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INFLUENCE OF THE SPACERS ON THE STABILITY OF CHANNEL COOLED SUPERCONDUCTING COILS

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In a previous paper¹, the thermal stability of channel cooled superconducting magnets was experimentally studied. Stable normal zones were observed within a range of currents and local disturbance energies. Usual theories fail to fully explain these results, owing to the fact that they take into account a heat transfer to liquid helium only function of the temperature of the conductor, whatever the position is. In a simplified theoretical analysis, it is shown that in a locally uncooled superconducting wire several stationary normal zones can exist. A criterion is derived that yields the recovery current as a function of the length of the uncooled region. Besides, a detailed numerical analysis determines the evolution of a normal zone in a channel cooled magnet. Theoretical calculations are compared with experimental results.

Introduction

The thermal behaviour of a liquid helium channel-cooled superconducting coil subjected to a local thermal perturbation has been studied experimentally. In addition to the influence of such parameters as current, magnetic field, magnitude of the perturbation and cooling, the tests showed how the width of the spacers bounding the helium channels affects the stability of the conductor.

The recovery current I_r for a given disturbance is defined as the largest current for which the normal region created by this disturbance will collapse. It decreases as the spacer width increases for a constant percentage of average wetted surface. Moreover for each value of the external magnetic field the recovery current I_r is smaller than the propagation current I_p , above which the normal zone propagates. Stable stationary normal zones were observed within the interval $[I_r, I_p]$.

These stationary normal zones are related to the presence of uncooled regions under the spacers. In fact this cannot be explained by existing theories wherein cooling and heat generation are assumed to be functions only of the temperature^{2,3}. For each value of current higher than the cold end recovery current defined in², this hypothesis leads to only one stationary temperature distribution with cold ends different from the stable uniform distribution $T = T_b$, bath temperature. This is the minimum propagation zone (MPZ) which is an unstable stationary solution of the heat equation. On the contrary in channel-cooled coils the heat exchange depends on the position along the conductor because of the existence of uncooled regions under the spacers.

Part one develops a simplified analysis which points out the influence of the width of an uncooled zone on the recovery current. For a given current it is shown that several stationary temperature distributions can exist. Some stationary solutions are stable and others are unstable. Different stability regions of the conductor are then defined.

In part two a detailed numerical analysis is presented to determine the transient temperature distribution along a conductor locally heated. Results are compared with the experimental data published in reference¹.

Simplified analysis

The conductor is regarded as an infinitely long homogeneous wire with its ends kept at the temperature of the bath. For clarity we consider only one spacer of width l and assume the exchange to be zero under it. A

thermal perturbation large enough to bring the conductor to normality occurs under the spacer.

Account is not taken of the temperature dependence of the thermal conductivity k , the electrical resistivity ρ , and the specific heat C of the wire. A linear relationship is used between the heat flux to helium per unit wetted area and the temperature difference between conductor and bath: $q = h(T - T_b)$. The current sharing zone is neglected.

With these hypothesis the equation governing heat diffusion in the conductor and the boundary conditions are written, after conventional normalization of the various quantities:

$$\begin{aligned} d^2T/dx'^2 &= d^2T/dx'^2 + g(T) - T \quad \text{if } x' > x_s \quad (1) \\ d^2T/dx'^2 &= d^2T/dx'^2 + g(T) \quad \text{if } 0 \leq x' < x_s \quad (2) \\ T(x' \rightarrow \infty) &= 0 \quad \text{and} \quad (dT/dx')_{x'=0} = 0 \end{aligned}$$

where:

$$\begin{aligned} \tau' &= (ph/CA)t, \quad x' = (ph/kA)^{1/2}x, \\ A & \text{ is area of cross section, } p \text{ is cooled perimeter,} \\ T &= (T - T_b)/(T_c - T_b) = q/(h(T_c - T_b)), \\ g &= 0 \quad \text{if } T < 1, \\ g &= -\alpha i^2 \quad \text{if } T > 1, \quad i = I/I_c, \\ \alpha & \text{ is the stability parameter}^4. \\ x_s & \text{ is the normalized half-width of the uncooled region} \\ & \text{centred in } x = 0: x_s = (ph/kA)^{1/2}l. \end{aligned}$$

