INIS-mf--6345

ROYAL INSTITUTE OF TECHNOLOGY

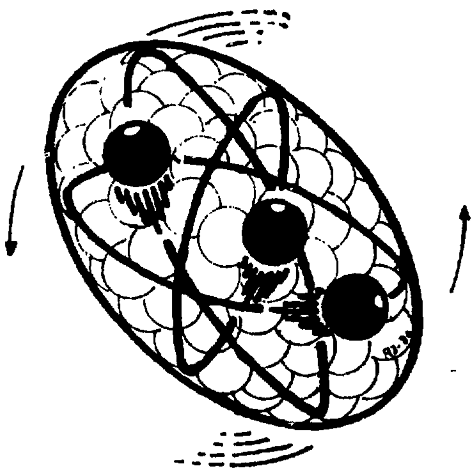
DEPARTMENT OF THEORETICAL PHYSICS

PARTICLE-ROTATION COUPLING IN ATOMIC NUCLEI

by

Jan Almberger

Stockholm 1980



S-100 44 STOCKHOLM — SWEDEN

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Akademisk avhandling
som med tillstånd av Kungl. Tekniska Högskolan
i Stockholm framlägges till offentlig granskning
för avläggande av teknisk doktorsexamen i teoretisk
fysik onsdagen den 21 maj 1980, kl 10.00 i Kollegie-
salen, Administrationsbyggnaden, KTH, Valhallavägen 79,
Stockholm. Avhandlingen försvaras på engelska.

Stockholm 1980

Almberger, J.: Particle-Rotation Coupling in Atomic Nuclei, 1980, Stockholm.
Royal Institute of Technology, Stockholm.
Department of Theoretical Physics.

ABSTRACT

In the last years an increased interest in the rotational nuclei has been spurred by the new experimental high-spin activities and by the possibilities for lower spins to interpret an impressive amount of experimental data by some comparatively simple model calculations. This thesis aims at a discussion of the particle modes of excitation for rotational nuclei in the pairing regime where some puzzles in the theoretical description remain to be resolved.

A model comparison is made between the particle-rotor and cranking models which have different definitions of the collective rotation. The cranking model is found to imply a smaller value of the quasiparticle spin alignment than the particle-rotor model. The reason lies in the unphysical neglect of the core recoil in the cranking approximation, where total angular momentum is not conserved.

Rotational spectra for both even and odd nuclei are here investigated with the use of the many-BCS-quasiparticles plus rotor model. This model gives an accurate description of the ground and S-bands in many even-even rare- earth nuclei. However, the discrepancies for odd-A nuclei between theory and experiments point to the importance of additional physical components. Therefore the rotationally induced quadrupole pair field is considered. This field has an effect on the low spin states in odd-A nuclei, but is not sufficient to account for the experimental data.

Another topic which is considered in this thesis is the interaction matrix element in crossings for given spin between quasiparticle rotational bands. The matrix elements are found to oscillate as a function of the number of particles, thereby influencing the sharpness of the backbending. Finally the low-spin continuation of the S-band is studied and it is shown that such states can be populated selectively by means of one-particle pickup reactions involving high angular momentum transfer.

Descriptors: Nuclear deformation, Nuclear rotation, Coriolis interaction, Spin alignment, Quadrupole pair field, Band crossing, Many-quasiparticle band, Particle-rotor model.

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1. INTRODUCTION

Nuclear physics is the science of the structure and the properties of atomic nuclei. There lies an obvious fascination in dealing with these dense lumps of protons and neutrons, the properties of which are so essential for the existence of the universe.

While the smallness of the nucleus poses a major challenge to the experimental physicists, the problems for the theorists are different. A reasonable characteristic of the nuclear system is that it contains too many particles, yet too few. There are too many nucleons to allow for the simultaneous description of all degrees of freedom, but there are not enough nucleons to allow for statistical methods. Furthermore, the fundamental interactions binding the nucleons together are not very well understood.

It is therefore no surprise that a manyfold of nuclear models has been used to encompass the by now overwhelming amount of experimental information. A nuclear model is most often based upon classical concepts, borrowed from other fields of physics, and with some quantum-mechanical frosting on top of it. There lies a great deal of physical knowledge in the various models as well as in the relations between the models and to more fundamental concepts.

In the present thesis some very simple-minded physical notions are discussed. It is therefore my purpose, on the following pages, to present an elementary introduction into the physics of nuclear rotations. Many nuclei are deformed, with spheroidal shapes, and may rotate slowly in space. It is of interest to study the particle motion inside such nuclei, since one may learn not only about the rotational motion itself but also about other features of the nuclear structure.

