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DEPOLARIZATION DUE TO THE RESONANCE TAIL DURING A FAST RESONANCE JUMP*

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ABSTRACT

In this paper we examine the mechanism of depolarization due to a fast resonance jump. We find that the dominant effect for cases of interest is not dependent on the rate of passage through resonance, but rather on the size of the resonance jump as compared to the width, ε, of the resonance. The results are applied to a calculation of depolarization in the AGS at Brookhaven National Laboratory.

INTRODUCTION

The major problem which must be overcome for the acceleration of polarized protons is the resonant depolarization that occurs when the perturbing fields, as seen by the particles, contain components with frequency equal to the spin precession frequency. The strongest of these resonances occurs when

$$\kappa = kP \pm \nu \quad \text{"Intrinsic Resonance"} \tag{1}$$

where $\kappa \equiv \gamma(g/2-1)$ is the spin precession frequency, k is an integer, P is the periodicity of the accelerator, and ν is the vertical betatron tune.

The standard method[1,2] for dealing with these so called intrinsic resonances is to "jump" them by changing the tune abruptly as κ approaches a resonance during normal acceleration. To study the depolarization due to such a resonance jump, we will consider the following realistic model: let κ_0 be an isolated resonance, and let $\kappa - \kappa_0$ vary in a three step process:

$$\kappa - \kappa_0 = \begin{cases} -\infty \text{ to } -\delta, \text{ very slowly, adiabatically} \\ -\delta \text{ to } \delta, \text{ fast jump, } d(\kappa-\kappa_0)/d\theta = \alpha \\ \delta \text{ to } \infty, \text{ again adiabatically.} \end{cases} \tag{2}$$

THE SPIN EQUATION

It is well known[3,4] that the equations of motion for the spin of particle during acceleration can be written, with the aid of spinors, as

$$\frac{d\psi}{d\theta} = i/2\, H\psi, \qquad H = \begin{pmatrix} -\kappa & \varepsilon e^{-i\kappa_0\theta} \\ \varepsilon^* e^{i\kappa_0\theta} & \kappa \end{pmatrix} \tag{3}$$

where θ is the turning angle, ε is the width of an isolated resonance and κ_0 is the frequency of the resonance. The spin vector is given by

$$S_i = \psi^\dagger \sigma_i \psi \tag{4}$$

where σ_i is the appropriate Pauli matrix. Equation (3) is formally just the Schrödinger equation with Hamiltonian H.

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SOLUTIONS TO THE SPIN EQUATION

A. $(\kappa-\kappa_0)$ = constant. The solution of the spin equations for a beam circulating at constant energy, κ = a constant, is useful in understanding the spin behavior in the neighborhood of a resonance. Transform Eq. (3) to a new rotating coordinate system;

$$\psi = e^{-i\kappa_0\theta\sigma_z/2}\varphi \quad \Rightarrow \quad \frac{d\varphi}{d\theta} = \frac{i}{2}\begin{pmatrix} -\delta & \varepsilon \\ \varepsilon & \delta \end{pmatrix}\varphi \tag{5}$$

where $\delta \equiv \kappa - \kappa_0$, and we have let ε be real and positive for convenience. We diagonalize and obtain the following normal modes:

$$\varphi_+(\delta) = \begin{pmatrix} \sqrt{(\lambda+\delta)/2\lambda} \\ -\sqrt{(\lambda-\delta)/2\lambda} \end{pmatrix} e^{-i\lambda\theta/2}, \quad \varphi_-(\delta) = \begin{pmatrix} \sqrt{(\lambda-\delta)/2\lambda} \\ \sqrt{(\lambda+\delta)/2\lambda} \end{pmatrix} e^{i\lambda\theta/2} \tag{6}$$

where $\lambda = +\sqrt{\varepsilon^2+\delta^2}$. The general solution is given by

$$\varphi = C_+\varphi_+ + C_-\varphi_- \quad , \quad |C_+|^2 + |C_-|^2 = 1. \tag{7}$$

