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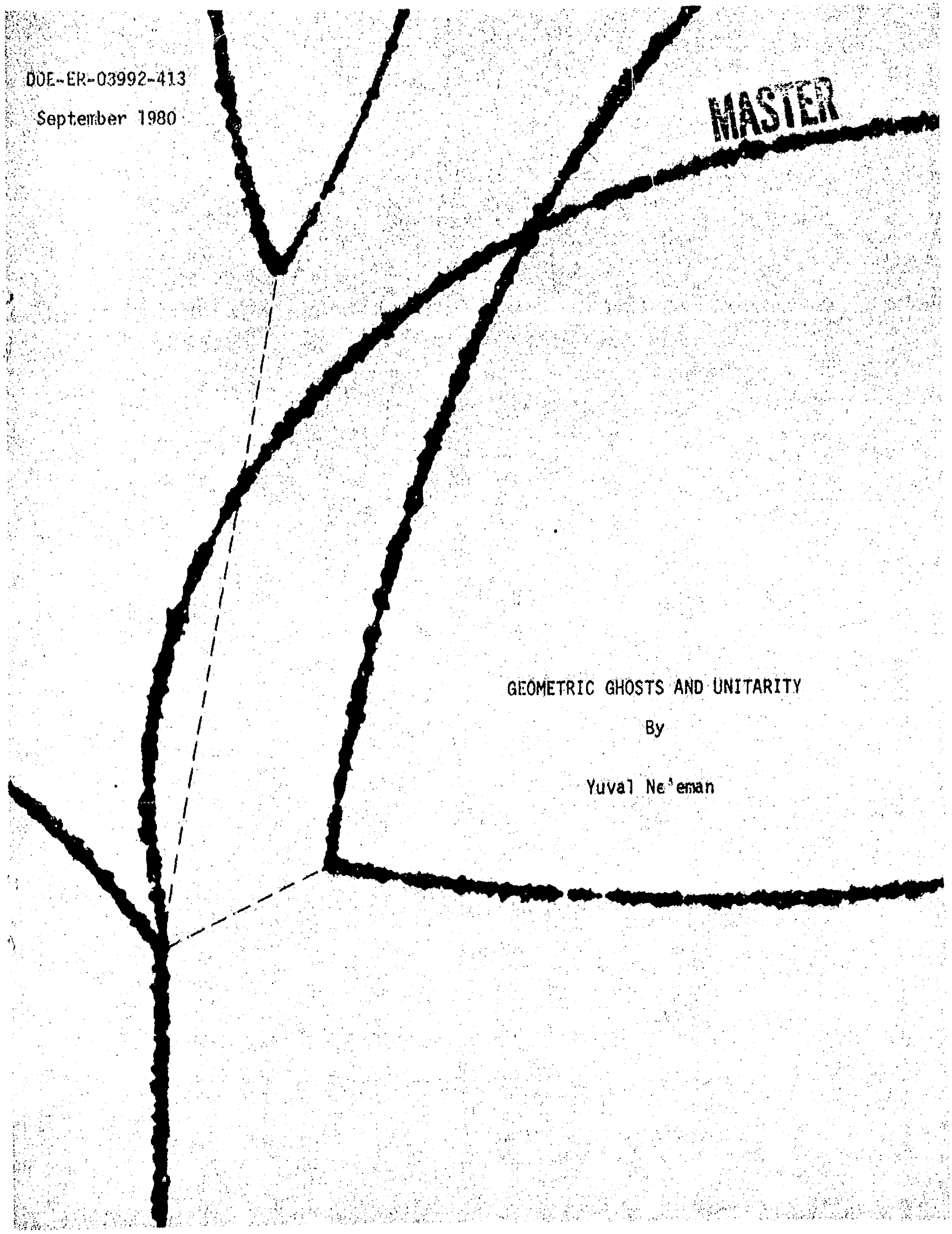

GEOMETRIC GHOSTS AND UNITARITY

By

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ABSTRACT

We review the geometrical identification of the renormalization ghosts and the resulting derivation of Unitarity equations (BRST) for various gauges: Yang-Mills, Kalb-Ramond and Soft-Group-Manifold.