The thesis consists of the following papers:

I J. Almberger, I. Hamamoto and G. Leander,
"Spin alignment in the particle-rotor and cranking models",
Nucl. Phys. A333 (1980) 184

II J. Almberger,
"The rotationally induced quadrupole pair field in the particle-rotor model", preprint TRITA-TFY-80-3

III J. Almberger, I. Hamamoto and G. Leander,
"Y_{21} pairing and the particle-rotor description of ^{167}Er",
Proc. Int. Conf. on nuclear behaviour at high angular momentum, Strasbourg, 1980

IV J. Almberger, I. Hamamoto and G. Leander,
"Many-BCS-quasiparticle effects in rotational spectra",
Physica Scripta (to be published)

V J. Almberger, I. Hamamoto and G. Leander,
"Band crossing in the particle-rotor model",
Phys. Lett. 80B (1979) 153

VI J. Almberger, I. Hamamoto, G. Leander and J.O. Rasmussen,
"Theoretical rotational signatures for neutron pickup reactions to $(i_{13/2})^2$ bands",
Phys. Lett. 90B (1980) 1

2. AN ELEMENTARY PRESENTATION

2.1 Nuclear models

The nuclear shell model has undergone many changes since it was originally put forward in 1949. However, the basic idea [Ha49,Ma49] has survived. The neutrons and the protons are assumed to move independently in a spherically symmetric potential well, representing their average interaction with all the other particles. A nucleus is built up by filling the successive quantum levels of this potential well with neutrons and protons until there finally emerges a number of closed shells and some "loose" nucleons in unfilled shells. In particular the numbers of nucleons needed to fill the major shells are required by definition to be the "magic numbers" for which the nuclei in nature exhibit particular stability.

However, the internucleon interactions always give rise to additional correlations. It was first suggested by Rainwater [Ra50] that permanently non-spherical shapes of nuclei might result from the polarizing action of one or several loosely bound nucleons on the rest of the nucleus. Such deformation effects seemed necessary in order to resolve the long-standing problem of some particularly large electric quadrupole moments in the rare earth region. Evidently the coupling between nucleons is such that their individual quadrupole moments tend to add coherently.

Clearly the concept of nuclear deformation is related to collective motion of the nucleus as a whole. It is e.g. evident that a spheroidal quantummechanical object can rotate in space around an axis perpendicular to its symmetry axis. Such a rotational motion is in fact easily identified by spectroscopy and the first experimental indications on rotational spectra and the very fast electric quadrupole transitions between the rotational states were collected by Bohr and Mottelson [Bo53a,Bo53b].

The two seemingly incompatible views of the single-particle model and the collective picture are brought together in the unified model developed by A. Bohr [Bo52] and applied by Bohr and Mottelson [Bo53c]. It is assumed that the nucleons move in a common, slowly changing potential. If the nuclear potential changes sufficiently slowly, there is an approximate separation of the nuclear motion into intrinsic and collective motions. The first of these represents motion of individual nucleons in a static potential, while the second involves motion of the nuclear core as a whole, for example, vibrations around the nuclear shape and rotations perpendicular to the symmetry axis.

When the nuclear shape is deformed with an axial symmetry and stable against vibrations, which is often the case for nuclei with many nucleons outside of closed shells, the collective motion may be represented by the Hamiltonian

$$H_{rotor} = \frac{\bar{R}^2}{2\mathcal{J}} \tag{2.1}$$

Here $\bar{R}$ is the collective angular momentum vector oriented perpendicular to the symmetry axis and $\mathcal{J}$ is the moment of inertia relative to the rotational axis.

For an odd nucleus, the motion of the rotor must be coupled to the intrinsic motion of the odd nucleon, in the static deformed potential which is generated by the particles in the core. The best known formulation of the deformed single-particle model involves the modified harmonic oscillator potential considered by Nilsson [Ni55].

The unified model also allows for more than one particle in the intrinsic states, but the independent particle model is in this respect not sufficient. It can e.g. not explain the striking difference in the properties of even-even nuclei as compared to their odd-even neighbours. The former always have a larger separation energy of the last nucleon, a considerable energy gap to its first intrinsic excitation and a 0^+ ground state. This points towards a pairing of the nucleons such that the total angular momentum of the pair is zero, at least in the ground state. In order to excite a single nucleon one would have to break a pair which is costly in energy.

The similar kind of energy gap in superconductors is successfully explained by the BCS theory [Ba57], on the assumption that a weak attractive force exists between two electrons. Following this Bohr, Mottelson and Pines [Bo58] pointed out that the mathematical approach developed for the understanding of superconductivity might also be applied to the pairing force in nuclei.

The idea behind the BCS theory for nuclei is to describe the intrinsic system in terms of independent fermions (the BCS quasiparticles) also when the pairing force is included. These BCS quasiparticles then obtain energies so as to directly account for the differences between odd and even nuclear spectra mentioned above. Furthermore the wave-functions differ from those of the single-particle motion in the deformed potential well, which one has to keep in mind when calculating matrix elements between the intrinsic states.

