THE RECURSIVE MONTE CARLO METHOD FOR DEEP-PENETRATION PROBLEMS

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ABSTRACT

The Recursive Monte Carlo (RMC) method developed for estimating importance function distributions in deep-penetration problems is described. Unique features of the method, including the ability to infer the importance function distribution pertaining to many detectors from essentially a single M.C. run and the ability to use the history tape created for a representative region to calculate the importance function in identical regions, are illustrated. The RMC method is applied to the solution of two realistic deep-penetration problems—a concrete shield problem and a Tokamak major penetration problem. It is found that the RMC method can provide the importance function distributions required for importance sampling, with accuracy which is suitable for an efficient solution of the deep-penetration problems considered. The use of the RMC method improved, by one to three orders of magnitude, the solution efficiency of the two deep-penetration problems considered: a concrete shield problem and a Tokamak major penetration problem.

1. INTRODUCTION

The Recursive Monte Carlo (RMC) method is being developed for the estimation of adjoint functions distributions in general three-dimensional geometries; these adjoint distributions are aimed for importance sampling in the course of solution of deep-penetration problems using Monte-Carlo (M.C.) techniques. The idea for a RMC method to calculate adjoint functions was first proposed by Steinberg, Kalos and Troubetzkoy who also looked at its feasibility for the solution of a simple one-dimensional (slab geometry) problem. Their effort was focused on an attempt to develop an automatic algorithm for the generation of equi-

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importance surfaces in general 3-D problems - a task which was found to be inefficient\(^2\). Our approach is to divide the system, intuitively, into regions of convenient geometry, and estimate the value of the importance function on the surfaces separating these regions. More on the background of the RMC method, its theoretical foundation and strategy for implementation can be found in Ref. 3. That reference considers monoenergetic examples only. The primary purpose of this work is to illustrate the applicability of the RMC method for the solution of realistic multi-group deep-penetration problems.

For the convenience of the reader we start with a brief review of the theoretical foundation of the RMC method (Section 2) and the strategy for its application (Section 3). The RMC method is then applied for the solution of two deep-penetration problems having streaming ducts: the right circular concrete shield problem studied by Tang et al.\(^4\) (Section 4) and a blanket-shield problem with a major penetration as encountered in Tokamak reactor designs\(^5\) and proposed as a benchmark problem\(^6\) (Section 5).

All of the RMC calculations reported on are performed using the recently developed REMOP code.\(^7\) The detectors' responses are calculated with MORSE\(^8\) (using the RMC results for importance sampling). The computer times quoted are of a CYBER-73 computer.

2. THEORETICAL FOUNDATION

Consider radiation transport problems the objective of which is to find the value of a performance parameter (or a detector response, or system characteristics) that can be expressed as

\[ R = \int \chi(p)f(p)dp \]  

where \( \chi(p) \) is the neutron birth-rate density distribution - the density of neutrons coming out from collisions per unit phase space volume, dp, at \( p \equiv (r,E,\Omega) \), per unit time; it is the solution of the Boltzmann equation

\[ \chi(p) = S(p) + \int \chi(p')K(p'\to p)dp' \]  

in which \( S(p) \) is the source density distribution, \( K(p'\to p) \) is the Boltzmann kernel which can be expressed as

\[ K(p'\to p) = T(r'\to r)C(E'\Omega' \to E\Omega) \]  

where \( T(r'\to r|E\Omega') \) is the transport kernel and \( C(E\Omega' \to E\Omega|r) \) is the collision kernel. Finally

\[ f(p) = T(r\to r_d|E\Omega) \eta(E,\Omega) \]  

where \( \eta(E,\Omega) \) is the response function (or efficiency) of the detector and \( r_d \) denotes a position vector in the active volume of the detector.
The optimum biasing function for the solution of Eq. (2) for the purpose of calculating the performance parameter of Eq. (1) is (see, for example, Ref. 9-11) the importance function - the solution of the adjoint equation

\[ \chi(p) = f(p) + \int K(p-p')\chi(p')dp' \]  

In terms of this importance function the performance parameter of Eq. (1) can be expressed as

\[ R = \int \chi(p)S(p)dp \]  