In the special case where $x_s = 0$, a solution of (1) gives the cold end recovery current determined by $\alpha i_r^2 = 2$. For a current i such that $i_r < i < 1$ there is a single non-uniform temperature distribution called MPZ of resistive length: $2x_1 = -\text{Log}(1 - 2/\alpha i^2)$. The normalized enthalpy difference e_{MPZ} between $T = 0$ and the MPZ temperature distribution is equal to:

$$e_{MPZ} = \int_{x' = -\infty}^{x' = +\infty} \int_0^{T_{MPZ}} dT dx' = 2 \alpha i^2 x_1$$

Stationary normal zones

Resolution of the stationary equations leads to four types of stationary solution noted ①, ②, ③, and ④, different from the trivial state $T = 0$, as shown in Fig.1. As x_s tends towards zero solution ① tends towards MPZ.

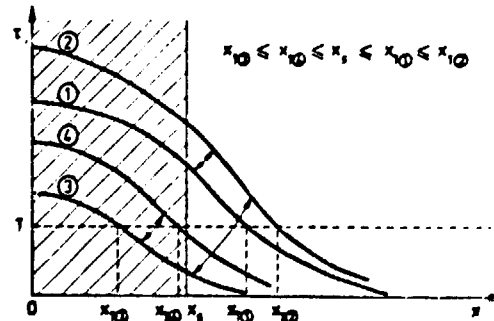


Fig.1 - Stationary temperature profiles in the presence of an uncooled region.

Zero, one, or two solutions exist according to the x_s , i , and α values. The conditions for the existence of these solutions are the following:

$$\begin{aligned}
\alpha i^2 > 2; \alpha i^2 x_s < 1 &: & \text{solution } \textcircled{1} \\
\alpha i^2 x_s > 1 &: & \text{solution } \textcircled{3} \\
\alpha i^2 < 2; \alpha i^2 x_s > 1 &: & \text{solution } \textcircled{2} \text{ and } \textcircled{4} \\
\alpha i^2 x_s < 1 &: & \\
\alpha i^2 (x_s^2 + 1) > 2 \text{ and } x_s < 1 &: & \text{solutions } \textcircled{1} \text{ and } \textcircled{2} \\
\alpha i^2 (x_s^2 + 1)^2 > 4 \text{ and } x_s > 1 &: & \text{solutions } \textcircled{3} \text{ and } \textcircled{4} \\
\alpha i^2 (x_s^2 + 1) < 2 \text{ and } x_s < 1 &: & \text{no solution} \\
\alpha i^2 (x_s^2 + 1)^2 < 4 \text{ and } x_s > 1 &: & \text{no solution}
\end{aligned}$$

The results are plotted in the $(\alpha i^2, x_s)$ diagram in Fig.3, which is discussed in detail below.

Stability regions

Let $T_{st}(x')$ be a stationary solution of the heat equation with boundary conditions. Let us consider the variable regime problem with the initial condition: $T(x', 0) = T_{st}(x')$. T_{st} is asymptotically stable if, for a very small disturbance on the initial condition, the solution of the unstationary problem converges towards T_{st} as time t tends towards infinity. In other words:

$$\begin{aligned}
&\text{if } T(x', 0) = T_{st}(x') + dT(x', 0), \\
&\text{and } \lim_{t \rightarrow \infty} |T(x', t) - T_{st}(x')| = 0, T_{st} \text{ is stable.}
\end{aligned}$$

A stability region may be assigned to any stable stationary solution $T_{st}(x')$. This region is defined by all initial perturbations such that $T(x', t) \rightarrow T_{st}(x')$.