Summing up this introductory review it has been stated that, apart from generating the spherically symmetric shell model potential, the most important effects of the internucleon interactions are the quadrupole shape deformability and the pairing of nucleons. These two effects are put 'on the same footing' in the pairing-plus-quadrupole model [Mo58,Be59]. Here the quadrupole interaction is responsible for the large nuclear deformations and other shape collective effects. It is a force with a range of the order of the nuclear radius and can therefore correlate the motion of nucleons on opposite sides of the nucleus. Moreover, due to the long range it will tend to dominate for nuclei with many particles outside of closed shells. Consequently such nuclei are often well deformed and may exhibit collective rotational bands. The pairing force, on the other hand, is of shorter range and favours spherical symmetry since it couples pairs of nucleons in spherical states. As a consequence a broad region of nuclei around the magic nuclei are spherical. The pairing force is also responsible for differences in properties between even and odd nuclei.

The intrinsic states for the many-body system governed by the quadrupole and pairing interactions may be approximately described by the motion of independent fermions in the deformed or spherical average potential. For simplicity, in the following elementary presentation, the rigorous distinction between the particle and quasiparticle concepts will not always be made.

2.2 Nuclear rotations

In the previous section some plausible arguments were put forward for the existence of collective effects based upon the nuclear shape degrees of freedom. However, a more fundamental description and understanding of deformed shapes in nuclei is outside the scope of the present work. Instead we shall proceed on an emphirical basis, firstly establishing from experimental results the existence of even-even rotors. Then the main attention is focused on the decisive competition between the rotationally induced forces and the potential binding forces, both of which are experienced by the particles in the rotating nuclear system. A broad variety of situations is envisaged, so one could hope to learn something interesting about the collective rotational mode and the particle degrees of freedom.

Experimental level structures for low-lying states in even-even nuclei normally show a remarkable simplicity. In particular, for nuclei with many nucleons outside of closed shells the lowest excited states are quite accurately described by the simple formula

$$E_{rotor} = \frac{\hbar^2}{2\mathfrak{J}} I(I+1) \qquad (2.2)$$

where $\mathfrak{J}$ denotes the effective moment of inertia and I the even integer angular momentum. The form (2.2) is the familiar expression, obtained from the Hamiltonian (2.1) for a purely collective rotation of a spheroidal core. Stated in more general terms, the observed pattern of the collective rotational band infers the shape and other properties of the rotating body. We shall here be mainly interested in the regions of spheroidal nuclei, such as the rare earths and the actinides.

In the non-rotating ground state of the spheroidally deformed even-even nucleus all nucleons are paired with opposite angular momenta. This conclusion can be drawn even from the independent particle model, but in the BCS theory a sizeable energy gap relative to the excited intrinsic states is also established. Quite often, however, one finds in the rotating even-even nucleus a number of excited levels rather close to the lowest rotational states. These other excitations are then presumably of collective nature, corresponding to shape vibrations around the rotating core.

In an odd nucleus, where an additional nucleon is present, also the one-quasiparticle excitations come low in energy. There are normally many close-lying quasiparticle levels in the non-rotating intrinsic system. But the particle is also subject to inertial forces due to the rotation. For the classical particle-rotor system (cf. paper I, p.196) these can be divided into the Coriolis and centrifugal forces and an additional force due to the explicit time dependence of the rotational frequency vector $\bar{\omega}(t) \equiv \mathfrak{J}^{-1} \bar{R}(t)$. The quantum mechanical core has quantized states of motion, and the corresponding forces are expressed by interactions, between the particle and the rotational degrees of freedom [Bo 52]. Here, as in the literature, the particle-rotation coupling is often denoted the Coriolis interaction since in the rotating nuclei, where the particle motion is very fast and the collective rotation relatively slow, effects resembling those of the classical Coriolis force will normally be the most prominent. However, other physical situations will be envisaged, the discussion of which is actually part of the essence of this thesis.

The Coriolis interaction acts to align the particle angular momentum $\bar{j}$ with the angular momentum $\bar{R}$ of the rotating core, which is always oriented perpendicular to the core symmetry axis. In contrast, for the non-rotating spheroidal core, $\bar{j}$ is in precession around the symmetry axis. The particle will then in the general rotating case occupy an orbital as determined by the competition between the Coriolis interaction and the deformed potential which is coupling the particle to the core.