Suppose the importance function distribution is known on a surface A (in the configuration space) which separates the detector from the volume V (represented by the radius vector \( r \); see Fig. 1). Dividing the spatial integration in Eq. (5) into the two regions V and V1 we find

\[ \chi^*(r,E,\Omega) = S^*(r,E,\Omega;r'' \cap A) + \int dr'/dE'/dQ' T(r\rightarrow r'|E\Omega)C(E\Omega+E'^\Omega|r') \chi^*(r',E',\Omega') \]  

where the first term on the right side of Eq. (7) is obtained as follows:

\[ \chi^*(r,E,\Omega) = \int dr't(r\rightarrow r'|E\Omega)T(r\rightarrow r'|E\Omega)C(E\Omega+E'^\Omega|r') \chi^*(r',E',\Omega') + f(r,E,\Omega) \]

\[ = U(r\rightarrow r''|E\Omega)[\chi^*(r'',E,\Omega) - f(r'',E,\Omega)] + f(r,E,\Omega) = U(r\rightarrow r''|E\Omega)X^*(r'',E,\Omega) \]

\[ \equiv S^*(r,E,\Omega;r'') \]  

and

\[ U(r\rightarrow r''|E\Omega) \equiv T(r\rightarrow r''|E\Omega)/\Sigma_t(r'',E) \]  

The adjoint source term \( S^*(r,E,\Omega;r'') \) is the importance of neutrons coming out at \( (r,E,\Omega) \) which reach the reference surface A uncollided.

Consider now the importance function at phase space point \( p_0 \in V' \) expressed as a detector response [the equivalent of Eq. (6)],

\[ \chi^*(p_0) = \int_V \chi^*(p)\delta(p-p_0)dp \]  

Fig. 1. A schematic illustration of the division of the configuration space to subregions. The importance function on surface A dividing V and V' is known.
We can express it using the distribution function formulation [the equivalent of Eq. (1)] getting

\[ X^*(r_0) = \int d\Omega \int dE \int d\Omega' (r \cdot E, \Omega; r_0) S^*(r_0, \Omega; r') \]

where \( J \) is the solution of the equation for the distribution function due to the point source at \( p_0 \),

\[ J(p; p_0) = \delta(p - p_0) + \int dp' J(p') K(p' \rightarrow p) \]

In other words, Eqs. (11), (12), (7) and (10) are the equivalent of, respectively, Eqs. (1), (2), (5) and (6). Eqs. (11) and (12) imply that the value of the importance function at phase space point \( p_0 \) can be calculated by solving the forward (or distribution function) equation in subregion \( V' \) subjected to a source \( \delta(p - p_0) \), and weight the flux of neutrons which reach, uncollided, the reference surface \( A \) with the value of the importance function on that surface (assumed known).

The value of the importance function averaged over a phase space region \( \Delta p = (\Delta r, \Delta E, \Delta \Omega) \), (with \( \Delta r \in V' \)),

\[ \chi^*_{\Delta p} = \frac{\int d\Omega \int dE \int d\Omega' X^*(r_0, E, \Omega; r_0)}{\int d\Omega \int dE \int d\Omega'} \]

can similarly be calculated from the expression

\[ \chi^*_{\Delta p} = \int d\Omega \int dE \int d\Omega' J_{\Delta p} (r, E, \Omega; r') S^*(r_0, \Omega; r') \]

where \( J_{\Delta p} \) is the solution of the forward equation

\[ J_{\Delta p} (p) = S_{\Delta p} (p) + \int dp' J_{\Delta p} (p') K(p' \rightarrow p) \]

subjected to an external source of neutrons uniformly distributed in \( \Delta p \),

\[ S_{\Delta p} (p) = 1/\lambda p \equiv [(\Delta r)(\Delta E)(\Delta \Omega)]^{-1} \]

3. THE RMC METHOD

The RMC method consists of the following ingredients: (1) The system is divided into relatively small geometrical regions, typically one mean-free-path (mfp) in thickness. (2) The forward transport equation is solved for each region subjected to an isotropic source of neutrons distributed uniformly on the surface of the region farthest from the detector. The histories of the source neutrons are followed throughout the volume of the region and a buffer zone adjacent to it (from the other side of the source surface). (3) The average importance of the source neutrons is calculated by summing the probabilities of the neutrons coming out of collisions in the region (and the buffer zone) to cross, uncollided, the preceding surface (obtained from forward Monte Carlo calculations) weighted with the value of the importance function at the
crossing point, which is known from the previous step. The result is the average importance function of the source neutrons; it is assigned to the source surface which provides the "reference" surface for the next step.

