra are due to hydrogen contamination of the target (ref. 7).
Fig. 7. Angular distribution of the E2 portion of the spectra from fig. 6. The data are compared to an L=2 DWBA calculation normalized to the indicated EWSR depletions (ref. 7).
Fig. 8. \((e,e')\) spectra for 120 MeV alphas on \(^{24}\text{Mg}, ^{26}\text{Mg}, ^{28}\text{Si}\) and \(^{40}\text{Ca}\) (ref. 8). The arrows are located at the excitation energy \(63 \times A^{-1/3}\) MeV.

4. Isovector giant quadrupole resonance

As pointed out earlier, excitation of \(T=1\) (isovector) states via hadron inelastic scattering is much weaker than excitation of \(T=0\) states. Charge exchange reactions excite \(T=1, S=0\) states and there is evidence\(^{12}\) from \((n,p)\) measurements that the GDR is excited. However, charge exchange reactions have yet to show excitation of the isovector giant quadrupole \((L=2, T=1, S=0)\). On the other hand, the electromagnetic \((e,e')\) interaction provides equal strength into both \(T=1\) and \(T=0\) excitations (all other things being equal, e.g. - EWSR, \(L, E_x\)). For the lighter nuclei \((A \lesssim 40)\) particle capture reactions provide information on isovector strength, however most of the data on the \(T=1\) GQR have come from inelastic electron scattering and indeed none from hadron reactions. The left side of fig. 10 shows inelastic electron scattering spectra\(^{13}\) from \(^{60}\text{Ni}\). The broad peak observed at \(\sim 32\) MeV of excitation is identified as the \(T=1,\) GQR. This identification is supported by comparison of cross sections for the peak with calculated \(E_2\) form factors as shown on the right side of fig. 10 for \(^{58}\text{Ni}\) and \(^{60}\text{Ni}\).

Plotted in fig. 11 are the systematics for the \(T=1,\) GQR. The data are all from inelastic electron scattering and are taken from ref. 14. Although much less data are available than for the \(T=0\) GQR the systematics are very similar. The excitation energy follows the systematic energy \(130 \times A^{-1/3}\) MeV for heavier nuclei while for lighter nuclei the energy drops off. The width of the resonance increases rather smoothly as the nuclear mass decreases. The available results show that \(\sim 100\%\) of the \(T=1,\) E2 EWSR is located in the resonance peak. The trend observed in the \(T=0,\) GQR for smaller depletion of the EWSR for nuclei
Fig. 9. Systematics for the excitation energy, width and sum rule depletion of the isoscalar giant quadrupole resonance. The data are mostly from refs. 4 and 5.

5. Isoscalar giant monopole resonance

Although the most thoroughly studied of the new giant resonances is the GQR, the resonance which has generated the most interest is the monopole. We discuss here the isoscalar giant monopole resonance (GMR) \((L=0, T=0, S=0)\). Observation of the monopole, "breathing", or compressional mode of nuclear excitation is of special significance because knowledge of its excitation energy provides direct information on the nuclear compressibility. During the past few years several candidates for the E0 resonance have appeared but most have not withstood the test of further measurements. These early measurements are discussed in ref. 4. It is important to note that some early indirect evidence for an E0 resonance\(^{15,16}\) placed the monopole excitation at an energy of \(\sim 80 \times\)
Fig. 10. Left: Inelastic electron scattering from $^{60}$Ni. The broad peak at ~ 32 MeV of excitation is assigned as isovector quadrupole resonance (ref. 13). Right: Form factors for the isovector E2 states observed (ref. 13) in $^{58}$Ni and $^{60}$Ni. The solid curve is for the Goldhaber-Teller model while the dashed curve is from the Myers-Swiatecki prescription.

$A^{-1/3}$ MeV, a value in agreement with those from the recent more direct observations I will describe.