Experimentally a broad variety of spectra can be identified [St75]. In odd-A nuclei level schemes ranging from the type $I(I+1)$, characteristic of the strong coupling limit, to rotational aligned (decoupled) bands with spectra described by $R(R+1)$ have been observed. In the former case the particle is strongly bound to the deformed potential and therefore contributes both to the total angular momentum $\bar{I}$ and to the rotational energies. In the latter case the odd particle contributes to $\bar{I}$ but not to the rotation which is carried out solely by the core having (semiclassically) $R = |I - j|$. Normally the outcome of the competition between the nuclear potential and the Coriolis interaction is intermediate, varying, e.g., within a rotational band or between rotational bands of neighbouring odd nuclei. The analysis of such rotational bands may then offer an important tool for more detailed investigations of the basic physics.

For higher angular momenta the particle-rotation coupling becomes larger so as to eventually bridge the pairing energy gap. The Coriolis effects are particularly large for particles which are moving fast on the surface of the nucleus. In the deformed regions there are often high-j particles filling some of the uppermost energy levels in the single-particle potential. It is now well established that the first distinct structural change along the yrast line* in many even-even nuclei is connected with the rotational alignment of such high-j quasiparticles [St72].

The rotation aligned two-quasiparticle band may cross the ground state band since the pair of high-j quasiparticles in the aligned state contributes a large part of the total angular momentum at little cost of intrinsic excitation energy. Experimentally one normally follows the yrast cascade, measuring therefore transitions in the two distinct bands. The discontinuous behaviour of the yrast transitions in the band crossing region has been called the "backbending phenomenon" (cf. Figs. 1 and 4 of Paper IV). It was first

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* "yrast" is a Swedish word which makes foreigners somewhat dizzy. The yrast line connects the lowest states for each angular momentum.

detected by the Stockholm group [Jo71] but was originally associated with a somewhat different physical phenomenon.

Analogously one can think of other crossings between aligned few-quasiparticle configurations in both even and odd nuclei, some of which have been detected in experiments. In conclusion, the rotational mode makes it possible to study the BCS few-quasiparticle states. This can then, to the extent that the rotation is understood, give additional insight into the structure of nuclei. The theoretical analysis is, in the present thesis, mostly carried out within the many-BCS-quasiparticles plus rotor model, which is just a special case of the unified model as outlined above.

Before entering into the more detailed discussion of the various papers, we shall briefly consider some additional features. In the more careful empirical analysis of rotational bands in even-even nuclei the energy expression (2.2) can be regarded as the leading order rotational contribution, while deviations occur due to the influence of the particle-rotation coupling on the particles of the core.

The classical Coriolis force acts so as to align the particle angular momenta along the axis of rotation and will therefore oppose the pairing force as well as redistribute the particles in space. The centrifugal force tends to push the particles away from the rotational axis. As a consequence the physical parameters describing the core and the average nucleon potential obtain a dependence upon the state of rotation. In particular for rather low angular momenta, the rotational perturbations on the core of well-deformed nuclei are associated mainly with changes in the pair correlations [Mo60]. The latter have an influence not only on the quasiparticle excitations but also on the moment of inertia of the core [Be59], which in the ground state is reduced with roughly 50 % as compared to the rigid rotor value appropriate to the independent nucleon system.

Another question to consider is if and when the particle-rotation coupling is strong enough to change the basic symmetries of the rotating body and of the average nucleon potential. One can envisage a breakdown of the pairing as well as potentials for different spheroidal and non-spheroidal shapes [Mo60, Bo75]. In such cases, major modifications are expected in the rotational spectrum and in the nucleon coupling scheme. The exploration of the high angular momentum domain ($I \gtrsim 20\ \hbar$) where such "phase transitions" are foreseen is at present one of the most exciting fields in experimental and theoretical nuclear research [Bo77].

3. DISCUSSION AND CONCLUSIONS

For the relatively low-spin part ($I \lesssim 20\ \hbar$) of the yrast line the competition between the rotational forces and the internal forces and fields offers additional possibilities to study the nuclear structure in and near the ground state. This is in short the essence of the previous elementary presentation and serves as the basic motivation for the present thesis.

Two different models are employed here, both implementing the collective rotation as an a priori assumption. The particle-rotor model describes a partly filled high-j valence shell coupled to an axially symmetric rotor in a manner which conserves the total angular momentum. In the cranking model the same high-j valence shell is used, and the average potential which binds the particles rotates in space with a fixed rotational frequency vector. The cranking description of the rotation is semiclassical and the total angular momentum is not conserved. Consequently the rotational mode is treated differently in the two cases and it becomes meaningful to compare the models in order to learn more about their physical relevance. Such a comparison is made in the present work, for the first time in a systematic manner.

The cranking model employed here is quite conventional [Ha76,Be79], but the particle-rotor model is treated more consistently than any earlier applied many-quasiparticle version within the BCS formalism [St72,Fl76]. No technical approximations are introduced here, apart from a numerically controlled truncation of the basis space. To achieve this goal the BCS many-body method is developed in detail which also makes it straightforward to include arbitrary additional fields and interactions in the particle-rotor model Hamiltonian.