(4) This procedure is repeated recursively, starting with the closest-to-detector region and continuing towards the external source. The surface averaged values of the importance functions are then used for importance sampling in the course of the M.C. calculation (forward) of the detector response.

In the following subsections we shall discuss a number of issues related to the practical application of the RMC method.

3.1 Region Geometry

Consider the system illustrated schematically in Fig. 2a. The problem is to estimate the adjoint function distribution corresponding to the detector throughout the system (up to the external source surface). Towards this end the system is divided (fictitiously) by surfaces, as illustrated in Fig. 2b. Let us suppose that the importance function is known on surface $A_3$ and examine the procedure to be used for calculating the average importance function on surface $A_4$.

Following the formulation of Sec. 2 we are to assign a uniform isotropic source of neutrons to surface $A_4$ and solve the forward Boltzmann equation subjected to that fictitious source throughout the volume of the system excluding that part between reference surface $A_3$ and the detector. Such an undertaking is both impractical and
unnecessary. Remembering that the solution looked for is the value of
the importance function on $A_4$, and realizing that neutrons reaching a
few mean-free-path from $A_4$ (away from the detector) have a very low
probability to reach reference surface $A_3$ (and hence, to contribute to
surface $A_4$ importance function) it is apparent that it is sufficient to
consider a truncated $V'$, as illustrated in Fig. 2c. This truncated $V'$
consists of the geometrical region (between $A_4$ and $A_3$) and a buffer
zone (from the other side of $A_4$).

The larger the thickness of the geometrical region the longer it will
take to get the importance function on $A_4$ with a given level of accuracy
but the smaller number of recursion steps will be necessary for the
solution of a given problem. Similarly, the thicker the buffer zone the
longer it will take to follow the histories of a given number of source
neutrons; beyond a certain thickness the added accuracy resulting from a
further increase in the buffer zone thickness becomes negligible.

Investigating the effect of the region and buffer zone thickness on
the accuracy and efficiency of the RMC method (considering monoenergetic
problems) we found that region and buffer zone thicknesses of the order
of one mean-free-path appear to be near the optimum. Recently we got an
indication that for certain problems it might be possible (and efficient)
to do without a buffer zone. Further investigation is needed before the
optimal thickness of the region and buffer zone for different problems
could be identified.

### 3.2 Sample Size and Statistical Accuracy

A question of primary concern to the practicality of the RMC method
is that of the propagation of statistical errors in problems having a
large number of recursion steps. Figure 3 compares the importance
function calculated with the RMC method for a simple one-dimensional deep-
penetration problem with the results from deterministic calculations
obtained with ANISN. The system consists of a homogeneous sphere having a
central spherical detector surrounded by a 22 mfp thick shield (extending
from a radius of 28 cm to 200 cm). The problem is monoenergetic with
isotropic scattering. An $S_8$ (symmetric) angular quadrature was used for
the ANISN calculations. For the purpose of the RMC calculations the
system is divided into 50-1/2 mfp thick regions. The solid angle is
divided into the eight quadratures used for the ANISN calculations thus
enabling a direct comparison between the RMC and the ANISN results. A
buffer zone, 1 mfp in thickness, is attached to each region. The detector
response function was taken to be unity. The importance function on the
closest-to-detector reference surface was obtained from a simple ANISN
run. (It can also be calculated, straightforwardly, by hand; note that
only angular components of the importance function pointing towards the
detector need to be known.) The sample size of the neutron source
assigned to each surface for the RMC calculations was 3000. Figure 3
shows a very good agreement between the RMC and the ANISN results
throughout the system, with no indication for propagation of errors. It
ought to be mentioned, though, that we did observe propagation of errors
when using, for the same problem, a sample size of 1500 neutrons and volumetric (rather than surface) sources.

