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by

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ABSTRACT:

A formalism is developed which makes it possible to consider the influence of the rotationally induced quadrupole pair field and corresponding quasi-particle residual interactions within the particle-rotor model. The Y_{21} pair field renormalizes both the Coriolis and the recoil interactions.

1. INTRODUCTION

In the particle-rotor description [1] of rotating nuclei the valence particles are subject to Coriolis, centrifugal and recoil forces. Many features of the band structures based on one- and two-quasiparticle intrinsic states can be explained in this way. In particle-rotor model calculations for odd mass deformed nuclei a long-standing problem [2] is that a considerable ad hoc attenuation of the Coriolis interaction seems necessary. This is evidenced by the band-mixing determined from E2 transitions and by level energy calculations. The particle-rotor model would still be useful on a quantitative footing only if it were possible to implement the necessary improvements by introducing additional terms in the Hamiltonian without too much extension of the basis space.

In this paper the formal framework for such calculations is put forward, with a particular emphasis on the rotationally induced quadrupole pair field. Such a field must exist in deformed rotating nuclei if the total pair field results from a rotationally invariant interaction [3], and it has earlier been suggested as a possible source of Coriolis attenuation [4]. The possible influence of quasiparticle residual interactions is also briefly mentioned.

2. THE MODEL

A framework for the inclusion of additional fields and two-body interactions in the valence space of the particle-rotor model is outlined in ref. 5. The basic Hamiltonian is

$$H_{PR} = \sum_{i} (\varepsilon_i - \lambda) a_i^+ a_i + \frac{1}{2}\Delta \sum_{i} s_i (a_i^+ a_{\bar{i}}^+ + a_{\bar{i}} a_i) + \Theta^{-1} \bar{R}^2$$

$$= H_o + \Theta^{-1} \bar{R}^2 \qquad (2.1)$$

and solutions are sought with the aid of the many-BCS-quasiparticle basis. In (2.1) H_o is the intrinsic Hamiltonian consisting of the deformed Nilsson potential and a BCS pair field. The notation is that of ref.5. In particular s_i is the time-reversal phase factor and $s_i|\bar{i}\rangle = T|i\rangle$. The last term in (2.1) is the bare core rotational energy, which gives couplings between the rotational mode and the intrinsic degrees of freedom (the Coriolis term) as well as additional intrinsic correlations (the recoil term).

Extensions of the basic model are possible along different lines but we shall here be concerned with the possibility of modifications induced specifically by the rotation in the pairing part of the intrinsic Hamiltonian. Such an extension has previously been considered mainly in a cranking model formulation [3,4,6].

Generally, additional rotation-particle couplings may be introduced by

$$H = H_{PR} + H_{coupling} (\bar{R}, \text{intr.}) \qquad (2.2)$$

A guideline for the nature of such couplings, originating from interactions among all nucleon degrees of freedom, would have to be a microscopic formulation of the nuclear structure problem incorporating the same concept of a collective rotation as the particle-rotor model.

Another line of investigation would be to include explicitly the quasiparticle residual interactions

$$H_o' = H_o + H_{res} \text{ (intr)} \qquad (2.3)$$

The picture of independent BCS quasiparticles in the non-rotating system may work rather well for states with less than two quasiparticles, but in case of states with two or more quasiparticles one may have to consider quasiparticle interactions. For example, quasiparticle excitations in even nuclei may be lowered into the 2Δ energy gap on account of quasiparticle interactions.

In the rotating system the states with two or more BCS quasiparticles play a role for the ground state rotational bands but also many quasiparticle rotational bands have been experimentally identified [7]. In this situation it might be appropriate to consider the modified intrinsic Hamiltonian, where H_{res} includes the residual interactions. General methods are developed in ref. [5] for the treatment of one- and two-particle interactions. It is important to realize that it may then be necessary to renormalize the parameters of H_o. For example, a residual interaction may give undesired changes of the one-quasiparticle energies in the intrinsic frame. One way to correct for this is by an a priori diagonalization of H_o' in the BCS 1+3 quasiparticle basis. The quasiparticle energies E_i' going into the a priori calculation, and ultimately into the particle-rotor model calculation, should then be adjusted so as to approximately reproduce the original one-BCS-quasiparticle spectrum E_i. It is then other properties of the new quasiparticles which may be different from those of the BCS quasiparticles.

The latter line of extension of the basic particle-rotor model need not be considered further in the present note since it is in principle straightforward apart from the renormalization problem and the question of the explicit form and strengths of the residual interaction.

