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VIOLATIONS OF THE CALLAN-GROSS RELATION AS FUNCTION OF X,Q^2 FROM QCD [x]

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ABSTRACT We consider the behaviour of $R(x,Q^2) \sim \sigma_L/\sigma_T$ as $x \to 0,1$ from QCD. The simplest parametrisation compatible with the behaviour found (exact at $x = 0,1$) is

$$R(x,Q^2) = \frac{4\alpha_c(Q^2)}{3\pi}\left\{\frac{1-x}{2+\lambda_s}\left[1+\frac{3n_f}{2}(1-x)^2\,\frac{d^{+}(1+\lambda_s)-D^{11}(1+\lambda_s)}{(3+\lambda_s)\,D^{12}(1+\lambda_s)}\right]\right.$$

$$\left.+\frac{x}{1+\nu_0-(16/3\beta_0)\log\alpha_c(Q^2)}\right\}(1-x); \quad \text{as } Q^2\to\infty.$$

d^{+}, D^{ij} are the largest eigenvalue and the matrix elements of $D = -\gamma_0(n)/2\beta_0$ evaluated at $n = 1+\lambda_s$. $\tilde{\nu}_0 = 2.1 \pm 0.2$ and $\tilde{\lambda}_s = 0.37 \pm 0.07$ are known from existing fits to F_2 alone. For $0 \lesssim X \lesssim 0.1$ our calculation predicts $R(X,Q^2) \simeq (1.8 \pm 0.3)/\log Q^2/\Lambda^2$; experimentally, $R^{exp}(X,Q^2) = (1.2 \pm 0.4)/\log Q^2/\Lambda^2$.

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The Callan-Gross relation[1], originally based on the parton model, predicts that, in (say) electroproduction, $\sigma_L/\sigma_T = 0$. When QCD corrections are taken into account this relation gets modified by terms of order $\alpha_c(Q^2) = 12\pi/\{(33-2n_f)\log Q^2/\Lambda^2\}$, $\Lambda = 0.45$ GeV. We define $R \sim \sigma_L/\sigma_T$ to measure the violation of the Callan-Gross relation:

$$R(x,Q^2) = \frac{F_2(x,Q^2) - F_1(x,Q^2)}{F_1(x,Q^2)} \tag{1}$$

with F the usual electroproduction structure functions. From QCD we then have[2,3]

$$F_2^{NS}(x,Q^2) - F_1^{NS}(x,Q^2) = \frac{4\alpha_c(Q^2)}{3\pi}\int_x^1 dy\, \frac{x^2}{y^3}\, F_2^{NS}(y,Q^2); \tag{2.a}$$

$$F_2^S(x,Q^2) - F_1^S(x,Q^2) =$$

$$= \frac{4\alpha_c(Q^2)}{3\pi}\left\{\int_x^1 dy\, \frac{x^2}{y^3}\, F_2^S(y,Q^2) + \frac{3}{2} n_f \int_x^1 dy\, \frac{x^2}{y^3}\left(1-\frac{x}{y}\right) F_2^G(y,Q^2)\right\} \tag{2.b}$$

where F^S is the quark singlet, F^G the glue and F^{NS} the non-singlet structure functions. In the available literature[3,4] R is usually calculated by assuming approximate functional forms for the F_i^a, $i = 1,2$; $a = S,G,NS$ at a given Q_0^2 and using the QCD evolution equations to calculate the F_i^a, and hence R, numerically at other Q^2. In this note we remark that one can obtain exactly the behaviour of $R(x,Q^2)$ at the endpoints $x = 0,1$ from QCD. This will incidentally allow us to write a simple and explicit parametrization, depending on quantities obtained from fits to F_2 alone, which is exact at $x = 0,1$ and a good approximation at all other x. We will carry out the analysis for proton targets and e, μ projectiles; the extension to ν scattering or other targets is straightforward. It is convenient to consider separately the contributions to R of F_S and F_G in (2.b), writing R_S and R_G.

Behaviour at the endpoints x = 0,1.- As $x \to 1$, both singlet and nonsinglet parts of the F_j behave as[5,6] ($j = S,NS$) $\alpha_c^{-d_0}(1-x)^{\nu_j(\alpha_c)}/\Gamma(1+\nu_j(\alpha_c))$, with $\nu_j(\alpha_c) = \nu_{j0}-[16/(33-2n_f)]\log \alpha_c$; but the glue function vanishes faster by a power of 1-x. If $\nu_0 \simeq \nu_{NS0} \simeq \nu_{S0}$ then we can write :

$$R_{S+NS}(x,Q^2) \underset{x\to 1}{\simeq} \frac{4\alpha_c(Q^2)}{3\pi}\,\frac{1-x}{1+\nu(\alpha_c)}$$

$$R_G(x,Q^2) \underset{x\to 1}{\simeq} \frac{4}{5\pi}\alpha_c(Q^2)\,\frac{n_f\,(1-x)^3}{(1+\nu(\alpha_c))(2+\nu(\alpha_c))(3+\nu(\alpha_c))}\,\delta$$

$$\nu(\alpha_c) = \nu_0 - \left[16/(33-2n_f)\right]\log\alpha_c(Q^2)$$

where $\delta = F_S/(F_S + F_{NS})$ as $x \to 1$.

