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QCD CONSTRAINTS FOR THE ELECTROMAGNETIC FORM FACTOR OF THE PION

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ABSTRACT

Using the modulus representation, we derive constraints for the behaviour of the electromagnetic form factor of the pion in the time like region $[1\ \mathrm{GeV}^2, +\infty[$, from information given by perturbative QCD in the space like region $[-\mu^2, -\infty[$. A phenomenological μ dependent upper bound for the exponent of the first non leading logarithmic correction is deduced. Restrictions and problems of the method are discussed.

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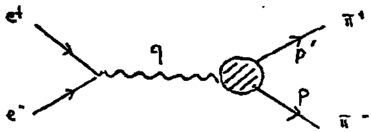
I. THEORETICAL RESULTS

1) General Results

We just recall that, in the reaction



we define the electromagnetic form factor F_π of the pion [1] by

$$\langle \pi^+ \pi^- | J_\mu(0) | 0 \rangle = (p' - p)_\mu F_\pi(q^2) \qquad (1)$$

where $\quad q = p' + p \qquad (2)$

and J_μ is the electromagnetic hadronic current.

Then we can define a function F [3] of the complex variable $s = q^2$. such that

$$\begin{cases} F_\pi(s) = \lim\limits_{\varepsilon \to 0^+} F(s + i\varepsilon) & \text{for } s \geq 4 m_\pi^2 \\ F_\pi(s) = F(s) & \text{for } s < 4 m_\pi^2 \end{cases} \qquad (3)$$

where m_π is the mass of the pion, F(s) is a real $(F(s^*) = F^*(s))$ analytic function in the complex s plane, with a cut on the real axis from $s = 4 m_\pi^2$ to $s = +\infty$, and with eventual singularities in the complex plane, we shall hereafter suppose not to exist.

Because of the strong interactions involved in the $\gamma\pi\pi$ vertex, it is only in the deep euclidian region that we might hope perturbative QCD [2] be meaningful and give some insight in the behaviour of F_π. Such results have been deduced recently [8] [9].

and we shall try to combine them with analyticity properties so as to see what are the consequences for the time-like region.

2) What information do we have on the pion form factor ?

i) There exist rather precise experimental results [4] [5] [6] from $s = 4m_\pi^2$ to $s = 1$ GeV^2 approximately. They may quite well be fitted either by Gounaris - Sakurai [7] -type formulae (modified to account for ω-ρ interference [4]) from $s = 0{,}3$ GeV^2 to $S = 1$ GeV^2, or from threshold to 0,3 GeV^2 by simple inverse power law (see ref. [6] on inverse electroproduction data).

ii) For $q^2 \to -\infty$, recent QCD computations [8] [9] predict

$$F_\pi \underset{q^2 \to -\infty}{\sim} \frac{4 f_\pi^2}{b\, Q^2 \ln \frac{Q^2}{\Lambda^2}} \left(1 + \frac{B}{\left(\ln \frac{Q^2}{\Lambda^2}\right)^\eta} + \dots \right) \qquad (4)$$

the exponents η 's are theoretically predicted to be directly related with the standard non singlet anomalous dimensions in QCD [8]

$$\eta_n = \frac{C_F}{\beta} \left\{ 1 + 4 \sum_2^{n+1} \frac{1}{k} - \frac{2}{(n+1)(n+2)} \right\}$$

$$C_F = \frac{4}{3} \qquad \beta = 11 - 2/3\, N_f \qquad (5)$$

the coefficients B's are still unknown.

One has

$$Q^2 = -q^2$$

$$f_\pi \simeq 93 \text{ MeV} \qquad (6)$$

$$b = \frac{11 - \frac{2}{3} N_f}{16\pi^2}$$

we shall take here the number N_F of quarks flavours equal to 4, and the invariant QCD scale [2]

$$\Lambda = 0{,}5 \text{ GeV}$$

Such an asymptotic estimate will be used from $q^2 = -\mu^2$ to $q^2 = -\infty$.