3. ROTATION-INDUCED PAIR FIELD

3.1. PAIR CORRELATIONS IN THE ROTATING SYSTEM

In non-rotating nuclei the most important part of the particle-particle correlation is expressed by the pair-correlations involving pairs of time-conjugate nucleon states $|i\rangle$ and $T|i\rangle$. A rotation of a nucleus where such pair-correlations are present induces a slow collective flow in the intrinsic coordinate system. Consider now short-range attractive residual interactions having a local Galileian and rotational invariance, i.e. in particular they are invariantly described in any coordinate system which follows the local collective flow. Note that a δ-force is such an interaction but the ordinary monopole pair force is not. This is because the latter force in the intrinsic system connects only K=0 pairs. The intrinsic system is instantaneously fixed in space, and if there is a collective rotation the same pairs moving with the collective flow have also a K=1 component in the intrinsic frame. An invariant interaction would therefore have to involve also K=1 pairs.

When the interaction has the desired symmetry, a pair corresponds to two orbits with an opposite motion in the moving frame defined by the collective flow, and in the instantaneously fixed intrinsic system these orbits are no more conjugate under time reversal. The rotational motion thus induces in the intrinsic system a pair density which is not time-reversal invariant. To first order the rotational perturbation produces a K=1 pair density, which will have a Y_{21} symmetry if the non-rotating nucleus has a Y_{20} space deformation.

Let us now review the way it was introduced by Hamamoto [8] into the cranking Hamiltonian, where the rotational perturbation is given by $-\omega J_x$. This pair density has to be taken into account as a field in the intrinsic Hamiltonian.

By expressing the two-body interaction in terms of field couplings, it is seen that whereas in the non-rotating nucleus only the $K^\pi=0^+$ pair exists, a $K^\pi=1^+$ pair field proportional to ω starts to appear as soon as the system rotates [9]. The strength of the Y_{21} pair field can be related to the interaction strength by a self-consistency argument (see eqs. 3.13, 3.14).

Consider now a short-range attractive interaction of the type

$$V(\bar{r}_1,\bar{r}_2) = \sum_\alpha (-G_\alpha)\; g_\alpha^*(\bar{r}_1)\; g_\alpha(\bar{r}_2) \tag{3.1}$$

where $\{g_\alpha\}$ is a complete orthonormal set. If G_α is independent of the interaction V is precisely a δ-force. As a schematic force we shall presently consider the nodeless parts of a complete set

$$g_\alpha \equiv g_{\lambda\mu} \sim r^\lambda Y_{\lambda\mu}(\theta,\varphi) \tag{3.2}$$

of which the monopole part gives the commonly used monopole pair force. Since the main effect is expected to be associated with a nodeless radial function, the simple choice (3.2) should, however, provide a useful indication of the effects to be expected.

We shall write the force (3.1) in occupation number space

$$V = \frac{1}{4} \sum_{ijk\ell} V_{ijk\ell}\; a_i^+\; a_j^+\; a_\ell\; a_k \tag{3.3}$$

Provided there exists a local pair density, it is relevant to express the matrix elements in the "particle-particle channel" as

$$V_{ijk\ell} = \sum_{\lambda\mu} (-G_\lambda) \langle\, i|g_{\lambda\mu}^*|\bar{j}\,\rangle\langle\,\bar{\ell}|g_{\lambda\mu}|k\,\rangle\; s_j\; s_\ell \tag{3.4}$$

The "field approximation" corresponds to a substitution

$$a_i^+ a_j^+ a_\ell a_k \rightarrow a_i^+ a_j^+ \langle a_\ell a_k \rangle + \langle a_i^+ a_j^+ \rangle a_\ell a_k \qquad (3.5)$$

Contributions of V to the number-conserving fields are thereby neglected. Then the monopole pair field is

$$V_o = -\frac{\Delta}{2} \sum_{i,j} \langle i|\bar{j}\rangle s_j (a_i^+ a_j^+ + a_j a_i) \qquad (3.6)$$