The efficiency of the importance sampling technique (judged by the computer time required for the estimation of the detector response at a given degree of accuracy) depends on the accuracy of the importance function used for biasing. The higher the accuracy required, the more time is needed, however, for the adjoint calculation. The efficiency and usefulness of the RMC method should therefore be judged by the overall computation time needed for the two phases of the solution - the RMC calculation of the importance function distribution, and the calculation of the detector response (using importance sampling). We shall refer to the overall procedure as the "RMC procedure." The first phase will be referred to as the "adjoint calculation" whereas the second phase as the "detector calculation."

To examine the sensitivity of the efficiency of the RMC procedure to the accuracy of the importance function distribution we define a figure of merit,

$$\gamma = \sigma^2 T$$  \hspace{1cm} (17)$$

where $\sigma^2$ is the variance of the final result for the detector response, and $T$ is the overall computation time, such that

$$T = T_a + T_d$$  \hspace{1cm} (18)$$

where $T_a$ and $T_d$ are the time required for the calculation of, respectively, the importance function and the detector response.

In conventional applications of

![Graph](image-url)
M.C. techniques, in which $T$ stands for the time it takes to perform the M.C. calculations (the equivalent of $T_d$ in the RMC procedure), $Y$ is constant. The figure-of-merit was calculated as a function of two parameters: the batch size of source neutrons used for the RMC calculation ($S_a$) and for the detector response ($S_d$).

The parametric study was performed for a shielding problem similar to (although smaller than) that considered by Tang, et al.\(^4\) The shield, illustrated in Fig. 4, is a right circular cylinder of a uniform homogeneous composition having a central duct along its axis. A plane isotropic source is located at the base of the shield, from the other side of the detector. The problem was treated as monoenergetic with isotropic scattering. The cross sections were taken to be $\Sigma_t = 1.0 \text{ cm}^{-1}$ and $\Sigma = 0.9 \text{ cm}^{-1}$.

![Fig. 4. The geometry of the 2-D shield problem of Ref. 4.](image)

For the purpose of the recursive solution, the shield is divided into 28 half-mfp thick regions, by planes perpendicular to the cylinder axis (see Fig. 4). Each plane is partitioned, somewhat intuitively, into five sections, a thru e, as illustrated in Fig. 4. In setting this partition we have taken into account the anticipation that most of the contribution to the flux in both detectors is likely to come through the axial duct and through regions adjacent to it. The distinction between the right and left sides of the system is to enable accounting for the asymmetry of the problem in the case of the side detector (considered in Ref. 3, but not in the present example). The first step in the solution is to calculate the importance function distribution on spherical surface $A_1$ centered in the axial detector (see Fig. 4). This is done with a simple ANISN run.

The importance function in the five sections of surface $A_2$ is then calculated using $A_1$ as the reference surface. A one mfp thick buffer zone is attached to the region for the adjoint calculations. The computation time for the average importance of the five sections of surface $A_2$ was 1 min. 8 sec., when using a source of 1500 neutrons per section. Geometrical imaging\(^3\) (see also Sec. 3.3.2) was applied to the next 27 regions which are identical to the first one. The time per recursion step (just for weighting a given histories tape with the appropriate adjoint function) was 4.18 sec. for the 1500 n/section source. The last two regions are different from the reference region by the thickness of their buffer zone. We could still apply, nevertheless, the histories tape of the reference region to the edge region, by ignoring the contribution to the importance function, of neutrons colliding in the missing sections of
the buffer zone. The total computation time needed for the calculation of the importance function was 2 min. 53 sec. for the 1500 n/section source case. Without geometrical imaging the solution would have taken 11 times longer. The importance function distribution thus calculated is used for biasing of the external neutron source, for the Russian roulette, splitting, as well as for the generalized exponential transform. For this purpose, the average importance function calculated for a surface is assigned to the volume extending 1.4 mfp in both sides of the surface.