3) The modulus representation [10] [11]

We shall just explain here the way we incorporate the information given by perturbative QCD. The reader interested in detailed computations will refer to the appendix or to ref. [10] .
In addition to the cut from $s = 4 m_\pi^2$ to $+\infty$, we introduce a fictive cut from $s = -\mu^2$ to $-\infty$ (we shall come back later to that precise point). A first conformal mapping

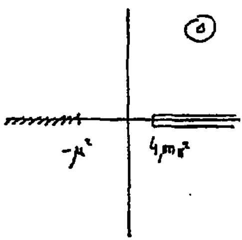
$$t = \frac{s}{s - s_0} \tag{7}$$

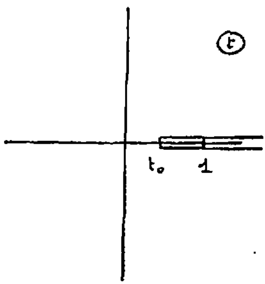
$$\text{with } s_0 = -\mu^2 \tag{8}$$

enables us to transform the two previously disconnected cuts into just one, from t_0 to $+\infty$, with

$$t_0 = \frac{4 m_\pi^2}{4 m_\pi^2 + \mu^2}$$

F(s) gives a new function $\tilde{F}(t)$.





the points $4m_\pi^2$, s_0, $\pm\infty$, are respectively mapped into t_0, $+\infty$ and $+1$.
This is the starting point of the usual technique of the modulus representation of which we shall only give here the main ideas.

A second mapping transforms the cut t plane into the unit z disc, such that the cut is mapped into the unit circle, and $\tilde{F}(t)$ into $f(z)$ defined on the disc. Then, as soon as we know the modulus of $f(z)$ on the circle (equivalent to knowing the modulus of F_π on the cut), and the positions of its zeros on the disc, we may write an explicit representation of $f(z)$ in the whole disc, up to a normalization factor. This last information is provided by the condition

$$F(0) = 1 \tag{9}$$

As far as we are concerned, the absence of any information about the zeros of F transforms this representation into an inequality, at the origin of the bound.
The final constraint is (see Appendix)

$$\frac{2m_\pi\mu}{\pi}\int_{4m_\pi^2}^{+\infty}\frac{ds'\,\ln|F_\pi(s')|}{s'\sqrt{(s'+\mu^2)(s'-4m_\pi^2)}} \geq \frac{2m_\pi\mu}{\pi}\int_{-\infty}^{-\mu^2}\frac{ds'\,\ln|F_\pi(s')|}{s'\sqrt{(-\mu^2-s')(4m_\pi^2-s')}} \tag{10}$$

and is the more general result we deduce, independent of any expressions for F_π.

Let us remark that when we take the limit $\mu\to\infty$, we find the condition

$$\frac{2 m_\pi \mu}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \; \frac{\ln \left(|F_\pi(s')| \right)}{\sqrt{s' - 4m_\pi^2}} \geqslant 0 \qquad (11)$$

which is formula (4) of ref. [13] .

II. PHENOMENOLOGICAL COMPUTATIONS

1) Reexpression of the constraint

As we want to deduce results for F_π in the region $[1\ \mathrm{GeV}^2, +\infty[$, we split the integral in the left hand side of (10) into two parts

$$\frac{2m_\pi\mu}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\,\frac{\ln|F_\pi(s')|}{\sqrt{(s'+\mu^2)(s'-4m_\pi^2)}} = \frac{2m_\pi\mu}{\pi}\int_{4m_\pi^2}^{1\,\mathrm{GeV}^2} + \frac{2m_\pi\mu}{\pi}\int_{1\,\mathrm{GeV}^2}^{\infty}$$

$$= J(\mu) + K(\mu) \qquad (12)$$

If we call

$$I(\mu) = \frac{2m_\pi\mu}{\pi}\int_{-\infty}^{-\mu^2}\frac{ds'}{s'}\,\frac{\ln|F_\pi(s')|}{\sqrt{(-\mu^2-s')(4m_\pi^2-s')}} \qquad (13)$$

then (10) may be reexpressed as

$$K(\mu) \geq I(\mu) - J(\mu) \qquad (14)$$

which is a constraint on F_π is the $[1\ \mathrm{GeV}^2, +\infty[$ region.

2) Method of Computations

There is no problem to evaluate $J(\mu)$ (see below), which extends in the domain where well known theoretical predictions fit well experimental data.

To evaluate $I(\mu)$ we shall use (4), introducing so two a priori unknown parameters B and η.

We parametrize F_π in K by choosing to use the analytic continuation of (4) with the <u>same</u> B and η.
This is only justified at infinity, that is why we just call it a parametrization.
(14) is now an implicit equation

$$F(\mu, B, \eta) \geqslant 0 \tag{15}$$

For each η and B, we have in each domain $[4 m_\pi^2, 1\ \mathrm{GeV}^2[$ and $[1\ \mathrm{GeV}^2, +\infty[$ a curve for F_π. These two curves must coincide at $s = 1\ \mathrm{GeV}^2$, $F_\pi = 1{,}45$ (point well in accordance with experiment), which gives us a new relation

$$g(B, \eta) = 0 \tag{16}$$

Combining (15), (16) we end up with a constraint

$$G(\mu, \eta) \geqslant 0 \tag{17}$$

which turns to be a μ dependent upper bound for η.