By using the coupling constant G_o of the monopole pair force, we may write

$$\Delta = \frac{G_o}{2} \sum_{k,\ell} \langle \bar{\ell}|k\rangle s_\ell \langle 0| a_\ell a_k |0\rangle \qquad (3.7)$$

for the BCS vacuum state $|0\rangle$. In principle, also the g_{20} interaction will contribute to the K=0 pair field for a nucleus with the Y_{20} deformation but this contribution is here assumed to be included in Δ and the possible orbit dependence of Δ is, therefore, neglected. For the Y_{21} interaction the problem is equivalent, in the present investigation, to considering the field from the interaction

$$- G_2 \, g^* (\bar{r}_1) \, g(\bar{r}_2) \qquad (3.8)$$

with $g = 1/\sqrt{2} \, (g_{2,1} + g_{2,-1})$. Then the pair field associated with the $K^\pi=1^+$ pair density is in analogy to (2.8)

$$V_{21} = - \frac{\Delta_{21}}{2} \sum_{i,j} \langle i|g^*|\bar{j}\rangle s_j (a_i^+ a_j^+ + a_j a_i) \qquad (3.9)$$

The $K^\pi=1^+$ pair deformation parameter is

$$\Delta_{21} = \frac{G_2}{2} \sum_{k,\ell} \langle \bar{\ell}|g|k\rangle s_\ell \langle a_\ell a_k \rangle_\omega \qquad (3.10)$$

where $\langle \ldots \rangle_\omega$ expresses the expectation value in the rotationally perturbed ground state $|\rangle_\omega$.

3.2 A MICROSCOPIC FORMULATION

An estimate of the strength of the Y_{21} pair field can be made in the cranking model [8]. For this purpose, consider the cranking

Hamiltonian

$$H_c = H_o - \omega J_x + V_{21} \tag{3.11}$$

where H_o is that of eq. (2.1) describing a system of independent BCS quasiparticles with energies E_i. To lowest order in ω

$$|\rangle_\omega = |0\rangle + \frac{1}{2}\sum_{k,\ell} f_{k\ell}|k\ell\rangle \tag{3.12}$$

where the two-BCS-quasiparticle amplitudes are given by

$$f_{k\ell} = \omega \frac{<k|j_x|\bar{\ell}> s_\ell (u_k v_\ell - v_k u_\ell)}{E_k + E_\ell} + \Delta_{21} \frac{<k|g^*|\bar{\ell}> s_\ell (u_k u_\ell + v_k v_\ell)}{E_k + E_\ell} \tag{3.13}$$

The selection rule for $f_{k\ell}$ is $\Omega_k + \Omega_\ell = \pm 1$. The Y_{21} pair deformation from eqs. (3.10) and (3.12) becomes

$$\Delta_{21} = \frac{G_2}{2} \sum_{k,\ell} <\bar{\ell}|g|k> s_\ell \, f_{k\ell}(u_k u_\ell + v_k v_\ell) \tag{3.14}$$

Equations (3.13) and (3.14) define a self-consistency problem which can easily be solved by eliminating $f_{k\ell}$. The solution is

$$\frac{\Delta_{21}}{\omega} = \frac{\frac{G_2}{2}\sum_{k,\ell} \frac{<\ell|g|k><k|j_x|\ell> (u_k v_\ell - v_k u_\ell)(u_k u_\ell + v_k v_\ell)}{E_k + E_\ell}}{1 - \frac{G_2}{2}\sum_{k,\ell} \frac{|<\ell|g|k>|^2 (u_k u_\ell + v_k v_\ell)^2}{E_k + E_\ell}} \tag{3.15}$$

The expression on the right hand side can be simplified by assuming a δ-force for V, i.e. by putting $G_0 = G_2$, and using some additional approximations as discussed by Hamamoto. In particular for a quadrupole deformed harmonic oscillator potential one can use the exact relation

$$\langle i|g|j\rangle = \gamma(\varepsilon_j - \varepsilon_i)\langle i|j_x|j\rangle \tag{3.16}$$

where γ is proportional to the quadrupole deformation. Then a relevant parameter is

$$D = 2 \frac{\Delta_{21}}{\omega} \cdot \gamma \cdot \Delta \tag{3.17}$$

which was estimated by Hamamoto to be

$$D \sim 0.15 - 0.20 \tag{3.18}$$

for the rare earth nuclei when $\Delta = 1.0$ MeV. Note that $\omega \cdot D$ is proportional to the rotational frequency ω, to the quadrupole deformation and to the monopole pair deformation Δ.