The results obtained for the central detector are summarized in Fig. 5. The figure of merit is found (Fig. 5) to have a clear minimum; to be strongly correlated with $S_a$; and almost independent of $S_d$. The $Y$-$S_a$ plane of Fig. 5 is divided into two distinct domains: in the high $S_a$ domain ($S_a > (S_a)_{opt}$) the improvement in the accuracy of the importance function distribution does not improve the efficiency of the importance sampling so that $T$ and therefore $Y$, increase essentially linearly with $S_a$. In the $S_a < (S_a)_{opt}$ domain, on the other hand, the accuracy of the importance function distribution degrades significantly with the decrease in $S_a$, causing a dramatic loss in the accuracy of the calculated detector response. At the optimum, corresponding in the problem considered to $S_a_{opt} \approx 1500$, the time required for the adjoint calculation is smaller than the time required for the calculation of the detector response.

An important question associated with the practical application of the RMC method is how to determine, a priori, the optimal source batch size for the adjoint calculation. This bath size is expected to be problem dependent. In all the problems we have solved so far with the RMC procedure, we found that a bath size of 1500 neutrons was adequate. Much more numerical experimentations, covering a wide variety of problems, is necessary, nevertheless, before a reliable recipe could be determined.
3.3 Special Features of the RMC Method

3.3.1 Simultaneous Solution of Multi-Detectors Problems

To each detector in a given system there corresponds a different importance function distribution. Consequently, to find the importance function pertaining to N different detectors in a given system, it is necessary to repeat the solution N times. The RMC method, on the other hand, enables getting the solution for all N detectors from, essentially, a single run (provided these detectors can be enclosed by the first reference surface).

In the RMC method the adjoint calculations consist of two phases: (1) the calculation of the probability of the source neutrons to reach the reference surface (as a function of location on that surface and of direction of arrival), and (2) weighting this "arrival probability" with the corresponding value of the importance function. The first phase is detector independent; once we know the arrival probabilities pertaining to a given detector, we know these probabilities for all other detectors in the same system (provided the system is divided the same way for all different detectors). The importance function distribution pertaining to each of the detectors can then be calculated just by weighting these arrival probabilities with the appropriate importance functions. The detector dependency comes only through the assignment of the importance function to the first reference surface.

3.3.2 Geometrical Imaging

When two regions are identical in geometry and composition (i.e., they are the image of each other), the histories of the source neutrons pertaining to each region (i.e., the arrival probability) are the same. Consequently, the histories generated for a representative region are directly applicable for all the images of that region. The application of this procedure will be referred to as geometrical imaging.

Geometrical imaging can significantly reduce the computation time for the solution of deep-penetration problems using the RMC method. The solution of the adjoint equation in certain deep-penetration problems can be transformed, with this feature, to the order of difficulty required for the solution of simple one mean-free-path type transport problems.

3.3.3 Estimation of the Importance Function in Low Importance Regions

In solving the adjoint equation for deep-penetration problems using the adjoint (conventional) M.C. method it is difficult to get a reliable estimate of the importance function in regions in which this function has relatively low values (usually the regions farthest away from the detector). Importance sampling applications require proper knowledge of the importance function throughout the system; too much a distortion in the adjoint distribution may lead to a significant error in the calculated detector response. (See also Sec. 42) The accuracy of the estimation of the detector response becomes more sensitive to the value of the
importance function in low-importance regions the higher is the relative flux of neutrons in these regions. (More details on this issue along with an illustrative example can be found in Ref. 12).

As the RMC method involves the solution of the forward equation corresponding to a source of neutrons assigned to each region (of phase space), this method "picks up" the value of the importance function in low importance region as reliably as it does in high importance regions. This feature is particularly important for problems characterized by relatively high fluxes in regions of relatively low importance.

4. A CONCRETE SHIELD PROBLEM

So far we have considered monoenergetic problems only. As the first illustration for the application of the RMC method to realistic multigroup problems, we apply it to the solution of the concrete shield problem of Tang et al.4 (Fig. 6). The problem is to find the total neutron fluence at the two point detectors. The 14 energy group structure along with the group constants library of Ref. 4 are used for the present study. The scattering anisotropy is described using the $P_3$ approximation.

