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Excitation and propagation of the fast wave in a two component non uniform plasma

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Recent experiments on TFR /1/ and Diva /2/ have shown that the efficiency of ICRF heating is strongly dependent on the isotopic composition of the plasma. Temperature increases of electrons and ions have been observed though the cyclotron resonances of deuterium or hydrogen were outside the plasma, with a relative concentration hydrogen/deuterium $\eta_p = 0.6$. Theoretical explanation involves the existence of the ion - ion hybrid resonance inside the plasma. Near this resonance the fast wave coupled by the antennas is converted to a slow wave (probably the shear Alfven-wave) which is strongly absorbed by cyclotron and Landau damping /3/. J. Jacquinot and al. /4/ have shown that the damping rate of the fast wave is in good agreement with a mode conversion process in the two ion hybrid layer. The purpose of this study is to compute the coupling of antennas in presence of plasma, and to derive the electric field distribution taking into account inhomogeneity in the magnetic field and in the density. The only calculations which have been down, up to now, were made under two kinds of assumptions : very low damping /5, 6, 7/ or very strong radial damping /8/. Our calculation takes into account the mode conversion as it affects wave propagation. This might be of great importance for large machines.

We consider a plane wave guide, limited by two walls of infinite conductivity, filled with a plasma (fig. 1). This plasma is confined by a magnetic field along the wave guide, with a linear variation across it :

$$\bar{B}_z = B_{zo} (1 - x/Ro)$$

We choose parabolic density profile

$$Ne(x) = Neo \left(1 - \frac{x^2}{R_p^2}\right),$$ Neo being the peak density ; $x = \pm R_p$ are the limits of the plasma.

Electromagnetic waves can be excited in the wave guide by the use of 2 antennas located on each side of the plasma at radius R_1 and R_2. I_1 and I_2 are the currents flowing along them in the y direction perpendicular to B_z and parallel to the wave guide walls. The plasma is homogeneous in y and Z directions. Making the Fourier transform of the current in the Z direction, we can solve the wave equation for each component in the K_z spectrum. We will get the field distribution for a given current by making the inverse Fourier transform.

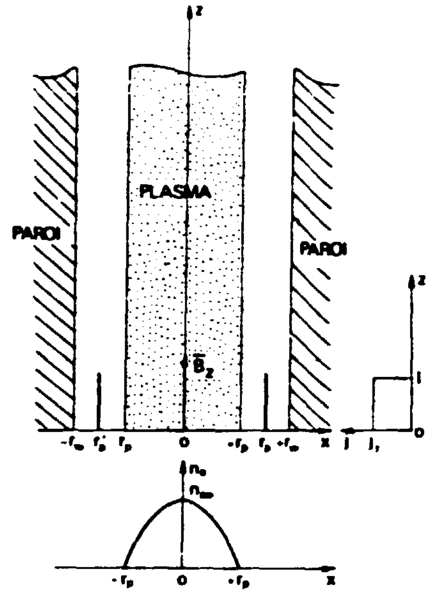


FIG. 1 : Schematic description of the wave guide. Density and current profiles

We define each component of the K_z spectrum by :

$$J_j\,(K_z) = \frac{I_j}{2\Pi\, l_j} \int_{-\infty}^{+\infty} J_j\,(z)\,\exp\,(-\,iK_zZ)\,dZ \qquad j = 1,2$$

I_j is the current flowing through the antenna located at R_j, l_j being its size in the Z direction, J_j (z) being the current profile of this antenna, normalised to unity. We neglect the electron inertia, then the component of the electric field along the magnetic field vanishes. The assumption is correct as far as the wave frequency is much smaller than the electron plasma frequency.