In conclusion, it is possible, within the cranking model, to take account of the rotationally induced quadrupole pair field by rewriting the cranking perturbation as

$$-\omega J_x + V_{21} = -\omega (J_x + F_x) \tag{3.19}$$

Then $J_x + F_x$ expressed in the BCS quasiparticle representation becomes

$$J_x + F_x = \sum_{i,j} \langle i | j_x | j \rangle (u_i u_j + v_i v_j)(1 - P_{ij})\, \alpha_i^+ \alpha_j + \\ + \frac{1}{2} \sum_{i,j} \langle i | j_x | \bar{j} \rangle s_j (u_i v_j - v_i u_j)(1 + R_{ij})(\alpha_i^+ \alpha_j^+ + \alpha_j \alpha_i) \tag{3.20}$$

where

$$P_{ij} = \frac{D}{2} \frac{(\varepsilon_i - \varepsilon_j)^2}{E_i E_j + (\varepsilon_i - \lambda)(\varepsilon_j - \lambda) + \Delta^2} \tag{3.21}$$

$$R_{ij} = \frac{D}{2} \frac{(\varepsilon_i - \varepsilon_j)^2}{E_i E_j - (\varepsilon_i - \lambda)(\varepsilon_j - \lambda) - \Delta^2} \tag{3.22}$$

3.3 CONTRIBUTIONS TO THE PARTICLE-ROTOR HAMILTONIAN

The similarity between the particle-rotor and cranking models will now be described by the correspondence

$$\theta^{-1} (R_+ J_- + R_- J_+) \leftrightarrow \omega J_x \tag{3.23}$$

where following the arrow to the right means a field approximation for the rotation and the assumption of a sharp rotational frequency [ ]. In (3.23) $R_{\pm}$ are the ladder operators of the bare core angular momentum. Then by analogy with the Y_{21} pair field in the cranking model, this "field" should be introduced in the particle-rotor Hamiltonian by the additional particle-rotation coupling

$$H_{coupl}\,(\bar{R}, \text{intr.}) = -\,\theta^{-1}\left[\tfrac{1}{2}\{R_-, F_+\} + \tfrac{1}{2}\{R_+, F_-\}\right] \qquad (3.24)$$

Here $F_{\pm}$ are the "ladder operators"

$$F_{\pm} = F_x \pm i F_y \qquad (3.25)$$

corresponding to F_x of (3.19). The anticommutator is used in (3.24) to ensure hermicity.

The rotationally induced quadrupole pair field implies in the intrinsic system a K-conserving term in addition to the $\Delta K=1$ term of the cranking model. To see this we write $\bar{R} = \bar{I} - \bar{J}$ where $\bar{I}$ is the total angular momentum operator and $\bar{J}$ belongs to the valence degrees of freedom. Then the particle-rotor Hamiltonian may be written

$$H = H_o + \theta^{-1}\bar{R}^2 + H_{coupl} = H_o + \theta^{-1} I_{\perp}^2 + H'_{Coriolis} + H'_{recoil} \qquad (3.26)$$

The new or "apparent" Coriolis and recoil interactions are

$$H'_{Coriolis} = -\,\theta^{-1}\left[I_+(J_- + F_-) + I_-(J_+ + F_+)\right] \qquad (3.27)$$

and

$$H'_{recoil} = \theta^{-1} J_{\perp}^2 + \tfrac{1}{2}\,\theta^{-1}\left[\{J_-, F_+\} + \{J_+, F_-\}\right] \qquad (3.28)$$

The matrix elements of the "apparent" recoil interaction within the BCS quasiparticle basis may be calculated by the general methods of ref. [5]. The basic ingredients of such calculations are summarized in table I where it is seen that the matrix elements of the unperturbed recoil term are modified by the P_{ij} and R_{ij} matrices of (3.21) and (3.22).

The corresponding matrix elements of the "apparent" Coriolis term can be trivially derived from (3.20).

4. DISCUSSION AND CONCLUSIONS

The rotationally induced Y_{21} pair field has in the particle rotor model some features which are analogous to the cranking model formulation [8]. We shall first review the main effects of these coupling terms, which may be read off from the "apparent" Coriolis interaction.

The quasiparticle number conserving part of the Coriolis term is attenuated by the additional factor $(1 - P_{ij})$. The diagonal elements of P_{ij} are zero, so e.g. the correction term to the decoupling parameter in $K = 1/2$ bands vanishes. Other elements of P_{ij} are small, and in fact too small to account directly for typical phenomenological attenuation factors [8]. P_{ij} takes on a maximal value D when the orbits i and j are on opposite sides of the Fermi surface with $|\varepsilon_i - \lambda| = |\varepsilon_j - \lambda| = \Delta$.