The first direct observation$^{17}$ of the new resonance that was later confirmed to be the GMR was made using inelastic scattering of 120 MeV alpha particles from $^{208}$Pb. Figure 12 shows spectra at two angles from the $^{208}$Pb($\alpha$,\$\alpha'$) reaction for 120 MeV incident alphas. The 14-degree spectrum shows the presence of what appears to be two broad peaks (indicated by the dashed lines) in the giant resonance region of the spectrum. The larger peak located at 11 MeV ($63 \times A^{-1/3}$ MeV) is the now familiar GQR peak. The smaller peak is located at 13.9 MeV or $\sim 80 \times A^{-1/3}$ MeV. This peak is near the energy of the GDR in $^{208}$Pb (13.6 MeV), however the isovector GDR will not be excited by the ($\alpha$,\$\alpha'$) reaction with nearly enough cross section to account for this new peak. A similar peak was also found$^{17}$ in $^{206}$Pb, $^{209}$Bi, and $^{197}$Au.

It was thus established that a here-to-fore not directly observed resonance peak was located at ~ 14 MeV for nuclei in the lead region. The obvious question was what is the nature of the peak? It is an unfortunate circumstance that the angular distributions for L=2 and L=0 excitation via the ($\alpha$,\$\alpha'$) reaction are identical in angular regions where the giant resonances are easily measured. This fact is demonstrated on fig. 13 which shows calculated angular distribution for the $^{208}$Pb($\alpha$,\$\alpha'$) reaction using 152 MeV alpha particles. The two curves are nearly identical$^{7}$ to angles as small as ~ 5 degrees with the L=0 curve having somewhat larger peak to valley ratios. The measured cross sections$^{7}$ for the two resonances show good agreement with the calculations, but not positive L=0 identification.

However, as is seen in fig. 13, at very small angles the two angular distributions are out of phase and convincing identification of the L=0 component could be made. Two groups have now provided measurements$^{18,19}$ on a large range of nuclei utilizing detection systems especially designed to study very small angle (down to zero degrees) inelastic alpha and helium-3 scattering.

Spectra from inelastic scattering$^{20}$ of 129-MeV alpha-particles from $^{144}$Sm and $^{154}$Sm at 0, 4, and 6 degrees are shown on fig. 14. It should be noted that use of small angle techniques does not eliminate the problem of excitation of
the underlying nuclear continuum. Some assumption about the shape and magnitude of the continuum underneath the resonances must be made in order to extract the resonance peaks from the spectrum.

Figure 15 shows the advantages to be gained with small angle measurements. The giant resonance peak, extracted from the continuum, is shown for the reaction \(^{90}\text{Zr} (^{3}\text{He}, ^{3}\text{He}')\) at several small angles. It is apparent that the centroid of the entire peak shifts and the width of the peak is not constant as the angle of observation varies, both effects being indicative of the existence of two different multipolarities within the single broad peak. The peak is shown decomposed into two components which clearly rapidly change relative magnitude with angle. The lower excitation peak is associated with the GQR while the higher excitation peak has an \(L=0\) angular distribution and is identified as the GJE.

Figure 16 shows a similar set of data and a similar analysis for the \(^{116}\text{Sn}(\alpha, \alpha')\) reaction for 129 MeV incident alphas. The angular distributions for the two components of the resonance peak are plotted in the lower part of the figure. The lower excitation peak has an \(L=2\) angular distribution while the
Fig. 12. Giant resonance spectra at 12 degrees and 14 degrees from the \(^{208}\text{Pb}(\alpha,\alpha')\) reaction for 120 MeV irrelevant alpha particles (ref. 17). The spectra are decomposed into two resonance peaks (\(E_x \sim 11\) MeV and \(\sim 13.9\) MeV).

Fig. 13. Calculated inelastic scattering angular distributions for 152 MeV alpha particle excitation of the GQR and GMR in \(^{208}\text{Pb}\) compared with data from ref. 7.

cross sections for the higher excitation peak follows the calculated \(L=0\) angular distribution. Such measurements have now clearly identified the peak seen earlier at larger angles as the isoscalar GMR resonance.