3) Numerical Computations

i) Evaluation of J

We split it into two parts

$$J_1 = \frac{2 m_\pi \mu}{\pi} \int_{4 m_\pi^2}^{.3\,\mathrm{GeV}^2} \frac{ds'}{s'} \frac{\ln |F_\pi(s')|}{\sqrt{(s'-4m_\pi^2)(s'+\mu^2)}} \tag{18}$$

where we use [6]

$$|F_\pi| = \frac{1}{1 - \frac{s}{m_\rho^2}} \qquad (19)$$

m_ρ = mass of the ρ meson = .775 GeV

and

$$J_e = \frac{2 m_\pi \mu}{\pi} \int_{.3\,\mathrm{GeV}^2}^{1\,\mathrm{GeV}^2} \frac{ds' \; \ln |F_\pi(s')|}{s' \sqrt{(s' + \mu^2)(s' - 4 m_\pi^2)}} \qquad (20)$$

here we use for (F_π) a Gounaris Sakurai [7] type formula, modified to account for $\omega-\rho$ interference [4]

$$|F_\pi(s)| = \left| \frac{m_\rho^2 \, (1 + d\,\Gamma_\rho / m_\rho)}{m_\rho^2 - s - i m_\rho^2 \frac{\Gamma_\rho}{\sqrt{s}} \left(\frac{k}{k_\rho}\right)^3} + \xi e^{i\varphi} \frac{m_\omega^2}{m_\omega^2 - s - i m_\omega \Gamma_\omega} \right| \qquad (21)$$

with m_ρ = .775 GeV $\qquad m_\omega$ = .783 GeV

Γ_ρ = .150 GeV $\qquad \Gamma_\omega$ = 10 MeV $\qquad$ (22)

m_π = .139 GeV

$$\xi = 6 \, \frac{\Gamma_{\omega \to 2\pi}^{1/2} \; \Gamma_{\omega \to e^+ e^-}^{1/2}}{\alpha \; m_\omega \; \beta_\pi^{3/2}} \; , \quad \beta_\pi = \frac{2 k_\omega}{\sqrt{s}} \qquad (23)$$

$$k = \left(\frac{s}{4} - m_\pi^2\right)^{1/2}, \quad k_\rho = \left(\frac{m_\rho^2}{4} - m_\pi^2\right)^{1/2}, \quad k_\omega = \left(\frac{m_\omega^2}{4} - m_\pi^2\right)^{1/2}$$

$$\alpha = 1/137 \qquad (24)$$

$$d = \frac{3}{\pi}\,\frac{m_\pi^2}{k_\rho^2}\,\ln\left(\frac{m_\rho + 2k_\rho}{2m_\pi}\right) + \frac{m_\rho}{2\pi k_\rho} - \frac{m_\pi^2\, m_\rho}{\pi k_\rho^3} \qquad (25)$$

with (22) , d = 0,48 , and we have taken

$$\varphi = 86° \simeq 1{,}5 \text{ rd} \qquad \Gamma_{\omega\to 2\pi} = 0{,}33 \text{ MeV} \qquad (26)$$

$$\Gamma_{\omega\to e^+e^-} = 7{,}6\ 10^{-4} \text{ MeV}$$

ii) Evaluation of I and K

As said before we take in I

$$F_\pi(s) = \frac{4 f_\pi^2}{b|s|\ln\frac{|s|}{\Lambda^2}}\left(1 + \frac{B}{\left(\ln\frac{|s|}{\Lambda^2}\right)^\eta}\right) \qquad (27)$$

and in K

$$F_\pi(s) = \frac{4 f_\pi^2}{b\, s\left(\ln\frac{s}{\Lambda^2} - i\pi\right)}\left(1 + \frac{B}{\left(\ln\frac{s}{\Lambda^2} - i\pi\right)^\eta}\right) \qquad (28)$$

For each value of η , eq.(16) turns out to be a second degree equation for B ; so, to each η are associated two values of B , one positive and one negative ; this is due to the fact that we deal only with the modulus of F_π . See in table (1) for some values of η , corresponding values of B.