3.1 On the Coriolis attenuation problem

The particle-rotor model, based upon the one-BCS-quasiparticle states, has for a long time been used to study odd-A nuclei at low angular momenta. A remarkable empirical experience is that the quasiparticle-rotation coupling is overestimated by a factor ranging up to 2 [St68]. Previous applications of the cranking model, on the other hand, suggest that this "Coriolis attenuation" anomaly does not arise [Ri74]. Instead an analysis in terms of the quasiparticle spin alignment (see below) exhibits a remarkable agreement, over a broad frequency region, between the cranking model and experiments [Be79].

In paper I of the present thesis these earlier findings are examined in a more consistent manner by assuming the same quasiparticle valence space and by choosing identical parameters in the particle-rotor and cranking models, thus limiting the difference to the description of the rotation. The aim of the examination is firstly to quantify the implications of the difference between the models and identify the physical mechanism behind it, secondly to make an extended investigation of the many-BCS-quasiparticles plus rotor model by a comparison with experiments up to high angular momenta.

Following conventional methods an analysis of physical quantities in terms of the rotational frequency ω is carried out, with a particular emphasis on the alignment of individual nucleon spins along the axis of rotation. For the cranking model the rotational frequency is an input parameter, but for the particle-rotor model and for the experimental spectra ω has to be calculated. The recipe used in paper I is based upon Hamilton's equation

$$\omega \equiv \dot{\theta} = \frac{\partial E_I}{\partial I_a} \qquad (3.1)$$

where E_I is the energy and I_a the rotation-aligned total angular momentum which is the variable conjugate to the angle θ. In this way ω can be derived from the calculated or measured level energies using a finite-difference approximation.

It is shown in paper I that the cranking model normally gives less spin alignment than the particle-rotor model, especially at low rotational frequencies. The reason for the difference is found to lie precisely in the cranking approximation where the fluctuations of the rotational frequency vector, caused by the recoil of the core against the odd high-j particle, is not taken into account. In the particle-rotor model, on the other hand, a recoil effect is present due to the sharing of the conserved total angular momentum between the core and the particle. A similar difference between the particle-rotor and cranking models is easily established in classical physics. The essence of paper I is to show that in the quantum-mechanical framework the recoil effect alone provides the dynamical forces that increase the spin alignment and in particular produce the strong spin alignment perpendicular to the core symmetry axis even in the absence of the average rotation. The latter effect can normally not be expressed as an increment to the average rotational frequency.

The main conclusion made in this part of paper I is that the cranking model does not contain any physical mechanism that can be used to explain the Coriolis attenuation problem. Rather, the recoil effect in the particle-rotor model is obviously physical.

The approximate solutions of the cranking model are very poor under conditions when the recoil is large. This is in particular the case for high-j one-quasiparticle bands at low rotational frequencies. However, the cranking model is expected to be more appropriate when the assumption of a sharp frequency vector is approximately fulfilled. This situation is well established for many other states in both even and odd nuclei. Then the cranking model provides an elegant and more general formulation of the same basic physics as in the particle-rotor model and has been much applied. It should also be mentioned that the conclusions of paper I do not rule out the existence in nature of some additional physical mechanism that in itself can justify the cranking model assumption also for low rotational frequencies.

The further aim of paper I is to make an extended investigation of the spin alignment in the many-BCS-quasiparticles plus rotor system by making a comparison to experiments for both even and odd nuclei. Such an investigation has previously not been made for the odd nuclear case.

First, however, the connections of the present particle-rotor model to the more commonly used one-BCS-quasiparticle plus rotor model are established for low rotational frequencies. In a second-order perturbation treatment of the Coriolis term it can be seen that the two models give equivalent results, apart from a small blocking effect, provided that the bare core moment of inertia is properly renormalized between the models. The complete diagonalization, made in paper I, confirms that the difference between the two approaches is indeed modest below the first band crossing. Only a small "Coriolis attenuation" effect is found from taking the many-BCS-quasiparticle states into account. For the narrow purpose of calculating the low-lying level energies in odd-A nuclei the many-quasiparticle approach to the particle-rotor model has therefore been shown, for the first time in paper I, to be redundant in the case of a high-j valence shell. However, the scope of this work is much more ambitious.