The small angle experiments and measurements using the \((p,p')\) reaction\(^{22}\) have now provided a significant body of information about the GMR in many nuclei. Perhaps the most interesting aspect of the GMR to be derived from these studies is the lack of observation of the monopole resonance in nuclei having \(A \leq 50\). There are no apparent theoretical reasons to suggest that the monopole should disappear in light nuclei. On the other hand we know the strength for other giant resonances becomes very fragmented in light nuclei. If such is the case for the GMR and the resonance is spread over many MeV of excitation energy in light nuclei, then present measurement techniques may not be sensitive enough to locate the \(E_0\) strength. Systematics of the monopole resonance are shown on fig. 17. The top plot shows the energy trend of the GMR. As has been the case for the resonances previously discussed, for heavier mass nuclei the resonance is located at a systematic energy, in this case \(\sim 80 \times A^{-1/3}\) MeV. For lighter nuclei the now familiar trend to lower excitation energy is observed for the GMR. The width of the monopole resonance shows a tendency to broaden as the nuclear mass decreases in agreement with the trend for other giant resonances. The sum rule depletion for the monopole resonance plotted at the bottom of fig. 17 shows that all of the \(T=0\), \(E_0\), EWSR is accounted for in nuclei as light as \(^{90}\text{Zr}\). However, less than half of the sum rule strength is observed for the three lighter nuclei that have been studied.
Fig. 14. Inelastic alpha particle spectra at very small angles for 129-MeV alpha-particles in $^{144}$Sm and $^{154}$Sm (ref. 20).
Fig. 15. Very small angle spectra from the reaction $^{90}\text{Zr}(^{3}\text{He},^{3}\text{He'})^{90}\text{Zr}^*$ for $E_{i}=108.5\text{MeV}$ incident $^{3}\text{He}$. The spectra are decomposed into a monopole (higher excitation) and quadrupole peak.
Fig. 16. Top; Giant resonance spectra from the reaction $^{116}\text{Sn}(\alpha,\alpha')$ for $E_\alpha = 129$ MeV. The nuclear continuum has been subtracted from the data. The peak is decomposed into monopole and quadrupole contributions (ref. 2).
Fig. 17. Systematics for the excitation energy width and sum rule depletion for the isoscalar giant monopole resonance. Data are from refs. 19, 20 and 22.
It now seems clear that the GDR has been identified and systematically observed in a wide mass range of nuclei. The question of the low sum rule depletion for mass 50-60 nuclei and the lack of observation of any E0 strength in lighter nuclei is yet be understood.

It is to be noted that the experimental results yield a value of $\sim 200$ MeV for the compressibility of nuclear matter.

6. Isoscalar giant octupole resonances

As described in the background discussion there should be two classes of octupole transitions, the $1\pi\omega$ and $3\pi\omega$ classes. As is the situation for quadrupole excitation, the low-lying collective $3^-$ states generally account for very little of the $T=0$, $E3$ EWSR thus raising the possibility that considerable octupole strength may be found in the $3\pi\omega$ transitions.

Recently, what seems to be a localization of $1\pi\omega$, $3^-$ strength has been observed by inelastic scattering of alpha particles from a large number of nuclei. Results from these measurements are shown on fig. 18. For most nuclei a broad peak is observed at the systematic energy of $\sim 32 A^{-1/3}$ MeV. Although for $^{197}$Au the peak falls considerably below this energy. Angular distributions for the peak cross section above the dashed line indicates an $L=3$ assignment for the excitation. Suggestions of such strength were also made through the $^{197}$Au($p,p'$) reaction and the $^{116}$Sn(e,e') reaction. The fraction of the EWSR depleted in the so-called low-energy octupole resonance is 10-20% for most of the nuclei studied. No LEOR is observed in $^{40}$Ca or $^{208}$Pb.