We are interested in the vertical projection of the spin, which is

$$S_z = \varphi^\dagger\sigma_z\varphi = (|C_+|^2 - |C_-|^2)\frac{\delta}{\lambda} + 2|C_+||C_-|\frac{\varepsilon}{\lambda}\cos(\lambda\theta + \Delta) \tag{8}$$

where Δ is just the relative phase of C_+ and C_-. Since Δ is different for different particles, the second term does not contribute to the average S_z. Notice that for pure states $S_z = \pm\,\delta/\lambda$ and for $\delta >> \varepsilon$, $S_z \to \pm 1$, a spin up state and a spin down state as one would expect. But if we begin with $S_z = +1$ and $\delta = -\infty$ and allow δ to vary only slowly (adiabatically), as we approach the resonance, $S_z \to \delta/\lambda$; and as we pass through the resonance (adiabatically), δ changes sign so that as $\delta \to +\infty$, $S_z \to -1$. This spin flip is just the so called "fast adiabatic passage through resonance" well known from the theory and practice of Nuclear Magnetic Resonance. The salient point here is that in spite of the apparant depolarization ($S_z = \delta/\lambda$) near the resonance, if we vary δ adiabatically, the depolarization is only apparant since the spin will reorient itself as $\delta \to \pm\infty$.

B. The Resonance Jump. First consider an instantaneous jump, $\alpha \to \infty$. In this case we need only the solutions given in Eq. (6). We choose an initial condition of $S_z(-\infty) = +1$; the corresponding spinor adiabatically transforms to $\varphi_-(-\delta)$. The final spinor after the resonance jump, at $\kappa - \kappa_0 = \delta$, is simply a linear combination of the basis spinors at δ;

$$\varphi(\delta) = C_+\varphi_+(\delta) + C_-\varphi_-(\delta) \tag{9}$$

so as we vary $\delta \to +\infty$ adiabatically we find

$$\varphi(+\infty) = \begin{pmatrix} C_+ \\ C_- \end{pmatrix} \tag{10}$$

and

$$S_z(+\infty) = \varphi^\dagger(+\infty)\sigma_z\varphi(+\infty) = 2|C_+|^2 - 1 \tag{11}$$

To calculate C_+ just notice that since the jump is instantaneous, the spinor is unchanged so that $\varphi(\delta) = \varphi_-(-\delta)$. From Eq. (9) and (11) we find

$$C_+ = \varphi_+^\dagger(\delta)\,\varphi_-(-\delta) = \delta/\lambda \tag{12}$$

and

$$S_z = (\delta^2-\varepsilon^2)/(\delta^2+\varepsilon^2) \tag{13}$$

So we are left with substantial depolarization even when the resonance is crossed instantly. To include the effect of α being finite is straight forward. Since we are considering a "fast" jump, a scattering matrix approach is appropriate. We quote the results here leaving the details to Ref. 5.

$$S_z = \frac{\delta^2-\varepsilon^2}{\delta^2+\varepsilon^2} - \frac{2\varepsilon^2\delta^2}{3\alpha^2}\,\frac{\delta^2}{\delta^2+\varepsilon^2}, \qquad (14)$$

where $\alpha \equiv d(\kappa-\kappa_0)/d\theta$. Restrictions: $(\delta^2/2\alpha) \ll 1$, valid to order (ε^2/α). Equation (14) is the final result for the depolarization due to a resonance jump. Table I shows the above result applied to a calculation of depolarization in the AGS at the largest intrinsic resonances below 26 GeV/c.

DISCUSSION

Consider the resonance at $\kappa = \nu$ in the AGS. From Table I we see that the residual polarization calculated with Eq. (14) is 0.969. However, the first term in Eq. (14) yields 0.970 while the second term gives only an additional - 0.001. So we find that the depolarization for this set of parameters is nearly rate independent. This does not mean that the rate of passage is unimportant since the dependence is $1/\alpha^2$. However, this does indicate that any attempt to improve the proposed program at the AGS should not concentrate on increasing the rate of passage through resonance, but rather should examine the feasibility of increasing the size of the resonance jump.

The depolarization due to a resonance jump has also been calculated by Turrin using a different model[6]. His results applied to the AGS yield depolarizations essentially identical to those in Table I, however; the present approach yields a simple physical picture and is easily generalized to an asymmetrical resonance jump. There is a spread in κ due to the spread in γ in the beam so that not all particles traverse the resonance symmetrically. As an example consider the resonance at $36+\nu$. At this energy $\Delta\kappa \simeq 0.1$ so that some particles jump the resonance from - 0.175 to + 0.075. Using the method developed here, applied to an asymmetrical jump, we find that the additional depolarization is only 3%, well within tolerable limits.

Table I

The Resonance Jumps

$\delta = 0.125$, $\alpha \equiv d\nu/d\theta = 0.0597$, $\nu = 8.75$.

$\kappa = kP \pm \nu$	γ res	ε	S_z, Eq. (14)
$0+\nu$	4.88	0.0154	0.969
$36-\nu$	15.20	0.0137	0.975
$36+\nu$	24.96	0.0266	0.911

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