

**MASTER**

DOE Contract No. DE-AC02-76EVO4116.M004 (FORMERLY EY-76-S-02-4116)

FINAL REPORT  
by  
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Brief Description of the Developed Methodology

During the contract period, a consistent formalism for the definition of the growth rates (and thus the doubling time) of breeder reactor fuel has been developed. This formalism was then extended to symbiotic operation of breeder and converter reactors. Further, an estimation prescription for the growth rate has been developed which is based upon the breeding worth factors. The characteristics of this definition have been investigated, which led to an additional integral concept, the breeding bonus.

The newly developed breeding description is based upon the idea of splitting the actual time dependency of fuel masses into

- a) the time dependence of "idealized strains" of fuel, and
- b) the introduction rate (or dates) of such fuel strains.

The idealized strains have an exponential asymptotic behavior which automatically yields the growth rate and thus the doubling time. The actual (non-exponential) time dependence is then obtained as superposition of (idealized) strains and the realistic strain start-up dates.

The general approach of this development proceeds through three stages:

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First, the detailed space and energy dependent description of the "strain" fuel cycle problem is condensed into an "integrated" fuel cycle description containing all features essential to the temporal development of the fuel mass and their isotopic composition. The result of this condensation is a set of ordinary differential equations for the temporal changes of the isotopic components of the fuel and two constraint conditions, through which the criticality of the reactor and the make-up density of the fuel is maintained.

Second, the integrated fuel cycle equations are solved numerically which yields the transition of the strain fuel into asymptotic growth. The asymptotic fuel composition and growth rate can be obtained directly by formulating and solving the characteristic equations of the set of ordinary differential equations.

The solution of the characteristic equations of the integrated strain-fuel cycle problem yields the desired consistent definitions of the growth rate and doubling time.

Third, the characteristic equations are converted in a corresponding adjoint problem, the solution of which yields the desired breeding worth factors. These breeding worth factors allow a fairly accurate estimation of the asymptotic growth rate and doubling time from statically calculated reaction rates of the fuel absorption and capture rates.

### Special Aspects and Publications

The description of the "Research to be Performed" lists in Appendix A several specific aspects which are discussed in the following:

The integrated fuel cycle model reflects the fuel loading schemes in a manner described in Ref. 1. Also the  $\beta$ -decay of  $^{241}\text{Pu}$  is included. Both effects are therefore reflected in the resulting growth rate definitions.

The blanket loading schemes affect the fuel growth rate primarily through the delay in the build-up of plutonium (fuel) inventory in the blanket. A special treatment of this delay has been developed and implemented in the growth rate definition. In effect, it amounts to a partial inclusion of the blanket inventory in the denominator of the growth rate formula (see Ref. 1).

The developed methodology lends itself readily to an extension to symbiotic operation of FBR and LWR (Refs. 7 and 1). It then describes also the transition from positive to negative growth rates with increasing LWR participation in the symbiotic scheme. A variety of joint FBR - LWR operation scenarios have been investigated, which indicates the vital role of the FBR for better fuel utilization (see Refs. 7 and 1).

The investigation of the dependency of the growth rate definitions on fuel compositions, as they appear in the transition from an arbitrary start-up fuel composition into equilibrium, led to the introduction of an additional integral fuel cycle concept, the breeding bonus (see Ref. 6).

The incremental savings in computer time and the associated losses in accuracy encountered in transitions between the four levels of doubling time definitions have been investigated and evaluated (see Ref. 9).

The growth rate definition developed in this research differs from the previously introduced formalism essentially in two respects: It includes the various special effects discussed above, and it is mathematically more elegant and thus more consistent due to the fact that the growth rate appear "naturally" as eigenvalue in the solution of the characteristic equations and need not be introduced in an ad hoc manner as in the earlier paper by Ott and Borg (Nucl. Sci. Eng. 62, 243, 1977).

The entire methodology developed in this research is presented in an invited survey paper in Progress in Nuclear Energy (Ref. 1).

List of publications see enclosure.

## LIST OF PUBLICATIONS

1. K. O. Ott and R. C. Borg, "Fast Reactor Burnup and Breeding Calculation Methodology," Progress In Nuclear Energy, Pergamon Press, (in print).
  2. P. J. Maudlin, R. C. Borg and K. O. Ott, "Transitory Fuel Growth Rates for Fast Breeder Reactors," Trans. Am. Nucl. Soc., 26, 235 (1977).
  3. K. O. Ott, R. C. Borg and P. J. Maudlin, "Theoretical Comparison of Various Fuel Growth Rate Descriptions," Trans. Am. Nucl. Soc., 26, 538 (1977).
  4. N. A. Hanan, R. C. Borg and K. O. Ott, "Interpretation of the Isotopic Breeding Worth Factors," Trans. Am. Nucl. Soc., 27, 250 (1978).
  5. P. J. Maudlin, K. O. Ott and R. C. Borg, "Instantaneous Fuel Growth Rates for Breeder Reactors," Nucl. Sci. Eng., 72, 140 (1979).
  6. K. O. Ott, N. A. Hanan, P. J. Maudlin and R. C. Borg, "Description of Breeding with Three Integral Concepts," Nucl. Sci. Eng., 72, 152 (1979).
  7. N. A. Hanan, "Breeding Description for Fast Reactor and Symbiotic Reactor Systems," Ph.D. Thesis, Purdue University (1978).
  8. P. J. Maudlin, "Fuel Cycle Kinetics," Ph.D. Thesis, Purdue University (1978).
  9. P. J. Maudlin, R. C. Borg and K. O. Ott, "Accuracy and Computational Time of a Hierarchy of Growth Rate Definitions for Fast Breeder Reactor Fuel", Nucl. Sci. Eng., 71, 202 (1979).
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