As $x \to 0$ the dominating piece is the singlet. We will assume a Regge-like behaviour for F_2^j, $j = S,G$ now :

$$F_2^j(x,Q^2) \simeq B^j(Q^2)\,x^{-\lambda_s}\,, \quad \lambda_s > 0 \qquad (4)$$

The proof that the λ_s are the same for $j = S,G$ and that λ_s is independent of Q^2 and strictly positive may be found in ref. 6 to leading and next-to-leading order in QCD. The phenomenological relevance of a behaviour like (4) is discussed there as well as in ref. 7. For behaviours alternative to (4), see refs. 6, 7 and below here.

The QCD equations for the moments of the F^j are[8]

$$\begin{pmatrix}\mu^S(n,Q^2)\\ \mu^G(n,Q^2)\end{pmatrix} = \exp\left\{\underset{\sim}{D}(n)\,\log\frac{\alpha_c(Q_0^2)}{\alpha_c(Q^2)}\right\}\begin{pmatrix}\mu^S(n,Q_0^2)\\ \mu^G(n,Q_0^2)\end{pmatrix}$$

where $\underset{\sim}{D}$ is related to the anomalous dimension matrix:

$$\underset{\sim}{D}(n) = -\underset{\sim}{\Gamma_0}(n)/2\beta_0 = \frac{16}{33-2n_f}$$

$$\times \begin{pmatrix} 3/4 + \frac{1}{2n(n+1)} - S_1(n) & \frac{3n_f}{8}\,\frac{n^2+n+2}{n(n+1)(n+2)} \\ \frac{n^2+n+2}{2n(n^2-1)} & \frac{9}{4}\left(\frac{1}{n(n-1)} + \frac{1}{(n+1)(n+2)} - S_1(n)\right) + \frac{33-2n_f}{16} \end{pmatrix};$$

(5)

$$S_1(n) = \sum_{j=1}^{n} \frac{1}{j} = n \sum_{K=1}^{\infty} \frac{1}{K(n+K)}$$

Let U(n) be the (in general, non-unitary) matrix that diagonalizes D(n), $d_\pm(n)$ its eigenvalues. By taking $n = 1+\lambda_s+\varepsilon$, $\varepsilon \to 0$ it follows that the combination

$$\begin{pmatrix} [\alpha_c(Q^2)]^{d_+(1+\lambda_s)} & 0 \\ 0 & [\alpha_c(Q^2)]^{d_-(1+\lambda_s)} \end{pmatrix} U^{-1}(1+\lambda_s) \begin{pmatrix} B^S(Q^2) \\ B^G(Q^2) \end{pmatrix}, \quad (6)$$

is independent of Q^2. For $\lambda_s \lesssim 0.7$, $d_+(1+\lambda_s) > 1+d_-(1+\lambda_s)$; hence, to leading order we may neglect $\alpha_c^{-d_-}$ versus $\alpha_c^{-d_+}$. *Solving for the* B^j *from (6), inserting into (4) and calculating* U$(1+\lambda_s)$ in terms of the D^{jk}, we find

$$F_2^j(x,Q^2) \underset{x\to 0}{\simeq} [\alpha_c(Q^2)]^{-d_+(1+\lambda_s)}\, x^{-\lambda_s}\, \hat{B}^j \quad , \quad (7)$$

with the $\hat{B}$ independent of Q^2 and

$$\hat{B}^G/\hat{B}^S = \frac{d_+(1+\lambda_s) - D^{11}(1+\lambda_s)}{D^{12}(1+\lambda_s)} \quad (8)$$

Using now (2.b), we finally get

$$R_S(x,Q^2) \underset{x\to 0}{\simeq} \frac{4}{3\pi}\alpha_c(Q^2)\,\frac{1}{2+\lambda_S}$$

$$R_G(x,Q^2) \underset{x\to 0}{\simeq} \frac{4\,\alpha_c(Q^2)}{3\pi}\,\frac{1}{(2+\lambda_S)}\cdot\frac{3n_f}{2}\;\frac{d_+(1+\lambda_S)-D^{11}(1+\lambda_S)}{(3+\lambda_S)D^{12}(1+\lambda_S)} \tag{9}$$

λ_S was obtained in refs. 6,7 from fits to F_2 alone:

$$\lambda_S \simeq 0.37 \pm 0.07$$

Eqs. (3), (9) fix R at x = 0,1 in terms of the two known parameters ν_0, λ_S.