The first phase of the solution is the estimation of the importance function distribution throughout the shield. Towards this end we divide the shield into 100 equally thick regions by planes perpendicular to the cylinder axis (as in Fig. 4). This gives a region thickness of 1.52 cm, which is smaller than the mfp pertaining to any of the energy groups (ranging from 2.36 cm to 8.37 cm). A buffer zone, 8.37 cm thick (i.e., equal to the largest mfp in this problem), is attached to each region for the purpose of the adjoint calculations (see Fig. 7). Each surface is partitioned into five sections (following the rationale described in Sec. 3.2) as follows (see also Fig. 7):

- section a: \[0 < R < 7.62\ \text{cm}\]
- sections b and c: \[7.62 < R \leq 15\ \text{cm}\]
- sections d and e: \[16 < R \leq 150\ \text{cm}\]

The solution starts with the calculation, using ANISN, of the importance function distribution on the spherical surface $A_1$ centered at the

Fig. 6. The geometry of the concrete shield problem.
The axial detector (see Fig. 7) using an adjoint source term of unity at the detector point, for each of the 14 energy groups. Then we calculate the importance function in the five sections of surface $A_2$ with $A_1$ serving as the reference surface (Fig. 7).

In order to save time following neutrons in relatively low importance regions of relatively thick buffer zones (which can be, in this problem, as large as 4 mfp) the buffer zone is divided into four subregions onto which splitting and Russian roulette is applied. An isotropic source is uniformly distributed on the surface of each section in each of the energy groups.

The sample size of the source neutrons was 21000 per section, corresponding to 1500 neutrons for each energy group. The time for calculating the average importance function on the first five sections (surface $A_2$) was 6 min 28 sec. Without Russian roulette and splitting, this computation time was longer by a factor of 2.6.

Capitalizing on the similarity of all the regions the system is divided to, we used the geometrical imaging procedure (see Sec. 3.3.2) for calculating the importance function in the rest of the regions. The last six regions (95-100) have a thinner than nominal buffer zone. We applied geometrical imaging to these regions as well, accounting for the difference in the buffer zone thickness by ignoring the histories in the "excessive zones" (see Sec. 3.1). The computation time for a single recursion step was approximately 12 sec, making the total time needed for estimating the importance function distribution pertaining to the central detector 26 min 18 sec.

The calculation of the importance function distribution pertaining to the side detector followed the same procedure, with only two small modifications: the first reference surface was $A_1'$ rather than $A_1$ (Fig. 7) and there was no need to calculate a new neutron histories tape for the first region - the tape created for that region during the calculation of the central detector importance function was used. Also notice that the importance function distribution on the reference surface $A_1'$ is identical to that on $A_1$ (remember that we are interested in the forward directions only). Using geometrical imaging, the total computation time needed for the estimation of the importance function distribution pertaining to the side detector was 20 min 3 sec.
The importance function distributions obtained for each of the detectors were then used during the calculation of the corresponding detector's reading for biasing of the external source, Russian roulette, splitting and for the generalized exponential transform. For this purpose, the value of the average importance function at a given surface is assigned to a region centered around this surface. To examine the sensitivity of the RMC procedure to the accuracy of the importance function distribution used for importance sampling, we also calculated the side detector response using the importance function distribution pertaining to the axial detector. Fifty batches of 400 source neutrons were used for the M.C. calculations of the response of each detector. No attempt was made to optimize the batch size used for the RMC procedure.

Table 1 compares the results obtained with the RMC procedure with "exact" results obtained from deterministic calculations (using the two-dimensional $S_n$ code DOT) and with the M.C. results of Ref. 4 obtained using the importance function distribution for the central detector calculated with DOT. The results from the RMC procedure are seen to agree well with those from DOT, when the appropriate importance function distribution is used for importance sampling. Moreover, the RMC calculated standard deviations for the central and the side detectors are, respectively, 1.8 and 9.3 times smaller than those obtained in Ref. 4 using the DOT importance function, while the overall effective computation time appears to be smaller.