THE CONVENTIONAL TREATMENT

The (geometric) Yang-Mills Lagrangian $L_{INV} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$ has no inverse Fourier-transform needed to write a propagator. It thus had to be supplemented by a gauge-fixing term $L_{FIX} = -\frac{1}{2} C^a C_a$ (with $C^a = \partial_\mu A^a_\mu$ for example) or more formally, the linear $L_{FIX} = -\sigma_a \Sigma^a$, with σ_a a (boson) scalar Lagrange multiplier, ensuring the appearance of a$\delta(\Sigma)$ in the functional integration. To ensure Unitarity (i.e. cancellation of contributions due to redundant components of A or its F) Feynman introduced ghosts[1]. The ghost Lagrangian $L_{GH} = \bar{X}\, \hat{m} X^a$ involves scalar ghost X^a and antighost $\bar{X}_a$ fields, $\hat{m}$ is given by the gauge variations of Σ^a (or C^a).

$$\delta A^a_\mu = D_\mu \varepsilon \quad \rightarrow \quad \delta \Sigma^a = \hat{m} \varepsilon^a \tag{1}$$

X^a abd $\bar{X}_a$ have to be assigned Fermi statistics, to produce a minus sign guaranteeing cancellations between their closed loops and the redundant contributions. The quantum Lagrangian

$$L = -\tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \sigma_a \Sigma^a + \bar{X}_a\, \hat{m}\, X^a \tag{2}$$

obeys Slavnov-Taylor invariance and provides generalized Ward-Takahashi identities. 't Hooft and Veltman then completed the proof of renormalizability, adding regularization. The conditions for unitarity have since been reduced to invariance under BRST (discrete) supertransformations[2]. Let Λ be a (constant) odd element in a Grassmann manifold, and take (δ is the gauge infinitesimal variation of (1)),

$$\varepsilon^a = X^a \Lambda \ , \ \Lambda^2 = 0 \ , \ \{\Lambda, X\} = \{\Lambda, \bar{X}\} = 0 \ ; \ s = -\frac{\partial}{\partial \Lambda} \delta \tag{3}$$

$$\therefore\ sA^a_\mu = D_\mu X^a ; \ sX^a = \tfrac{1}{2} f^a{}_{bc} X^a X^b = \tfrac{1}{2}[X,X]^a \ ; \ s\Sigma^a = \hat{m} X^a \tag{4}$$

Add the formal prescriptions $s\bar{X}_a = \sigma_a$, $s\sigma_a = 0$ (5)

$\therefore sL_{INV} = 0$; $sL_{FIX} = -\sigma_a \hat{m} X^a$; $sL_{GH} = \sigma_a \hat{m} X^a - \bar{X}_a s^2 \Sigma^a$; so that provided $s^2 A^a = 0$ (to make $s^2\Sigma^a$ vanish), which also implies $s^2 X^a = 0$ (using Jacobi's identity or algebraic closure of the δ variations), one has $sL = 0$. Note that in the more conventional quadratic L_{FIX}, $sC^a = \hat{m} X^a$, $s\bar{X}_a = C_a$, so that $s^2 \bar{X}_a = \hat{m} X^a \neq 0$. The transformation s, with s^2 vanishing on A^a_μ, X^a and (in the linear treatment only) on $\bar{X}_a$, has the features of an exterior differential.

THE GEOMETRICAL IDENTIFICATION

The geometrical identification of the ghosts, as vertical components of the connection P, a principal fiber bundle $(P, M, G\bullet)$ has been recently demonstrated[3]. Using x^μ as co-ordinates on the base manifold M and α^i $(i = 1..n)$ on the group fiber G, a section Σ may be

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given by σ^{-1} G, lifting the x^μ onto it. The dot ($\bullet$) represents right-multiplication $P \times G \to P$ so that $(p.g)\bullet g^{1} = p\bullet(gg^{1})$, $\forall\, p \in P$, $\forall g, g^{1} \in G$. To produce that action, the dot maps the n-dimensional Lie algebra A_G of G onto the differential operators of $P_\star$, the tangent manifold. The (quasi) inverse map (since $P_\star$ has 4 + n dimensions) is given (through contraction $dz^M \lrcorner \frac{\partial}{\partial z^N} = \delta^M_N$) by the connection w^a. Let $z^M(x^\mu, \alpha^j)$ represent local co-ordinates on P. On Σ,

$$w^a = \Xi^a_j\,(\alpha, x)\; d\alpha^i + \; A^a_\mu\,(\alpha, x)\; dx^\mu \qquad (6)$$