If the equation admits only one stable stationary solution $T_{st}(x') = 0$, all $T(x', t)$ solutions starting from whichever initial conditions will tend towards this stable state and the system will have a global stability. This is the case for the conductor working at currents below the cold end recovery current.

When several stable stationary solutions exist, they are approached by many other solutions starting from different initial conditions. Stability is then limited, and each stable solution governs a stability region defined by various initial conditions.

In the case of no spacer, when the current is higher than the cold end recovery current, the previously found stationary temperature distribution (MPZ) is unstable⁵. The stability region of the $T=0$ state should be defined by a set of functions. To simplify it is defined only by the enthalpy of the perturbations: the stability region is assumed to include all the perturbations of enthalpy smaller than the MPZ enthalpy. If the initial conditions lie outside the region, the solution will tend towards a "stationary" state described by a temperature distribution $T(x' - ct')$. This state is defined by an infinite normal zone whose front propagates at constant velocity c .

From the simplified analysis given above we can define all stationary solutions in the presence of an uncooled region. When $\alpha i^2 > 2$ we have two stationary states: $T=0$ (stable state), and state $\textcircled{1}$ or $\textcircled{3}$ according to the x_s value, the latter being unstable MPZ type stationary states. $\textcircled{1}$ and $\textcircled{3}$ states, which develop continuously at the boundary crossing $\alpha i^2 = 2$, are assumed to remain unstable when $\alpha i^2 < 2$. The new solutions $\textcircled{2}$ and $\textcircled{4}$, which appear for $\alpha i^2 < 2$, are then stable.

If the operating point is within the shaded region shown in Fig.3 stability is global; whatever the disturbance is the conductor returns to the superconducting state $T=0$. We can then define a cold end recovery current in the presence of an uncooled region of width $2x_s$ by:

$$\begin{aligned}
\alpha i_T^2 &= 2/(x_s^2 + 1) & \text{if } x_s < 1, \\
\alpha i_T^2 &= 4/(x_s + 1)^2 & \text{if } x_s > 1.
\end{aligned}$$

When $i > i_T$, the stability of the $T=0$ state is limited. As before in the presence of only one stable solution, we assume that the only parameter characterising the critical perturbations (stability region boundaries) when several stable solutions exist is the enthalpy of the unstable stationary solution. Consequently if the initial conditions lie within the stability region of $T=0$ (a perturbation $< e$ unstable solution) the conductor returns to the superconducting state. Otherwise, according to the existence of the stable stationary zone, it tends either towards this zone or towards an infinite normal zone. The influence of the uncooled zone width on the critical energy is shown in Fig.2.

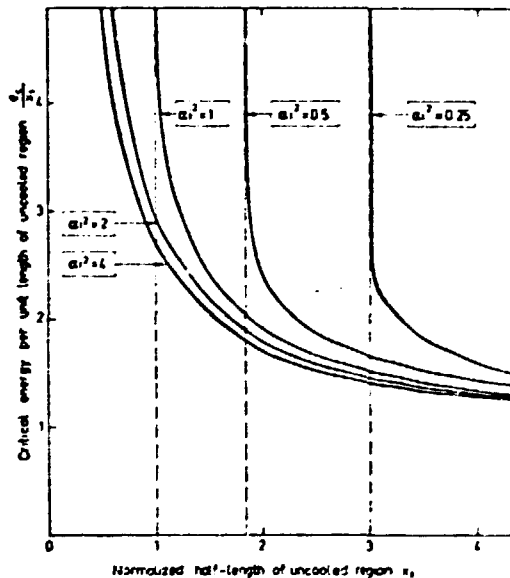


Fig.2 - Critical energy variation with the uncooled zone length.