Parametrization.- The simplest functional form compatible with (3), (9) is, for Q^2 large, $R = R_S + R_G$ (we forget the nonsinglet for the moment),

$$R(x,Q^2) = \frac{4\alpha_c(Q^2)}{3\pi}\left\{\frac{x}{1+\nu(\alpha_c)} + \frac{3}{5}\,\frac{n_f\,\delta\,(1-x)'x}{(1+\nu(\alpha_c))(2+\nu(\alpha_c))(3+\nu(\alpha_c))}\right.$$

$$\left. + \frac{1-x}{2+\lambda_S}\left[1+\frac{3n_f}{2}(1-x)^2\,\frac{d_+(1+\lambda_S)-D^{1}(1+\lambda_S)}{(3+\lambda_S)\,D^{2}(1+\lambda_S)}\right]\right\}(1-x); \tag{10}$$

For Q^2 small and medium, functions which are not pure singlet get a large contribution from the nonsinglet part for x small, but not very small. This may be taken into account by assuming the behaviour[6]

$$F_2^{NS}(x,Q^2) \underset{x\to 0}{\simeq} B^{NS}\left[\alpha_c(Q^2)\right]^{-D^{11}(1-\lambda)}\quad x^{\lambda}$$

$$\lambda = 1-\alpha_\rho(0) \simeq 0.55 \pm 0.05$$

This gives for "R_{NS}"(x,Q^2) the behaviour $(4\alpha_c/3\pi)/(2-\lambda)$, as $x\to 0$. How large x has to be for this to mask eq. (9) depends on Q^2, and the relative size $r = F^{NS}/F^S$. Considering the full $F = F^S+F^{NS}$ as $x \to 0$ we find that eq. (10) is modified to (we forget the negligible contribution of the glue as $x \to 1$)

$$R(x,Q^2) = \frac{4\alpha_c(Q^2)}{3\pi}\left\{(1-x)\ \frac{\frac{1}{2+\lambda_s}\left[1+\frac{3n_f}{2}(1-x)^2\frac{d_+(1+\lambda_s)-D_{11}(1+\lambda)}{(3+\lambda_s)\,D^{12}(1+\lambda_s)}\right]}{1+r\left[\alpha_c(Q^2)\right]^{\mu}x^{\lambda+\lambda_s}}\right.$$

$$+\ \frac{(1-x)}{2-\lambda}\ \frac{r\left[\alpha_c(Q^2)\right]^{\mu}x^{\lambda+\lambda_s}}{1+r\left[\alpha_c(Q^2)\right]^{\mu}x^{\lambda+\lambda_s}}\ +$$

$$\left.+\ \frac{x}{(1+\nu(\alpha_c))}\right\}(1-x) \quad ; \qquad (11)$$

$$\mu = d_+(1+\lambda_s) - D''(1-\lambda)$$

Discussion.- What is interesting about our formulas is that they are explicit and only contain parameters which were determined previously, so in fact they constitute a calculation of R, exact at the endpoints. A first test of the quality of our result is obtained by considering the x = 0 region at moderate Q^2 (n_f = 3). From eq. (11) we obtain

$$R(x,Q^2) \underset{x\to 0}{\simeq} \frac{1.8 \pm 0.35}{\log\frac{Q^2}{\Lambda^2}}$$

to be compared with the experimental value at small $x \lesssim 0.1$,

$$R^{exp} \simeq \frac{1.2 \pm 0.4}{\log\frac{Q^2}{\Lambda^2}}$$

measured in μ scattering at Fermilab[9]. This is a nice confirmation of the hypothesis[6,7] that, at large Q^2, the leading Regge trajectory is not the Pomeron but another one with intercept $\alpha_L(0) = 1+\lambda_s \simeq 1.4$. The predictions of eq. (10) and the experimental data[9,10] for a set of x, Q^2 values are shown in Fig. 1. The worst known parameter is $r = 12$ to 60. Luckily, the dependence of the prediction on r is not strong (cf., Fig. 1).