We remind the reader that although he may be used to A^a_μ being only x^μ dependent, any gauge transformation will introduce α^i dependence, through the homogeneous term $e^{i\alpha^i \lambda_i} A^a_\mu(x)\; e^{-i\alpha^i\lambda_i}$ (λ is a basis A_G). For Σ a global (trivial) horizontal section, $\Xi^a = \Xi^a_i\, d\alpha^i$ is the left-invariant Cartan 1-form. More generally, $\Xi^a = X^a$, the ghost field. To prove it, one writes the Cartan-Maurer structural equation on P. This corresponds to the statement that the curvature

$$\mathcal{F}^a = \tfrac{1}{2} F^a_{MN}\, dz^{[M}\, dz^{N]} \; = \left(\frac{\partial}{\partial z^M}\, w^a_N - \frac{\partial}{\partial z^N}\, w^a_M + \tfrac{1}{2} f^a{}_{bc} w^b_M\, w^c_N\right) dz^{[M}\, dz^{N]} \qquad (7)$$

has nothing but horizontal components $F^a = \tfrac{1}{2} F^a_{\mu\nu}\, dx^{[\mu} dx^{\nu]}$. Denoting by $\tilde{d}$ the full differential over z^M (more generally the exterior derivative, i.e. a curl when acting on a one-form, such as X^a) and by ℓ and d the α^i and x^μ components respectively, we have

$$\ell\delta = d\alpha^i \frac{\partial}{\partial \alpha^i}\delta \;,\; d\delta = dx^\mu \frac{\partial}{\partial x^\mu}\delta \;,\; \tilde{d}\delta = dz^M \frac{\partial}{\partial z^M}\delta = \ell\delta + d\delta \qquad (8)$$

$$d\alpha^i\, F^a_{i\mu} = 0 = \ell A^a_\mu - D_\mu X^a \;,\; d\alpha^{[i}\, d\alpha^{j]}\, \tfrac{1}{2} F^a_{ij} = 0 = \ell X^a - \tfrac{1}{2}\,[X,X]^a$$

i.e. the first two BRST equations (4), provided

$$\Xi^a \equiv X^a \quad , \quad \ell \equiv s \qquad (9)$$

indeed, $\tilde{d}^2 = d^2 = s^2 = sd + ds = 0$ from cohomology.

We now have to verify whether indeed the fermi statistics of X^a derive indeed from its being a 1-form: in non exterior-calculus words, a differential, anticommuting because of the antisymmetrization of differentials in a measure. Perturbation physics is described by the generating functional (G is the group volume)

$$e^{i\Gamma} = G \int \mathcal{D}\,(\phi)\; e^{iS_{INV}(\phi)} \text{ with } \phi = A^a_\mu \lambda_a dx^\mu \qquad (10)$$

with $S_{INV}(\phi) = \int L_{INV} d^4x$, the Yang-Mills action. We fix the gauge by inserting $1 = \int \delta(\Sigma)\, d\Sigma$; however, $d\Sigma = d\,{}^i\partial_i\Sigma = s\Sigma$ with no ∂_μ contribution through our choice of co-ordinates for Σ. Using the representation $\delta(\Sigma) = \int d\sigma e^{i\sigma\Sigma}$ and the Berezin intregral $s\Sigma = \int \hat{d}\bar{X} e^{i\bar{X}s\Sigma}$ (with $\int d\theta\, \delta(\theta) \equiv -\, i\frac{\partial}{\partial\theta}\, \delta(\theta)\big|_{\theta=0}$)

$$e^{i\Gamma} \; = \; \int \mathcal{D}(\phi, X, \bar{X}, \sigma)\, e^{iL} \qquad (11)$$

where we have replaced the group volume G by an integral over X, the measure on the fiber. The functional integral is indeed geometrical, and the fermi statistics of $X, \bar{X}$ derive from their nature as differentials or 1-forms.

Ojima has recently suggested a geometric derivation for (5). Complexifying the fiber G, one writes for (8) $\tilde{d} = s + r + d$, where $r = d\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}$ and $rs + sr = 0$, $r^2 = 0$. Note that since spin and statistics do not co-relate, the r equations may violate CPT and differ from the s; so does $\bar{X}$ differ from X in its BRST variations. The resulting new (rr), (rs) equations are

$$r\bar{X} = \tfrac{1}{2}[\bar{X},\bar{X}] \;;\quad rX + s\bar{X} + [X,\bar{X}] = 0 \qquad (12)$$

Fixing $s\bar{X}_3 = \sigma_3$, the second equation reads $rX = -\sigma - [X,\bar{X}]$. Applying s to the (rr) equation, we have $r\sigma = -[\sigma,\bar{X}]$. We now apply s to both sides of the (rs) equation. The left hand side is $-rsX = -\tfrac{1}{2}r[X,X] = +[\sigma,X] + [[X,\bar{X}],X]$ where we have used the value rX again. For the right hand side, we have $-s\sigma - \tfrac{1}{2}[[X,X],\bar{X}] + [X,\sigma]$. The triple brackets cancel on both sides by Jacobi's identity, and we find $s\sigma = s^2\bar{X} = 0$. Note that $L_{FIX} + L_{GH} = -s\,(X\Sigma)$.