4. RESULTS, COMMENTS AND PROBLEMS

Results are summarized in fig. 2 and 2', respectively for a positive ($B > 0$) and negative ($B < 0$) non leading logarithmic correction to F_π .

We have made the exponent η vary from 0 to 2.5, range reasonably allowed by experimental data (see below and figs. 3 to 6), and drawn the corresponding curves for I-J and K versus μ .

Let us recall that, to each value of η , there corresponds by (16) one positive fixed value of B and one negative. From (14) the allowed values of η are those for which the corresponding K curve is above the I-J one.

Some remarks can immediately be done

i) unlike K, I-J depends very little on μ as soon as is big enough (say 100 GeV) . We find then the unambiguous condition

$$\eta \leq .2$$

Taking lower values of μ means trusting perturbative QCD in a bigger domain, putting so more and more constraint in the problem, and thus one might hope to improve the precedent bound.

ii) However, as μ decreases, bigger and bigger values of η seem to be allowed.

One cannot argue that this is the sign of perturbative QCD breaking down at such low μ 's, for the allowed domain for η should then shrink more (asking for contradictory conditions must result in the absence of any final solution).

The root of the problem may be found at the very beginning of our study, when we introduced a fictive cut from $-\mu^2$ to $-\infty$. In this domain we asked no more for F_π to be analytic ; consequently it is less and less constrained by analyticity as μ decreases. So, on one side we ask for perturbative QCD to be trustable in a bigger and bigger domain, while on the other side F_π has constraining properties in a shorter and shorter one.

Computations have shown that the second phenomenon has the more relevant consequences and that we cannot trust final results for $\mu <$ 100 GeV.

iii) One can of course criticize the use of a QCD inspired parametrizations for F_π in a time-like region down to $\Delta = 1$ GeV^2. The only justification is comparison with experiment.

On figs. 3 and 4 we have plotted F_π curves corresponding to formula (28) for η = .2 and 2.5, together with experimental data. We see that η lying between .2 and 2.5 is perfectly in agreement with the data.

iv) In the case where we do not include non leading terms for F_π in the space like region, the I-J curve doesn't change at high μ (Notice that $\mu \gtrsim$ 100 GeV is enough).

However the presence of such terms is crucial in the time-like region ; the non sensitivity of the space like integral to the exponent η for big values of μ required (see ii)) means that we essentially test the phenomenological time-like formula (28).

III. CONCLUSION

We think the conclusions of this paper are twofold :

- first experimental data above 1 GeV^2 for the electromagnetic form factor of the pion can very well be fitted by a "QCD inspired" parametrization, including non leading logarithmic corrections of the form $\frac{B}{(\log \frac{s}{\Lambda^2})^{\gamma}}$.

- secondly we have given a phenomenological upper bound for γ, which proves to be .2. By the same way we have been faced with the problem that, introducing a fictive cut, and thus loosing analyticity properties for F_π in the space-like region entails very drastic deviations and errors from intuitive assumptions. The way to tackle this difficulty is currently under investigations.

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APPENDIX

THE MODULUS REPRESENTATION

After the first mapping (7), a second mapping

$$\mathfrak{z} = \frac{\sqrt{\frac{t}{t_0}-1} - i}{\sqrt{\frac{t}{t_0}-1} + i} = \frac{\sqrt{\frac{s}{t_0(s-s_0)}-1} - i}{\sqrt{\frac{s}{t_0(s-s_0)}-1} + i} \qquad \text{(A1)}$$

or $$\Delta = \frac{4z\cdot(-\mu^2)\ \frac{4m_\pi^2}{4m_\pi^2+\mu^2}}{4z\ \frac{4m_\pi^2}{4m_\pi^2+\mu^2}+(1-z)^2}\tag{A2}$$

transforms the "cut" plane from t_0 to $+\infty$ into the unit disc $|z| \leq 1$ the "cut" $[t_0\ ,+\infty[$ being mapped into the circle $|z| = 1$ $F(\Delta)$ is successively transformed into $\bar{F}(t)$ and $f(z)$.