We derive the differential equation for the electric field in the y direction after Fourier transform.

$$(S - N_z^2)\,\frac{d^2}{d\,x^2}\,E_y^K + \frac{\omega^2}{C^2}\left[(S - N_z^2)^2 - D^2\right] E_y^K = 0 \qquad (1)$$

$$E_y^K = \frac{1}{2\,\Pi} \int_{-\infty}^{+\infty} E_y\,(x,\,z)\,\exp\left[-\,iK_z\right] dZ \qquad (2)$$

$N_z = K_Z\,\dfrac{C}{\omega}$, S and D being the usual terms of the dielectric tensor as given by STIX /9/

We solve equation (1) with the following boundary conditions :

$$E_y^K\,(\pm\,R_w) = 0 \qquad (3,1)$$

$$E_y^K\,(R_j^+) = E_y^K\,(R_j^-) \qquad (3,2)$$

$$\left(\frac{d}{dx}\,E_y^K\right)_{/R_j^-} - \left(\frac{d}{dx}\,E_y^K\right)_{/R_j^-} = i\,\omega\,\mu_0\,J_j\,(K_z) \qquad (3,3)$$

$\pm$ Rw are the abscissa of the walls

$$R_j^+ = \lim_{\substack{x \to R_j \\ x > R_j}} x \quad , \quad R_j^- = \lim_{\substack{x \to R_j \\ x < R_j}} x$$

The other components of the electric field and the components induction field are function of Ey and $\frac{d}{dx}$ Ey.

The Poynting vector in the X direction is given by :

$$P_x^K = \frac{i}{4\,\omega\,\mu\,o}\left[E_y \left(\frac{d}{dx} E_y\right)^* - E_y^* \left(\frac{d}{dx} E_y\right) \right] \qquad (4)$$

The star being for complex conjugate.

The wave equation is singular when the coefficient $S - N_Z^2$ vanishes. This point defines a resonance in W K B approximation. Near this point a cut off defined by $S - D = N_Z^2$ occurs. The fundamental assumption we have made is that the cold plasma theory correctly describes the mode conversion process. If we want to take into accout the hot plasma terms, we then need to solve a fourth order differential equation. But several authors /9, 10/ have shown that if the mode conversion occurs far enough from the cyclotron resonance, cold plasma theory describes correctly the propagation of the fast wave.

Around the point of mode conversion, the solution of equation (1) involves a logarithm. This solution can be written as /11/ :

$$E_y(x) = E_1\, x\, \psi_1(x) + E_2 \left[g_s\, x\, \psi_1(x) \operatorname{Log} x + \psi_2(x) \right] \quad (5)$$

ψ_1 and ψ_2 are real functions $\psi_1(o) = 1$, $\psi_2(o) = -1$

The causality determines on which side of the real axis the logarithm is choosen. The Poynting vector in x direction exhibits a discontinuity which is :

$$|\Delta P| = 2\, E_2 E_2^*\, \Pi\, |g_s| \qquad (6)$$

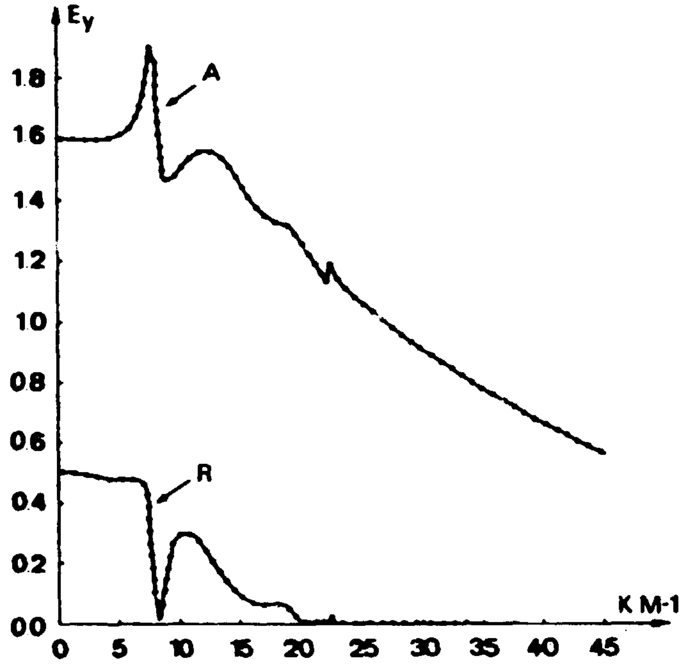
This discontinuity can be seen as an absorption : the energy of the wave is converted into a slow mode which is absorbed by Landau damping or cyclotron damping.