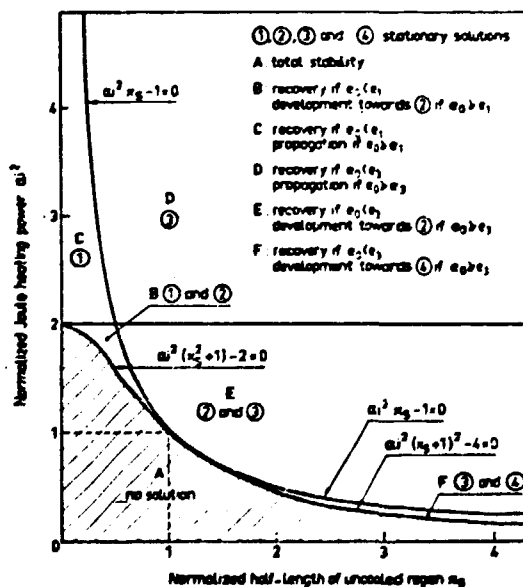


Fig.3 - Stability regions of a conductor.

Let e_0 be the normalized energy of the disturbance to which the conductor is subjected and e_m the enthalpy of the stationary distribution \textcircled{m} . The behaviour of the conductor is depicted in Fig.3. Development towards

⊕ or propagation means that the normal zone, created by the energy perturbation ϵ_0 , tends towards the stable stationary normal zone ⊖ or towards an infinite normal zone respectively.

Detailed numerical analysis

The previous considerations about stationary solutions fail to describe the time-dependent temperature distribution along the conductor. Moreover dynamic studies show that the stability is governed not only by the perturbation energy but also by the space and time distribution of this perturbation.

Calculation method

A numerical analysis was established from the unidirectional heat equation to simulate the development of a resistive zone in a superconducting composite cooled by helium channels perpendicular to the conductor. The model can account for the superconductor-matrix current sharing effect, the conduction along the conductor, a heat deposit $P(x,t)$ per unit conductor volume, the heat exchange $Q(T,x,t)$ (possibly periodic in space to simulate the presence of spacers), and the variations with temperature of all the properties of the materials. A numerical solution of the following equation is approached by a finite space difference method combined with an explicit Kutta Runge type time resolution method.

$$C \frac{dT}{dx} = d/dx \quad (k \frac{dT}{dx}) - u(T) \rho I^2/A^2 - Q + P$$

$$(dT/dx)_{x=0} = 0 \text{ and } T(x=L) = T_b$$

The characteristics of the conductor are described in reference¹. The electrical resistivity of the copper was measured as a function of magnetic field: $\rho_{cu} = (1 + 0.5 B) 10^{-10} \Omega \cdot m$. The thermal conductivity was measured under zero field and the Wiedemann Franz law is assumed valid whatever the magnetic field: $\kappa_{cu}(B) \rho_{cu}(B) = LT$ with $L = 2.4 \cdot 10^{-6}$. The electrical resistivity and specific heat variations with temperature are taken from the literature.

Development of the normal zone

Consider as a disturbance a uniform and instantaneous heat release of energy E_0 in the length $2x_0$ of conductor. The initial temperature distribution is an interval of amplitude $(T_0 - T_b)$. Figures 4 and 5 show, for different initial energies, typical time variations of the normal zone half-length x_n defined by $T(x_n) = T_c$, current sharing temperature. The average wet surface percentage is the same in both figures.

In the cases where the heat exchange is homogeneous along the conductor the normal zone collapses or propagates towards an infinitely large normal zone, according to the initial energy (Fig.4). For the type of disturbance studied the critical energy is 9.5 mJ (note MZL energy = 8.8 mJ). When the presence of spacers along the conductor is simulated by a periodic system of cooled and uncooled zones the temperature distribution tends towards the same stationary temperature profile for different initial perturbation energies (Fig.5). The critical energy (stability region limit of the $T = T_b$ state) is 3 mJ. When the perturbation energy is higher than this critical energy the normal zone tends towards a stable stationary normal zone of energy 6 mJ. The stability region limit of this stable zone lies around 30 mJ. For higher energies the normal zone propagates.