From precedently written analytic properties of F and from field-theoretically deduced bounds [12], one can show that $f(z)$ satisfies

$$\int_0^{2\pi} \ln^+ |f(re^{i\theta})|\, d\theta \qquad \text{bounded when} \quad r \to 1 \tag{A3}$$

where

$$\ln^+(x) = \begin{cases} \ln x & \text{if} \quad x > 1 \\ 0 & \text{if} \quad x \leq 1 \end{cases} \tag{A4}$$

and so may be factorised in the form [11]

$$f(z) = G(z)\, B(z)\ \frac{S_1(z)}{S_2(z)} \tag{A5}$$

with :

i)

$$G(z) = e^{i\gamma} \exp\left\{ \frac{1}{2\pi} \int_0^{2\pi} \frac{e^{i\theta}+z}{e^{i\theta}-z}\ \ln |f(e^{i\theta})|\, d\theta \right\} \tag{A6}$$

which takes into account all the information we have on the modulus of F_π on the cut.

ii) B(z) is the so-called "Blaschke product", [11] factorising all the zeros of f which are not on the cut

$$B(z) = z^m \prod_n \frac{|a_n|}{a_n} \left(\frac{a_n - z}{1 - a_n^* z} \right) . \tag{A7}$$

with m non negative integer, and

$$0 \leq a_1 \leq a_2 \ldots\ldots \leq 1 \tag{A8}$$

iii) $S_1(z)$ and $S_2(z)$ are singular inner functions of the form

$$S_\alpha(z) = \exp \left\{ - \int_0^{2\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} \, d\mu_\alpha(\theta) \right\} \tag{A9}$$

where $\mu_\alpha(\theta)$ are bounded singular non decreasing functions ($\frac{d\mu_\alpha(\theta)}{d\theta} = 0$ almost everywhere).

This contribution accounts for the eventual singularities and zeros of the form factor on the cut.

If we take for $\mu(\theta)$ the simplest singular functions

$$\mu(\theta) = c \; Y(\theta - \theta_0) \tag{A10}$$

(Y is the Heaviside step function), we see that

$$S(z) = \exp \left\{ -c \, \frac{e^{i\theta_0} + z}{e^{i\theta_0} - z} \right\} \tag{A11}$$

S(z) is then allowed to vanish for $z = e^{i\theta_0}$, which supposes the existence of a pole of F_{π} on the cut. Eliminating this possibility, we take

$$S_2 = 1 \tag{A12}$$

As F may have zeros on the cut, we are led to take

$$\mu_1 = \sum_i c_i \; Y(\theta - \theta_i) \tag{A13}$$

$$c_i \geq 0$$

so that :

$$S_2 = \prod_i \exp\left\{ -c_i \frac{e^{i\theta_i} + z}{e^{i\theta_i} - z} \right\} \tag{A14}$$

and we may finally write

$$f(z) = e^{i\gamma} \exp\left\{ \frac{1}{2\pi}\int_0^{2\pi} d\theta \; \frac{e^{i\theta}+z}{e^{i\theta}-z} \; \ln|f(e^{i\theta})| \right\} \times$$

$$z^m \prod_n \frac{|a_n|}{a_n} \left(\frac{a_n - z}{1 - a_n^* z} \right) \prod_i \exp\left\{ -c_i \frac{e^{i\theta_i}+z}{e^{i\theta_i}-z} \right\} \tag{A15}$$

We must then go back to the t plane and then to the s plane. The first step is done by using (A1), and the reality property of $\overline{F}$ when noticing that as θ goes from 0 to 2π, the cut is moved on twice, and leads to

$$\overline{F}(t) = \exp\left\{ -i \frac{\sqrt{t - t_0}}{\pi} \int_{t_0}^{\infty} dt' \; \frac{\ln|\overline{F}(t')|}{(t'-t)\sqrt{t'-t_0}} \right\} \times \overline{B}(t)$$

$$\times \prod_i \exp\left\{ i c_i \frac{\sqrt{\frac{t}{t_0}-1}\sqrt{\frac{t_i}{t_0}-1} + 1}{\sqrt{\frac{t_i}{t_0}-1} - \sqrt{\frac{t}{t_0}-1}} \right\} \tag{A16}$$

where $\bar{B}(t)$ is the expression of the Blaschke product in the t plane, and the t_i are the inverse transformed of $e^{i\theta_i}$ by (A1).