The modifications of the quasiparticle number non-conserving parts of $J_{\pm}$, expressed by the additional factors $(1 + R_{ij})$, are in general considerably larger. An important consequence of this is the large contribution to the moment of inertia known as the Migdal term.

Other properties of the Y_{21} pair field are peculiar to the particle-rotor formulation. The crucial role is played by the contributions to the recoil term. A difference to the cranking approximation lies precisely in the exact treatment, within the particle-rotor model, of the two-body recoil term [10]. Therefore a difference between the effects of the Y_{21} pair field in the two models might be due to this term.

Calculations in the spirit of the many-BCS-quasiparticle model, including the rotationally induced quadrupole pair field and the corresponding quasiparticle residual interactions, are in progress. In a forthcoming paper [11] the particle-rotor Migdal term and the rotational alignment of quasiparticles for both low and high spins will be considered.

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TABLE CAPTION

TABLE I — Basic matrix elements of the modified recoil term

$$B = \frac{1}{2}\{J_+, J_-\} + \frac{1}{2}\{J_+, F_-\} + \frac{1}{2}\{J_-, F_+\}$$

in the BCS quasiparticle basis. The $M_{ijk\ell}$ matrix element is short for the following expression of single particle matrix elements:

$$M_{ijk\ell} = \langle i|j_+|k\rangle\langle j|j_-|\ell\rangle + \langle i|j_-|k\rangle\langle j|j_+|\ell\rangle$$

For further explanation ref. [5] can be consulted.

TABLE I

$\langle 0\|B_{00}\|0\rangle$	$\frac{1}{2}\sum_{i,j} \|\langle i\|j_x\|j\rangle\|^2 \; (u_i v_j - v_i u_j)^2 \; (1 + 2R_{ij})$
$\langle p\|B_{11}\|q\rangle$	$\frac{1}{2}\sum_{k} M_{pk,kq} \; \{ (u_p u_k + v_p v_k) \; (u_k u_q + v_k v_q) \; (1 - P_{pk} - P_{kq}) +$ $+ (u_p v_k - v_p u_k) \; (u_k v_q - v_k u_q) \; (1 + R_{pk} + R_{kq}) \}$
$\langle pq\|B_{20}\|0\rangle$	$\frac{1}{2}\sum s_q M_{pk,k\bar{q}} \; \{ (u_p u_k + v_p v_k) \; (u_k v_q - v_k u_q) \; (1 - P_{pk} + R_{kq}) -$ $- (u_p v_k - v_p u_k) \; (u_k u_q + v_k v_q) \; (1 + R_{pk} - P_{kq}) \}$
$\langle pq\|B_{22}\|rs\rangle$	$M_{pq,rs} \; (u_p u_r + v_p v_r) \; (u_q u_s + v_q v_s) \; (1 - P_{pr} - P_{qs}) -$ $- M_{pq,sr} \; (u_p u_s + v_p v_s) \; (u_q u_r + v_q v_r) \; (1 - P_{ps} - P_{qr}) +$ $+ s_q s_r M_{p\bar{r},\bar{q}s} \; (u_p v_q - v_p u_q) \; (u_r v_s - v_r u_s) \; (1 + R_{pq} + R_{rs})$
$\langle pqr\|B_{31}\|s\rangle$	$s_q M_{rp,s\bar{q}} \; (u_r u_s + v_r v_s) \; (u_p v_q - v_p u_q) \; (1 - P_{rs} + R_{pq}) +$ $+ s_p M_{qr,s\bar{p}} \; (u_q u_s + v_q v_s) \; (u_r v_p - v_r u_p) \; (1 - P_{qs} + R_{rp}) +$ $+ s_r M_{pq,s\bar{r}} \; (u_p u_s + v_p v_s) \; (u_q v_r - v_q u_r) \; (1 - P_{ps} + R_{qr})$
$\langle pqrs\|B_{40}\|0\rangle$	$s_q s_s M_{pr,\bar{q}\bar{s}} \; (u_p v_q - v_p u_q) \; (u_r v_s - v_r u_s) \; (1 + R_{pq} + R_{rs}) +$ $+ s_r s_q M_{ps,\bar{r}\bar{q}} \; (u_p v_r - v_p u_r) \; (u_s v_q - v_s u_q) \; (1 + R_{pr} + R_{sq}) +$ $+ s_s s_r M_{pq,\bar{s}\bar{r}} \; (u_p v_s - v_p u_s) \; (u_q v_r - v_q u_r) \; (1 + R_{ps} + R_{qr})$