The approach for the experimental comparison is first to perform a least squares fit to the even-even nuclear level energies, using only parameters with a clear physical significance. In conclusion the available experimental data on alignment in the yrast sequences of ^{166}Yb and ^{168}Yb can be accounted for without Coriolis attenuation. In the odd-A nucleus ^{167}Yb the positive-parity high-j ($i_{13/2}$) rotational bands are experimentally known up to high spins and the corresponding theoretical level scheme, for angular momenta $I \geq j-1$, is calculated in paper I using the fitted model parameters of the even neighbours. It is then found that the present particle-rotor model implies a quasiparticle spin alignment in ^{167}Yb which at low rotational frequencies is roughly 30 % too large as compared to the experiments. At high frequencies, however, the particle-rotor model is seen to give an excellent description of the "favoured" band (the I=j,j+2,j+4,... spin sequence) even in the band-crossing region. It is therefore concluded in paper I that the Coriolis attenuation problem is peculiar to low rotational frequencies.

The spin alignment in the "unfavoured" band (the I=j-1,j+1,j+3,... level sequence) is not equally well reproduced by the model calculations. The difference in alignment to the "favoured" sequence appears to be too small in the model. Furthermore the experimental spin alignment in the "unfavoured" band exhibits an unexpected downward trend for increasing rotational frequencies.

The conclusions of paper I are interesting since the remaining discrepancies between theory and experiments indicate a possible importance of additional physical components in the particle-rotor model Hamiltonian. It is the objective of the papers II and III of the present thesis to consider a concrete example of such effects based upon the following well known arguments. In the BCS theory the intrinsic system is described by a Hamiltonian where the spheroidally deformed Nilsson potential and the monopole pair field manifest the quadrupole and pairing parts of the nucleonic correlations. If the system is subject to some perturbation, however, other aspects of the forces may come into play. This is the situation for a collective rotation in which case new fields, proportional in the lowest order to the rotational frequency, may be generated. It can therefore be deduced that e.g. a rotationally induced quadrupole (Y_{21}) pair field must exist in the rotating paired nuclear system [Bo75].

The Y_{21} pair field has previously been considered within the cranking model [Be61,Ha74] where it is found to contribute directly to the Coriolis interaction. Since the cranking model, as stated above, does not appropriately account for the physical recoil effect at low rotational frequencies we think it does not make sense to implement additional effects into the cranking model for the purpose of a discussion of the Coriolis attenuation anomaly. Instead, in paper II of this work, the Y_{21} pair field* is included, for the first time, in the particle-rotor model.

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* In the particle-rotor model the Y_{21} pair "field" becomes an additional coupling term between the particle and the core degrees of freedom.

The previous discussion of the Y_{21} pair field within the cranking model is applicable to other physical situations, when the recoil effect is not large. It is, however, also useful in the present context because it involves a microscopic formulation of the collective field whereby the field strength can be related to the quasiparticle interactions by a self-consistency argument. Paper II assumes the formal equivalence of the Y_{21} pair field between the particle-rotor and cranking models, together with the correspondence $\omega \leftrightarrow \mathcal{J}^{-1}\bar{R}$ between the rotational frequency in the cranking model and the core angular momentum operator in the particle-rotor model. It is further assumed that the value for the field strength, which is derived within the cranking model, can be adopted in the particle-rotor approach. No explicit calculations are performed in paper II, but it is remarked that the formal structure of the Y_{21} pair field, indicates that there may exist important contributions to the level energies at a low average rotational frequency in odd nuclei. The many-BCS-quasiparticle degrees of freedom are essential for this effect.

Paper III exhibits a schematic application, of the model put forward in paper II to some selected experimental data. The used strength of the Y_{21} pair field evaluated for a quasiparticle interaction of δ-type [Ha74], is probably an overestimate of the realistic value. The other parameters of the particle-rotor model are more appropriately chosen in a rough accordance with the scheme of paper I. Therefore the calculations in paper III can be used as an indication of the maximal influence of the rotationally induced quadrupole pair field.

The experimental data has been taken from ^{167}Er because here not only the energies of the three lowest-lying high-j rotational bands are known, but also the strengths of some electric quadrupole transitions between the bands have been measured [Ha80]. The latter quantities represent the most direct measure of the particle-rotation coupling. The conclusion made in paper III from the simultaneous consideration of both level energies and transition probabilities is, that the Y_{21} pair field may influence the coupling of an odd nucleon to a rotational core. When several bands are considered, however, it is seen that this effect alone cannot be used to remove the discrepancies between theory and experiments.

3.2 Additional features of the many-BCS-quasiparticles plus rotor model

Paper IV contains the more detailed description of the present particle-rotor model together with a survey of the many-body formalism necessary for the accurate solutions. Here a valence space constructed from the many-BCS-quasiparticle degrees of freedom is, for the first time, handled consistently within the particle-rotor model, and both even and odd nuclei are treated.

The BCS quasiparticle states, which constitute the intrinsic eigenfunctions for the non-rotating nucleus, can be employed in connection with a rotor in a rotationally invariant formalism, if the angular momentum is projected out by means of the rotational D-functions [Bo52]. In paper IV it is shown, by explicit calculations, that the high-j BCS quasiparticle states in this way provide a rapidly convergent many-body basis for the particle-rotor model over the whole low spin yrast region as long as the monopole pair field is well developed.