Finally, in the s plane, we get

$$F(s) = \exp\left\{ \frac{-i}{\pi} \sqrt{\frac{s_0(4m\pi^2-s)}{(4m\pi^2-s_0)(s-s_0)}} \int_{4m\pi^2}^{\infty} ds' \frac{s_0-s}{(s'-s_0)(s-s')} \frac{\ln|F(s')|}{\sqrt{\frac{s_0(4m\pi^2-s')}{(4m\pi^2-s_0)(s'-s_0)}}} \right.$$

$$\left. - \frac{i}{\pi} \sqrt{\frac{s_0(4m\pi^2-s)}{(4m\pi^2-s_0)(s-s_0)}} \int_{-\infty}^{-\mu^2} ds' \frac{s_0-s}{(s'-s_0)(s-s')} \frac{\ln|F(s')|}{\sqrt{\frac{s_0(4m\pi^2-s')}{(4m\pi^2-s_0)(s'-s_0)}}} \right\}$$

$$\times B(s) \times \prod_i \exp\left\{ i c_i \frac{\sqrt{\frac{s}{(s-s_0)t_0}-1}\sqrt{\frac{t_i}{t_0}-1}+1}{\sqrt{\frac{t_i}{t_0}-1}-\sqrt{\frac{s}{t_0(s-s_0)}-1}} \right\} \tag{A17}$$

We see that the $\int_{t_0}^{\infty}$ in the t plane has splitted into $\int_{4m\pi^2}^{\infty} + \int_{-\infty}^{-\mu^2}$ in the s plane.

ii) Normalization condition and final inequality

B is known to have the property [11]

$$B(0) < 1 \tag{A18}$$

and F_π must verify [1]

$$F(0) = 1 \tag{A19}$$

(from which m must be taken equal to zero).

Taking then the logarithm of the two members of (A19) for

$s = 0$

we get :

$$0 = \ln B(0) - \sum c_i + \frac{2m_\pi}{\pi}\left\{ \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\ln|F_\pi(s')| \ \mu}{\sqrt{(s'-s_0)\ (s'-4m_\pi^2)}} + \int_{-\infty}^{-\mu^2} \frac{ds'}{s'} (-1) \frac{\ln|F_\pi(s')| \ \mu}{\sqrt{(4m_\pi^2-s')(s_0-s')}} \right\}$$

(A20)

and, as $C_i \geqslant 0$

$$\frac{2m_\pi \mu}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\ln|F_\pi(s')|}{\sqrt{(s'+\mu^2)(s'-4m_\pi^2)}} \geqslant \frac{2m_\pi \mu}{\pi}\int_{-\infty}^{-\mu^2} \frac{ds'}{s'} \frac{\ln F_\pi(s')}{\sqrt{(-\mu^2-s')(4m_\pi^2-s')}} \quad \text{(A21)}$$

which is formula (10).

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FIGURE CAPTIONS

Figure 1. Graphical Transcription of Numerical Results
Case $B > 0$

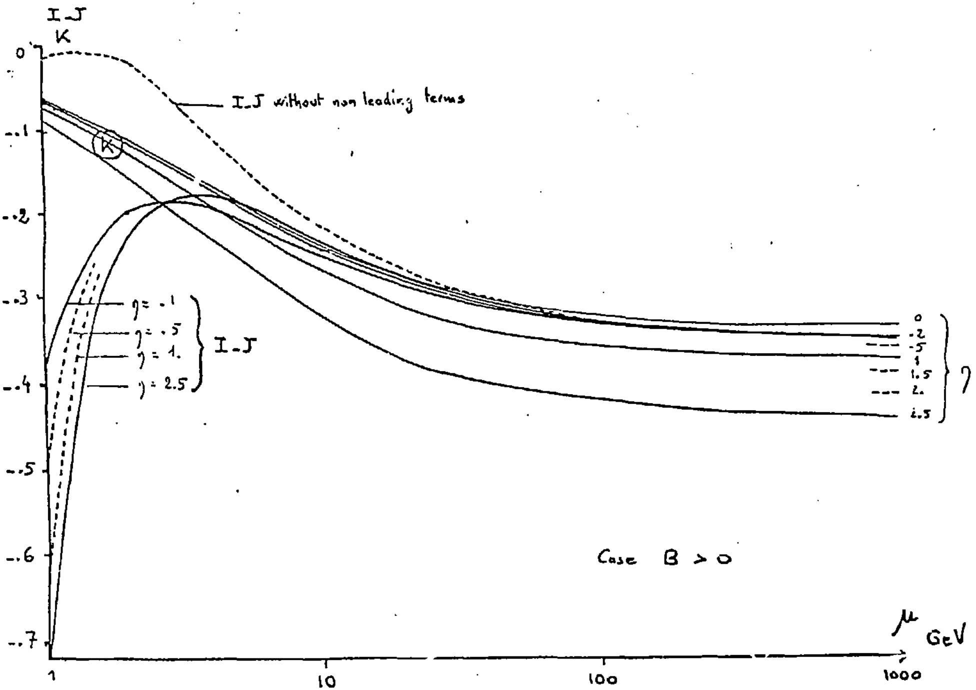
Figure 2. Graphical Transcription of Numerical Results
Case $B < 0$

Figure 3. Modulus of (F_π) according to in the Time-Like Region
Case $\eta = .2$
Comparison with Experimental Data.