We present some results which have been obtained with the typical parameters of T. F R /1/

$R_o = 98$ cm, $R_w = 25$ cm, $|R_1| = |R_2| = 22$ cm, $R_p = 21$ cm

$N_{eo} = 10^{14}$ cm^{-3}, $B_z = 4.4$ T, $f_o = 60$ MHz, $\eta_p = 0.2$

First consider the case when the antenna is on the high field side. On figure 2, we have plotted the real and imaginary parts of the Fourier spectrum of the electric field at $x = R_1$. We notice that the real part of the spectrum is very broad ($0 < K_Z < 23$ cm^{-1}). We have also to notice that near $K_{Zo} = 8.5$ m^{-1} the coupling is strongly reduced.



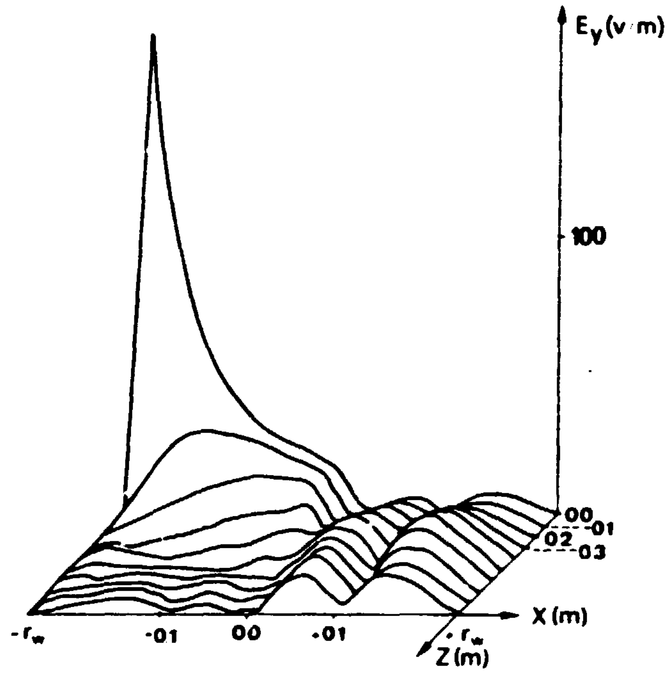


FIG. 2 : Fourier spectrum of the electric field at $X = R_1$. Antenna on the high field side.

FIG. 3 : Electric field distribution. Inverse Fourier spectrum of fig. 2.

Figure 3 represents the variation of the electric field E_y after the inverse Fourier transform. The wave experiences a very strong damping is the Z direction. On the high field side, the damping length is about 20 cm. So the field is localized under the antenna. Nevertheless, on the low field side, the electric field shows a well defined structure which appears as an eigenmode with a very much longer damping length. This eigenmode exists between the plane defined by $x = 0$ and the wall. With our plasma parameters, the hybrid layer defined by $S = 0$ occurs on that plane. As it is well known /2/, the hybrid layer acts as a reflector, for the wave coming from the low field side, and as an absorbant for the wave coming from the high field side.

Let us consider a wave whose electric field E_y is zero on the hybrid layer. This condition defines a finite valve of K_z, K_{zo}. For a wave coming from the low field side, with $K_z = K_{zo}$, the space between the low field side wall and the hybrid layer is seen as a resonant cavity. In addition if this wave satisfies resonance conditions, then an eigenmode is able to exist in this cavity.

Consider now a wave excited from the high field side, with $K_z = K_{zo}$. As $E_y = 0$ on the hybrid layer, formula (5) shows that $E_2 \equiv 0$. Consequently, from (6), there is no absorbtion for such a wave. So it crosses the hybrid layer and then can propagate in the cavity defined above. The E_y spectrum must show an almost null real part for this particular wave because the absorption is very low. As K_z is changing from K_{zo}, $|E_2|$ increases and absorption takes place.

On the other hand, an antenna on the low field side will see a completely different plasma. As the hybrid layer is a reflector, the E_y electric field has to be null on the hybrid layer to satisfy the boundary conditions of this particular cavity. So can we await a strong coupling around K_{z0} and no coupling anywhere else. This is what we observe on the E_y spectrum (fig. 4). The inverse Fourier transform of the electric fields shows a well defined structure of the wave which propagates far away from the antenna. The damping length is typically 2 meters. The absolute level of the electric field on the low field side of the hybrid layer is much more important than on the high field side (fig. 5).