More realistic calculations are carried out in paper IV, for the nucleus ^{164}Er. Here, as in paper I, only parameters with a clear physical significance are used except for an ad hoc Coriolis attenuation factor. The results show, however, that the attenuation in the even-parity spectrum of ^{164}Er is small or absent. The least-squares fit is carried out with 5 parameters to reproduce 17 experimental level energies in the ground state rotational band and the rotational aligned two-quasiparticle band (the S-band) of ^{164}Er. The situation is illustrated in figure 1 where it is seen that the particle-rotor model calculation provides an excellent description of the ground band and the S-band. Another less aligned two-quasiparticle band is also nicely accounted for.

In ^{164}Er the ground state band and the S-band are observed on both sides of the crossing point as an effect of their small interaction. The interaction between the crossing bands is also related to the sharpness of the "backbending" phenomenon. A "backbending" diagram exhibits the yrast level sequence in a semiclassical manner by plotting the effective moment of inertia

$$\mathcal{J}_{eff} \equiv \frac{\hbar^2}{2} \frac{\partial\sqrt{I(I+1)}}{\partial E_I} \qquad (3.2)$$

as a function of the rotational frequency squared. The form (3.2) is exact for a perfect rotor and $\mathcal{J}_{eff}$ can be calculated from the level energies using

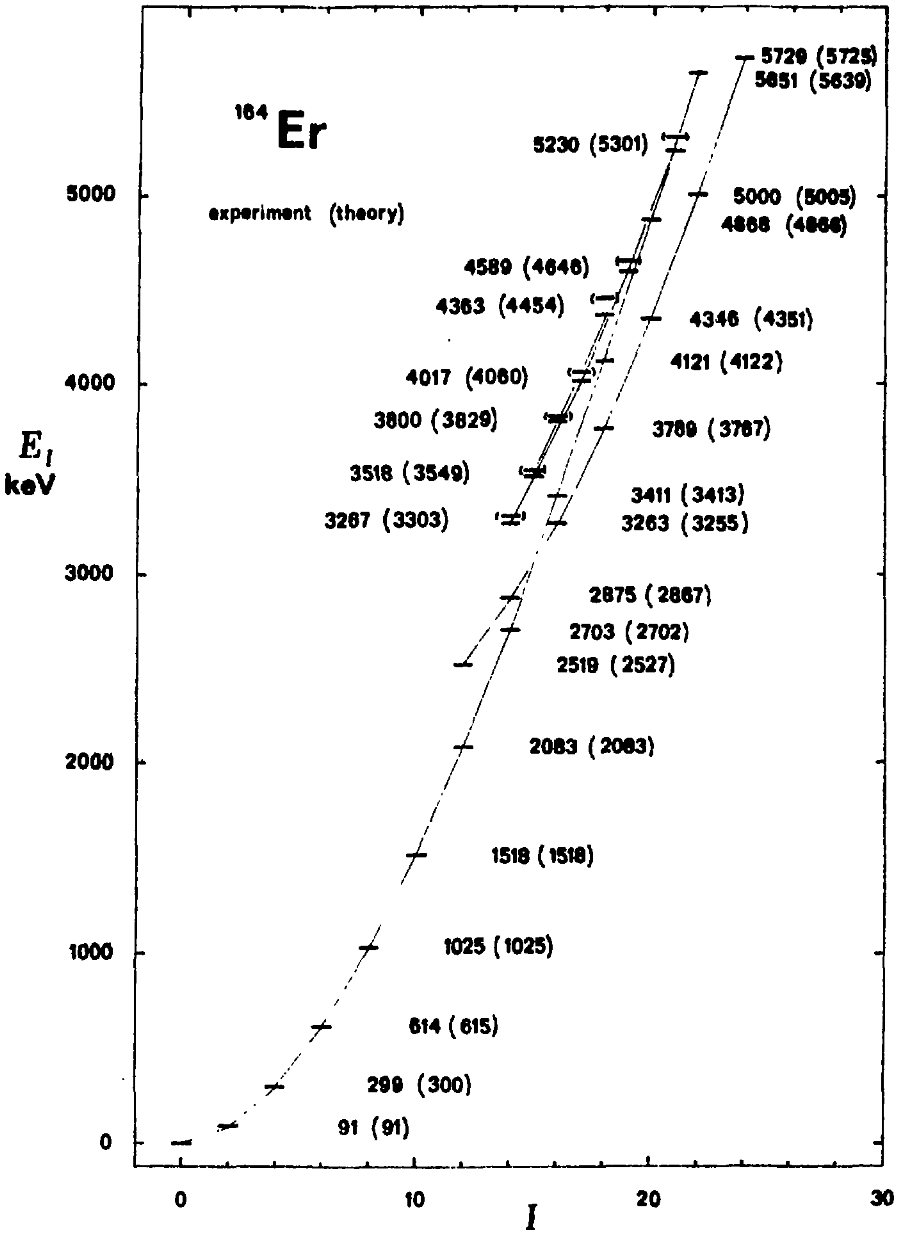

FIGURE 1 (from paper IV)