Figure 4. Modulus of (F_π) according to in the Time-Like Region
Case $\eta = 2.5$
Comparison with Experimental Data.

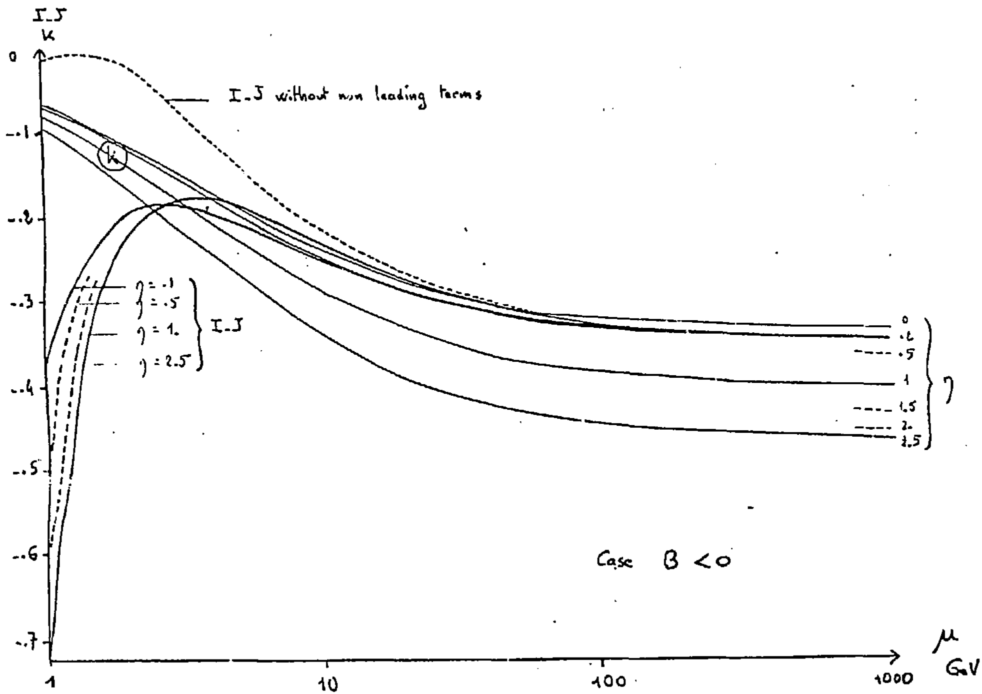
TABLE 1. Corresponding Values of B for Some Values of the Exponent η.

Footnote. In Figures 3 and 4, NOV 1972 refers to ref. [15] .
BCF 1973 refers to ref. [16] .



I-J
K
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
I-J without non leading terms
K
η = .1
η = .5
η = 1.
η = 2.5
I-J
0
.2
.5
1
1.5
2.
2.5
η
Case B > 0
μ GeV
1
10
100
1000

Fig. 1



I-J
K
0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
I-J without non leading terms
K
η = .1
η = .5
η = 1.
η = 2.5
I-J
η
Case B < 0
μ
GeV
1
10
100
1000