Within the past few months suggestions have been made for the systematic observation of a $3\pi\omega$ giant octupole resonance (GOR) through inelastic scattering of 800-MeV protons, $^{26}$) 172-MeV alpha-particles, and 110-140 MeV helium-3 particles. Figure 19 shows inelastic proton spectra (800 MeV incident protons) on $^{208}$Pb, $^{116}$Sn and $^{40}$Ca. In each case a peak is observed (labeled HEDR, high-energy octupole resonance) at an excitation energy above the GQR peak. The angular distributions for the peak are shown in fig. 20. The data agree with the very characteristic angular distribution calculated for $L=3$.

Figure 21 shows a spectrum from the $^{208}$Pb($\alpha,\alpha')$ reaction using 172-MeV alpha-particles. In addition to the GQR at 10.9 MeV and the GMR at 13.8 MeV the data indicate another peak which the authors subdivide into a peak at 17.5 MeV and one at 21.3 MeV. The angular distribution for the 17.5 MeV peak agrees very well with an $L=3$ DWBA calculation. (The authors assign the 21.3 MeV peak as an isoscalar dipole resonance [L=1, T=0, S=0].) The inelastic helium-3 measurements were made on a wider mass range of nuclei and the angular distributions of the peaks are again well described by $L=3$.

The results from these measurements are summarized on fig. 22. The resonance excitation energy follows the systematic trend of $\sim 110 A^{-1/3}$ MeV. The dashed line gives the value of 108.2 $A^{1/3}$ MeV calculated in ref. 11. The values from the $^3$He measurements are consistently somewhat higher than those from the other measurements. The resonance width again follows the familiar trend of increasing width with decreasing nuclear mass. The dashed curve for the value 140 $A^{-2/3}$ is from ref. 11 where the constant has been adjusted from the author's suggested value in order to fit the experimental results. There is clear discrepancy in the sum rule depletion as deduced from the ($\alpha,\alpha'$) and ($^3$He, $^3$He') results on the one hand and the ($p,p'$) results on the other hand. The EWSR strength depleted in giant resonances is inherently difficult to extract from inelastic hadron measurements because uncertainties in both the continuum shape and the model dependent calculations must be contended with. If one accepts, for example, the EWSR value for the $T=0$ GOR of $\sim 60\%$ for $^{208}$Pb from the ($\alpha,\alpha'$) measurement then all of the $T=0$ E3 sum rule strength is accounted for since $\sim 45\%$ of the strength lies in low lying states.

These results suggest that the GOR ($3\pi\omega$) has been located. However, as was the case ten years ago for the GQR and three years ago for the GDR, more data is needed to firmly establish the identity and systematics of this resonance.
Fig. 18. Inelastic alpha-particle spectra from several nuclei bombarded by 96 and 115 MeV alpha-particles. The low-energy octupole resonance is located above the dashed line (ref. 23).
Fig. 19. Inelastic proton from 800 MeV protons on $^{208}$Pb, $^{116}$Sn and $^{40}$Ca. The peak labeled HEOR (high energy octupole resonance) is interpreted as arising from excitation of the isoscalar giant octupole resonance ($3^+_2$) (ref. 26).

Fig. 20. Angular distributions for the giant octupole resonance as excited by 800-MeV protons (ref. 26).
7. Isoscalar_hexadecapole giant resonance

There has been no direct, (i.e. - peak observed in a spectrum), experimental evidence for an E4 giant resonance (4,0,0). RPA calculations for $^{208}$Pb have shown\(^\text{30}\) that the $2^{+}$ giant hexadecapole resonance (GHR) should occur at virtually the same excitation energy as the $2^{+}$ GQR. Other calculations\(^\text{31}\) on $^{208}$Pb indicate that $\sim 40\%$ of the L=4 EWSR should be located in the $2^{+}$ transitions and $\sim 60\%$ in the $4^{+}$ transitions. It was shown\(^\text{32}\) through microscopic DWBA calculations that the resonance cross sections from 61 MeV (p,p') and 96 MeV ($\alpha$,a') results on $^{208}$Pb were consistent with the existence of $\sim 20\%$ of the T=0 E4 EWSR within the (predominately) GQR peak. However, the inclusion of L=4 strength was based only on the total resonance cross section since the angular distributions could not definitively indicate a necessity for inclusion of GHR strength.