The energies of the observed positive parity levels in the nucleus ^{164}Er [Jo78] compared to the results of a particle-rotor calculation.

a finite-difference approximation. In a "backbending" diagram an abrupt change in the yrast level sequence is reflected in a sharply backbending curve while a less abrupt change in the level sequence gives an upbending behaviour.

Based on the cranking model, with a single j-shell, it has previously been shown [Be78] that the interaction matrix element between the ground state and the aligned two-quasiparticle configuration is an oscillating function of the chemical potential λ which vanishes for $(j-3/2)$ values of λ. The particle-rotor model, on the other hand, has a different description of the band-crossing process since here the two bands interact and cross for a given angular momentum I while the cranking model considers the mixing of the two bands for a given rotational frequency ω. It is not established what is the "fundamental" band-crossing process in nature.

In view of this the objective of paper V in the present thesis is to consider the interaction between the ground band and the S-band within the particle-rotor model. The interaction matrix element is taken to be half of the minimum energy separation between the two bands and is accurately determined by a variation of the model angular momentum through non-integer values. A rough agreement is then reached to the results of the cranking model. The nodes of the oscillating function are found to coincide with those of the cranking model while the peak heights are different.

The result of paper V is interesting since, in spite of the fact that the band-crossing process is fundamentally different in the particle-rotor and cranking models, the interaction between the crossing bands exhibits a common behaviour. The same oscillating behaviour is found also in other versions of the particle-rotor model which do not employ the BCS basis [Gr79,An80]. In a recent work [Hå80] the mechanism behind the interaction in the various models is briefly discussed. It is concluded that the mechanism is highly dependent on the representation chosen and what is defined as interaction free levels.

A comparison with the experimentally observed yrast spectra of rare-earth nuclei was made in ref. [Be78] and seems to confirm the oscillating character of the interaction between the ground and S-bands. However, a quantitative comparison of the peak heights which distinguish the interactions in the particle-rotor and cranking models can unfortunately not be made at the present stage.

Other band crossings than that between the ground band and the S-band in even-even nuclei have been considered earlier in a schematic manner [St75], but are also studied in paper V within the many-BCS-quasiparticles plus rotor model. It is then found, as a common feature, that the interaction matrix element between any crossing quasiparticle bands exhibit periodic fluctuations as a function of the number of valence particles.

In particular, for odd nuclei, the rotational aligned three-quasiparticle band crosses with the lowest one-quasiparticle band in a manner which oscillates out of phase with the lowest crossing in the neighbouring even nuclei. This means that the sharpest backbends or upbends for an odd nucleus would occur when the even neighbour exhibits a smooth crossing and vice versa. The available experimental information seems to confirm this point.

In paper VI the $i_{13/2}$ two-quasineutron states in the continuation of the S-band below the band crossing point are considered. It is shown that such states can be populated selectively by means of $\ell = 6$ neutron pickup reactions from an appropriate target having the odd neutron in an $i_{13/2}$ state. However, the population pattern for the various states is very sensitive to the position of the chemical potential λ.

The calculations naturally predict the largest probabilities for population of two-quasiparticle states where the collective angular momentum $\bar{R}$ is small. The total angular momentum of such states is essentially the sum of the angular momenta of the two quasiparticles, which can take any even value between 0 and 2j-1. The states favoured in the pick-up reactions, and which in the model occur at a roughly constant energy for a varying angular momentum, have then a structural similarity to the rotation aligned states of the S-band, but do not form a collective band.

Experimental investigations of the low spin continuation of the S-band are in progress. Going back to paper IV it is there remarked that the investigation of this low rotational frequency domain may provide additional valuable information on the Coriolis attenuation anomaly.

ACKNOWLEDGEMENTS

I wish to express my deepest gratitude to my friends, teachers and collaborators I. Hamamoto and G. Leander without whose inspiration, guidance and encouragement this work would not have been possible.

Also many pleasant times at NORDITA in Copenhagen, where much of the work has been done, are gratefully remembered.

Last but not least a mention is due to all my friends at the Department of Theoretical Physics and the Research Institute of Physics and in particular to Prof. J. Blomqvist, Prof. L. Hulthén, Prof. B. Nagel and Dr. S. Wahlborn for their support.

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