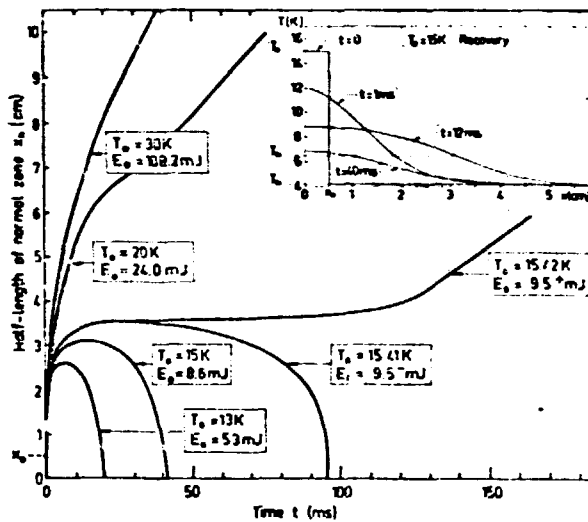


Fig.4 - Normal zone length versus time for different initial perturbations.

$$T_b = 4.2 \text{ K}, I = 500 \text{ A}, B = 5 \text{ T}$$

$$x_0 = 0.5 \text{ cm}, E_0 = 2Ax_0 \int_{T_b}^{T_0} C dT$$

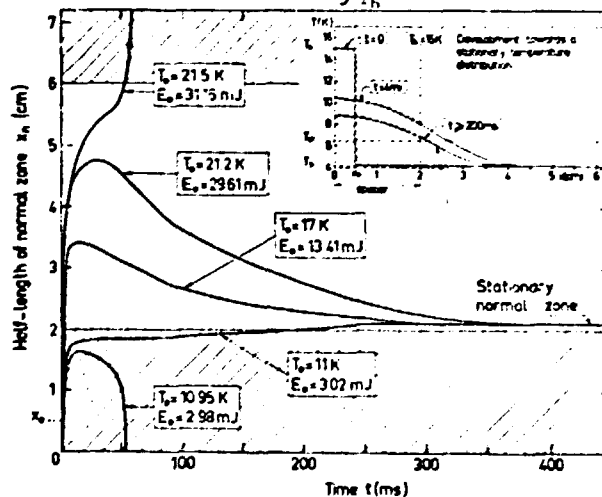


Fig.5 - Normal zone length versus time for different local perturbations under a spacer.

$$T_b = 4.2 \text{ K}, I = 500 \text{ A}, B = 5 \text{ T}$$

$$x_0 = 0.5 \text{ cm}, l = 4 \text{ cm}, E_0 = 2Ax_0 \int_{T_b}^{T_0} C dT$$

Comparison between theory and experiment

The voltage across the coil cooled by boiling liquid helium channels at 4.2 K is now compared with that calculated from the temperature distributions given by the numerical analysis. The experimental arrangement and the main results are described in reference¹.

In this experiment, a power supply delivers pulses of $\theta = 1$ - to 10-ms duration to drive a heater under a spacer. The insertion of an electrical insulator between the conductor and the heater introduces a thermal diffusion time constant T_d . This time constant was measured during other tests carried out on the same type of heater. The order of magnitude of T_d is 6 ms for a heat pulse of energy $F_h = 1$ mJ generated in the heater and 20 ms for 100 mJ. The heat flow transferred to the conductor volume $2Ax_0$ is then simulated in the computation by the theoretical law:

$$2Ax_0 P(t) = E_p / \theta (1 - \exp(-t/\tau_d)) \quad 0 \leq t < \theta$$

$$2Ax_0 P(t) = 2Ax_0 P(\theta) (1 - \exp(-(t-\theta)/\tau_d)) \quad t > \theta$$

Note: The time constant is an important parameter to study the recovery current for a small disturbance (0.1 to 10 mJ):