Fig. 2

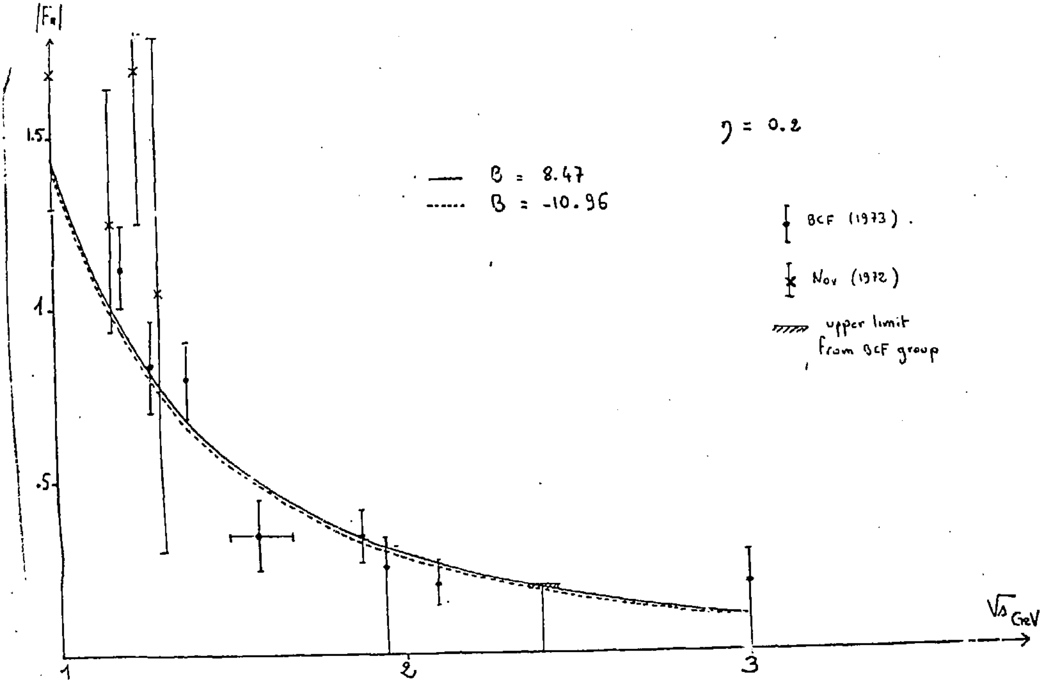

|Fπ|
1.5
1
.5
η = 0.2
B = 8.47
B = -10.96
BCF (1973)
Nov (1972)
upper limit from BCF group
√s GeV
1
2
3

Fig. 3

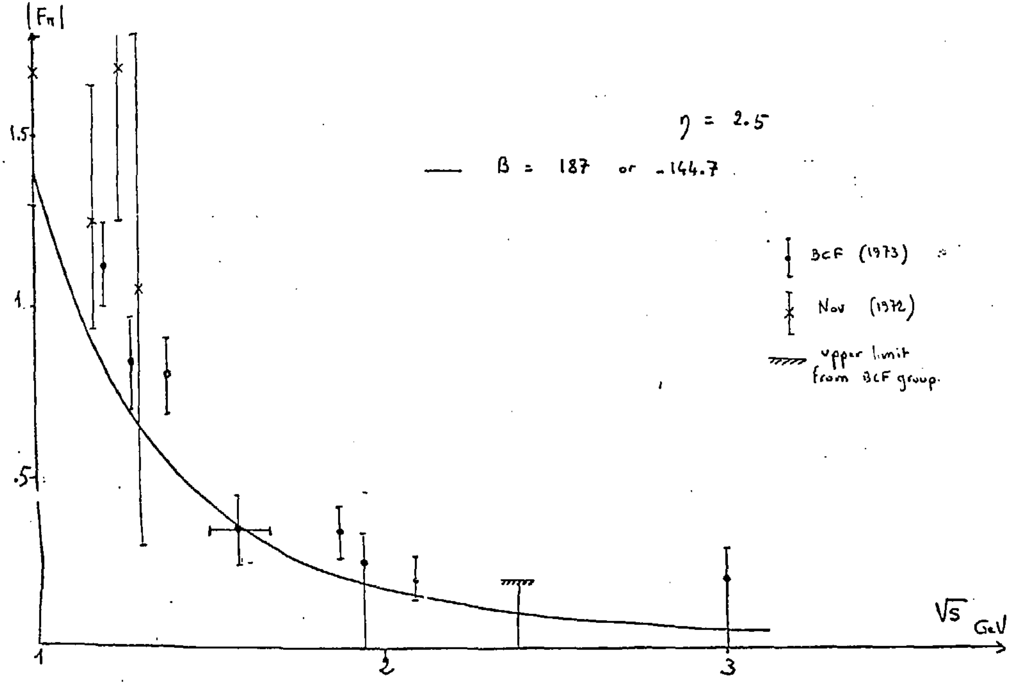

|Fπ|
1.5
1.
.5
η = 2.5
β = 187 or -144.7
BCF (1973)
Nov (1972)
upper limit from BCF group.
√s GeV
1
2
3

Fig. 4

- TABLE 1 -

VALUES OF B COEFFICIENTS CORRESPONDING TO
SOME VALUES OF η

-:-:-:-:-

η	0	.2	.5	1	1.5	2	2.5	3	4	5
$B > 0$	6.59	8.47	12.48	24.5	48.94	97.1	187	345.62	1059.54	3201
$B < 0$	-8.59	-10.96	-15.59	-27.28	-46.89	-81.18	-144.70	-268.86	-1034.13	-4305.94