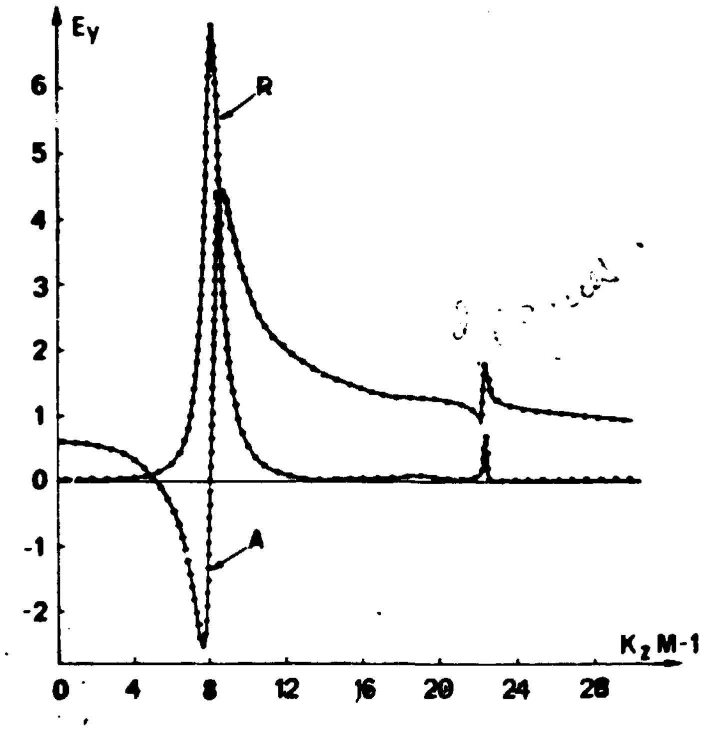


FIG. 4 : Fourier spectrum of the electric field at Z = R_2. Antenna on the low field side.

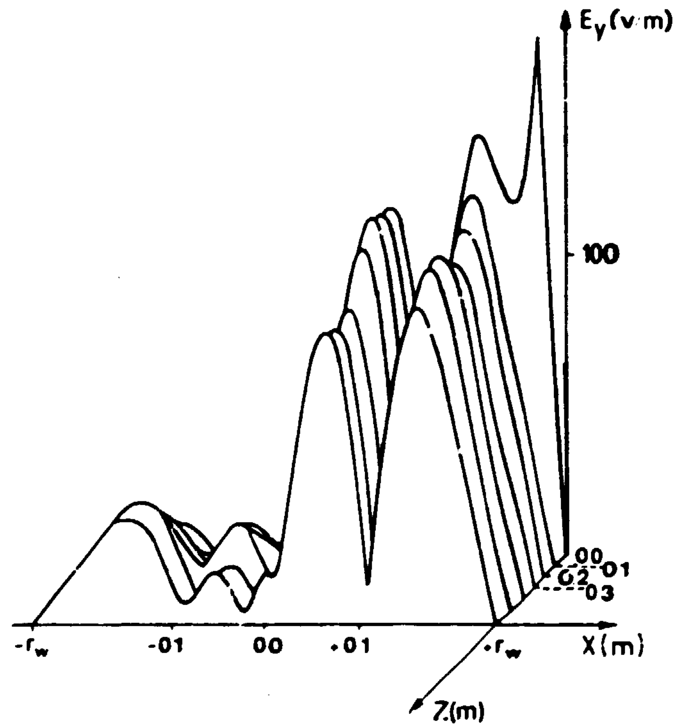


FIG. 5 : Electric field distribution from fig. 4.

As a conclusion, this model describes the propagation of the fast wave. From the value of the damping length (about 20 cm when the antenna is on the high field side), there is a good agreement with experimental observations / 14/. One point is of particular interest. For a finite value of K_z, an eigenmode can exist between the hybrid layer and the low field side wall. It means that the coupling of an antenna is drastically different when it is on one side of the slab or on the other one. Although this point needs further investigations, there are indices that this phenomenon really occurs in a tokamak plasma /13/. This is of great importance for large machines in which the antenna would be on the low field side.

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