One may wonder what happens if we relax the Regge-like hypothesis, eq. (4). We may consider, for example, the non-Regge behaviour discussed in ref. 11. In this case and after some work, we get that eq. (9) is replaced by

$$R_{n\text{-}R}(x,Q^2) \underset{x\to 0}{\simeq} \frac{5\alpha_c(Q^2)\,(\log 1/x)^{1/2}}{9\pi n_f \left\{a_0 - (\log \alpha_c)/(33-2n_f)\right\}^{1/2}} \tag{12}$$

a_0 is a parameter that could be obtained from low x fits to F_2. Comparison with experiment must wait until somebody performs such fits, providing us with the corresponding value of a_0.

Another question is the inclusion of effects like higher twists, target mass corrections and the like. We could have tried to take them into account as in ref. 4; but since the corresponding theory is not fully developed, we prefer to limit the validity of our calculations to a region where the correction be small: $x \le x_{max}$, with x_{max} such that $(\bar{\mu}^2/Q^2) \ll x_{Max}^{-1}(1-x_{Max})$, $\bar{\mu}$ a typical hadronic mass. Actually, all available experimental points fall well inside this allowed region.

We would like to finish by suggesting to experimentalists the use of eqs.[12] (10) to (12) to extract F_1, F_2 from the data by simultaneous fits to F_2, R which depend upon the same parameters, ν_0, λ_s. Particularly important are the values of R near x = 0, which provide stringent tests on second order QCD calculations and put tight constraints on the quark singlet and glue structure functions where they are largest. Clearly, these measurements are much easier to implement experimentally than the measurements of high moments ($n \geq 3$) of F_L recently proposed[13] as a test of F_2^G, since such measurements are sensitive to the $x \to 1$ region where F_L is extremely small, and, moreover, the contribution of F_2^G to F_L is much smaller than that coming from F_2^S.

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12.- In this context we note that although eq. (10) is only exact at x = 0,1, it provides a good approximation at all x. We can assume an explicit functional form for F_2 and using eqs. (2), get R exactly; the result lies slightly below that obtained with parametrizations (10), (11).

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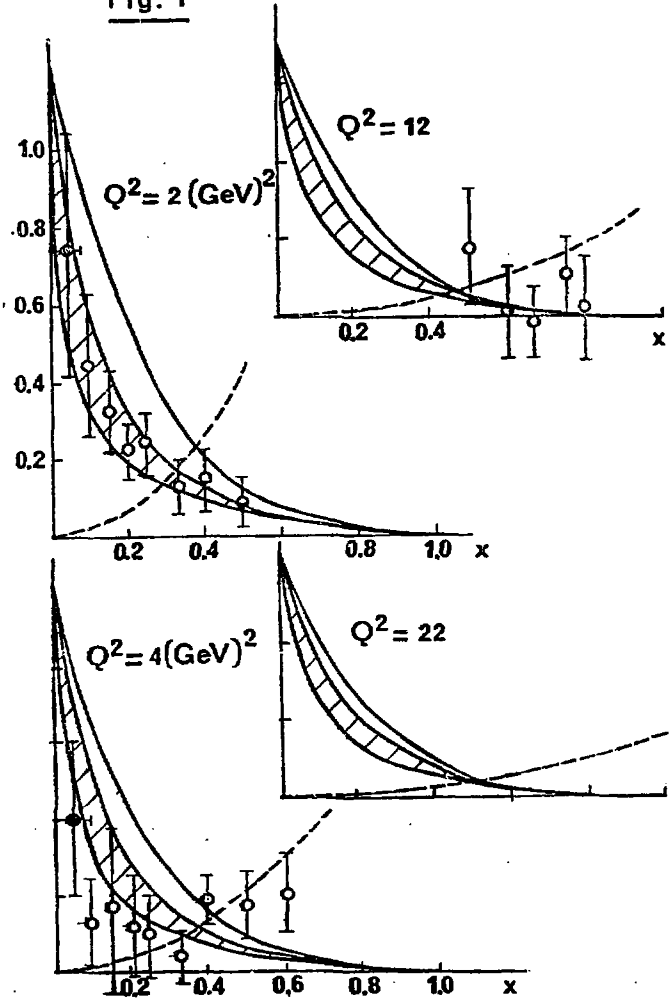
Figure Caption

Fig. 1.- Theoretical calculation of $R(x,Q^2)$ using (10), and experimental data from refs. 10, 11. $\Lambda = 0.45$ GeV, $\lambda_g = 0.37$. Solid line: eq. (10). Hatched area: eq. (11) with an r varying between 12 and 60. Dashed line : $4\,M_p^2x^2/Q^2 = R^{exp} - F_L/F_2$.

Fig. 1



Q²= 2 (GeV)²
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.8
1.0
x
Q²= 12
0.2
0.4
x
Q²= 4(GeV)²
0.2
0.4
0.6
0.8
1.0
x
Q²= 22