For the channels under investigation ($L/d_p = 56$) the minimum film boiling heat flux and the peak nucleate boiling heat flux are both 0.16 Wcm^{-2} for a uniformly heated channel⁶. For a partially heated channel a "chimney" effect can improve the heat transfer, which is enhanced to 0.35 Wcm^{-2} for the middle turn in accordance with⁷. The film boiling flux curve is adjusted so that the cold end recovery current measured experimentally fits with the theoretical recovery current obtained from the equal area criterion. The slope of the curve is found to be $0.1 \text{ Wcm}^{-2}\text{K}^{-1}$.

In the film boiling region, a transient heat transfer effect is introduced to account for the heat absorbed or released by a thin layer of helium vapour created at the surface of the conductor, as proposed in reference⁸:

$$q(T) = q_s(T) + a(T) \, dT/dt,$$

where $q(T)$ is the transient transfer and $q_s(T)$ the stationary transfer. We have used the empirical expression for $a(T)$ proposed in⁸.

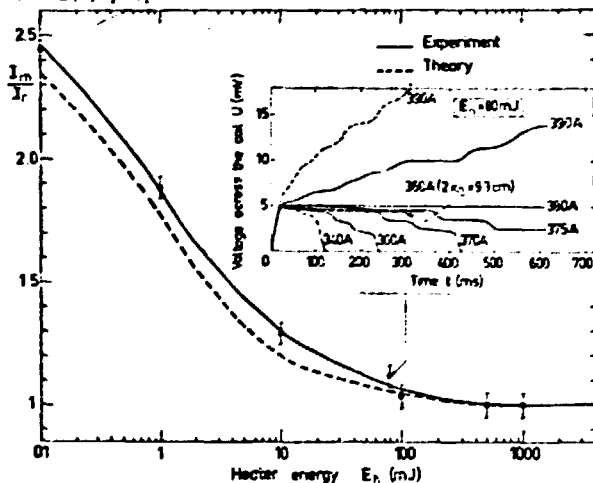


Fig. 6 - Ratio of the recovery current against localized disturbance to the cold end recovery current. Theory-experiment comparison.
 $T_b = 4.2 \text{ K}$, $l = 10 \text{ cm}$, $x_0 = 0.5 \text{ cm}$
 $B = 9 \text{ T}$ ($I_r = 330 \text{ A}$, $I_c = 900 \text{ A}$)

Figure 6 compares the measured and calculated recovery current values for different perturbation energies. Agreement is found to within 10%. In that Fig. some records of voltage versus time are also compared with computed values.

Furthermore, normal zone propagation speed experiments, not reported in¹, have been performed with the same coil. The calculated values of the velocity, which do not depend on the disturbance, agree with the measured values to within 20%.

Conclusion

In usual theories, the cooling is assumed to depend only on the conductor temperature. This is unrealistic to study the stability of a helium channel-cooled conductor where the heat exchange is periodic in space. A particularly difficult situation for the coil arises when local heating affects a spacer. We have shown that the stability region of the supercon-

ducting state is reduced by the presence of the spacers in comparison with the case where the average cooled perimeter is the same. In other words, a periodic system with a cooled perimeter fraction $f = 1$ and 0 is worse than $f = 0.5$ all along.

By a simplified analysis, in cases where the normal zone is created in an uncooled region of the conductor, we have determined the possibility of observing stationary normal zones since in this case the solutions can be thermally stable. Moreover, recovery current has been found to decrease as the spacer width increases.

A detailed numerical resolution of the heat equation has shown the different possible developments of a normal zone created under a spacer: resorption, trend towards a stationary normal zone, or propagation towards a very large normal zone, as a function of the initial conditions. A good agreement was found with the experimental values obtained on a small coil cooled by boiling liquid helium channels at 4.2 K.

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