Since $sL = 0$, and L is a 4-form over x^μ so that $dL = 0$ by saturation, we have geometric closure of L over P,

$$\tilde{d}L = 0 \qquad (13)$$

SUPERGROUPS AND SPONTANEOUS BREAKDOWN

In ref.[4] we have shown that for G a supergroup, $H \subset G$ its even subgroup (indices $a,b\ldots \in H$; $i,j\ldots \in G/H$)

$$\left.\begin{aligned} \omega^a &= \chi^a + dx^\mu A^a_\mu \\ \omega^i &= \eta^i + dx^\mu\ \xi^i_\mu \end{aligned}\right\} \qquad (14)$$

Due to the odd nature of G/H, the η^i are scalar boson fields, corresponding to a Goldstone-Higgs set. The ξ^i_μ is a vector-ghost set. This picture was used[4] to reproduce chiral spontaneously-broken $SU(3)_L \otimes SU(3)_R$, applying the supergroup Q(3). It may also provide the interpretation of SU(2/1) as a unified weak-electromagnetic theory (see our paper in session QFDIII of these Proceedings).

THE KALB-RAMOND FIELD

The usefulness of this geometric formalism was displayed in recent work on the $B^{\mu}_{\mu\nu}$ antisymmetric tensor gauge field[4]. Both the original Abelian model [4] and the more recent non-Abelian[5] have been shown to represent scalar fields, the latter a non-linear σ-model. The Lagrangian is here

$$L = \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma}\, B^a_{\mu\nu}F_{a\rho\sigma}(A+H) - \tfrac{1}{2}m^2\, H^a_\mu H^\mu_a - \tfrac{1}{4}F^i_{\mu\nu}(A)\, F^{\mu\nu}_i(A) \qquad (15)$$

$$B^a = \tfrac{1}{2}\, B^a_{\mu\nu}dx^\mu\wedge dx^\nu \;;\; H^a = H^a_\mu dx^\mu \;;\; A^i = A^i_\mu dx^\mu \;,\; A^m = 0 \qquad (16)$$

a denotes the adjoint representation of G; i the subgroup G′ and m the quotient G/G′.

The first attempts to supply ghosts and BRST equations was done with no geometric guidance and failed. Applying geometry, Thierry-Mieg produced straight-forward results[6]. For instance, the full B "connection" is

$$\tilde{B} = \tfrac{1}{2}B_{\mu\nu}dx^\mu\wedge dx^\nu + \tfrac{1}{2}\tilde{B}_{ij}d\alpha^i\wedge d\alpha^j + \tilde{B}_{\mu i}dx^\mu\wedge d\alpha^i \qquad (17)$$

and we see that we have as ghosts a scalar boson ϕ and a vector fermion ξ_μ as in (14),

$$\phi = \tfrac{1}{2}\tilde{B}_{ij}d\alpha^i\wedge d\alpha^j \quad;\quad \xi_\mu = \tilde{B}_{\mu i}d\alpha^i \qquad (18)$$

and the BRST equations represent the vanishing of all components of $\tilde{D}_{A+H}\tilde{B}$ other than the fully horizontal[6] (in $dx^\mu\wedge dx^\nu\wedge dx^\rho$).

THE SOFT GROUP MANIFOLD

To treat non-internal gauges (such as gravity and supergravity) we have introduced[7] the Soft Group Manifold (SGM). The SGM becomes a principal bundle upon application of the equations of motion (spontaneous fibration), with a subgroup $F \subset G$ as fiber[8]. We thus get pseudo-closure $d_{(F)} L = 0$.

More recently, we have checked[9] that the geometric identification of ghosts and of the BRST operator s holds over G (the SGM). We have used the algebra of Lie derivatives $L_{\tilde{c}}$ which holds off-mass-shell. However, the conventional quantum Lagrangian is not invariant under this algebra $L_{\tilde{c}} L = \tilde{c} \lrcorner dL + d(\tilde{c} \lrcorner L)$. The second term is a divergence and does not matter, but $dL \neq 0$ as long as L is not reduced by fibration to space-time only. Invariance has generally been obtained by adding a quartic ghost term in L, cancelling its vertical components[10].

The geometric derivation also fixes the "natural" commutation relations among the various ghosts and fields[11].

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