While the angular distributions for L=2 and L=4 excitations are not very different in low energy proton and alpha-particle inelastic scattering, the differences are very large for higher energy inelastic proton scattering. Figure 23 shows angular distributions for L=0, 1, 2, and 4 calculated using the DWBA for a state in $^{90}$Zr at $E_x = 14$ MeV. The differences between L=2 and L=4 are very large and it should be rather easy to detect a 20-40\% mixture of E4 strength in the GQR peak even with uncertainties on the data as large as 20-30\%.

Figure 24 shows inelastic proton spectra\(^\text{33}\) taken at TRIUMF using 200 MeV protons on $^{90}$Zr and $^{120}$Sn. The most obvious feature of these spectra which the cross section below $\sim 5$ MeV has been artificially suppressed, is the very prominent excitation of the "GQR". For $^{90}$Zr the 1$^{+}$, LEOR is also visible at a $\sim 7$ MeV of excitation. The preliminary cross sections extracted from these data indicate that 15-20\% of L=4 strength is needed to provide agreement with the measured angular distributions in $^{120}$Sn and $^{208}$Pb. However, for $^{90}$Zr no more than 10\% of the L=4 EWSR could be accounted for within the "GQR" peak. It should be noted that there may be considerably more L=4 strength present in this region of the nuclear continuum but it must be spread out rather than concentrated as for the GQR.

The location of the GHR is thus an unsolved question. If the strength is localized at all it will probably be situated at very high excitation energies. The calculations of ref. 11 place the L=4 strength at 150 A$^{-1/3}$ MeV or $\sim 25$ MeV in $^{208}$Pb. It is also very likely that the peak would be even broader than that reported for the GQR. Location of the GHR presents a considerable experimental challenge as does the location of even higher multipole resonances.

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**Fig. 21.** Giant resonance spectra from inelastic scattering of 172-MeV alpha particles from $^{208}$Pb. The resonance structure is decomposed into a GOR (10.9 MeV), GMR (13.8 MeV), a proposed giant octupole resonance (17.5 MeV) and an isoscalar giant dipole resonance (21.3 MeV) (ref. 27).
Fig. 22. Systematics of the excitation energy width and sum rule depletion for the isoscalar octupole resonance as proposed from the three indicated experiments (refs. 26, 27 and 28).
Fig. 23. Calculated angular distributions for L=1-4 excitations in the reaction $^{90}\text{Zr}(p,p')$ for $E_p = 200$ MeV.

Fig. 24. Inelastic proton scattering spectra from $^{120}\text{Sn}$ and $^{90}\text{Zr}$ bombarded by 200 MeV protons (ref. 33).
Through this rather rapid summarization of the experimental measurements of electric giant multipole resonances hopefully one sees that the field has progressed a great deal during the past ten years. The parameters of the giant quadrupole resonance are now firmly established by an extensive set of measurements. The GQR is providing a significant influence in other areas of nuclear physics. The monopole resonance has now been established and its observation has provided the first direct measure of the nuclear compressibility. A fairly strong case for the existence of a giant octupole resonance is now being made through a variety of hadron reactions.

However, we certainly have not exhausted the supply of giant multipole resonances. The newer techniques such as higher energy proton scattering, charge exchange reactions, heavy-ion scattering and pion reactions offer considerable hope for identifying new resonances during the next few years. We have come a long way in the past ten years but there is still a great deal to do in the giant multipole resonance field.

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