<table>
<thead>
<tr>
<th>Computation Method</th>
<th>Response of Axial Detector</th>
<th>Response of Side Detector</th>
<th>Importance Function</th>
<th>Detector Response</th>
<th>Computation Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_n$ (DOT)</td>
<td>$2.453 \times 10^{-9}$</td>
<td>$1.408 \times 10^{-11}$</td>
<td>-</td>
<td>-</td>
<td>50**</td>
</tr>
<tr>
<td>M.C. + Adjoint $S_n$ (DOT)</td>
<td>$3.093 \times 10^{-9}(+4.8%)$</td>
<td>$1.062 \times 10^{-11}(+43%)$</td>
<td>52**</td>
<td>60**</td>
<td>112**</td>
</tr>
<tr>
<td>M.C. + RMC (for the axial detector)</td>
<td>$2.540 \times 10^{-9}(+2.6%)$</td>
<td>-</td>
<td>26.3</td>
<td>19.3</td>
<td>45.6</td>
</tr>
<tr>
<td>M.C. + RMC (for the side detector)</td>
<td>-</td>
<td>$1.560 \times 10^{-11}(+4.6%)$</td>
<td>20.1</td>
<td>23.6</td>
<td>43.6</td>
</tr>
<tr>
<td>M.C. + RMC (for the axial detector)</td>
<td>-</td>
<td>$0.910 \times 10^{-11}(+50%)$</td>
<td>26.3</td>
<td>31.8</td>
<td>58.1</td>
</tr>
</tbody>
</table>

*Data from Ref. 4
**CPU-time of IBM 360/91

To the best of our knowledge, the CPU speed of the IBM 360/91 used in Ref. 4 is about a factor of 4.5 faster than that of the CYBER-73.
With the axial detector importance function used for importance sampling, the RMC standard deviation of the side detector response was a factor of 10.8 greater than when the side detector importance function is employed for importance sampling. A similar standard deviation for the side detector was obtained in Ref. 4. It is thus seen that, in the problem under consideration, the use of an importance function distribution from 2-D calculations for importance sampling in a 3-D problem (which differ from the 2-D problem in the location of the detector only) leads to a reduction in the overall efficiency of the M.C. calculation of the side-detector response, by about two orders of magnitude. This example indicates to the sensitivity of the M.C. method efficiency to distortions in the importance function used for importance sampling.

5. A TOKAMAK MAJOR PENETRATION PROBLEM

As a second test of the efficiency of the RMC method for the solution of realistic deep-penetration problems we applied it to a three-dimensional Tokamak blanket-shield major penetration problem recently studied by Jung and Abdou and proposed for a benchmark. A cut through the system considered is shown in Fig. 8 and the thickness and composition of the different zones are summarized in Table 2.

Table 2. Dimensions and Compositions of the Tokamak Blanket/Shield Major Penetration Problem

<table>
<thead>
<tr>
<th>Zone</th>
<th>Outer Radius (cm)</th>
<th>Thickness (cm)</th>
<th>Material Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210</td>
<td>210</td>
<td>Plasma</td>
</tr>
<tr>
<td>2</td>
<td>240</td>
<td>15</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>3</td>
<td>241</td>
<td>15</td>
<td>Vacuum</td>
</tr>
<tr>
<td>4</td>
<td>256</td>
<td>5</td>
<td>Boron Carbide</td>
</tr>
<tr>
<td>5</td>
<td>261</td>
<td>5</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>6</td>
<td>276</td>
<td>5</td>
<td>Boron Carbide</td>
</tr>
<tr>
<td>7</td>
<td>281</td>
<td>5</td>
<td>Stainless Steel</td>
</tr>
<tr>
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<tr>
<td>18</td>
<td>490</td>
<td>60</td>
<td>50% SS + 50% Cu</td>
</tr>
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</table>

Following Ref. 5, the toroidal geometry is approximated by a cylindrical one, with the X-axis designating the centerline. The first wall, blanket, shield and Toroidal Field Coils (TFC) are of cylindrical cross section in the Y-Z plane (see Fig. 8), and the vacuum duct cross section is in the X-Y plane. This duct, 20 cm in diameter, is lined with a 1 cm thick stainless steel tube. A 5 cm thick stainless steel disc provides an end-cap to the duct (Zone 25, Fig. 8). The TFC are divided into concentric rings, 5 cm thick each (Zones 18, 19, 20 etc.), bounded (following Ref. 5) by the y = ±100 cm planes (with the centerline of the duct being at y=0). The problem is to calculate the total flux in the end-cap (Zone 25) and in the two inner zones (Zones 18 and 19) of the TFC due to a uniform isotropic shell source of 14 MeV neutrons coming from the surface of the plasma region.
Twenty-two energy groups (the group structure for which is summarized in Table 3), $P_3$ scattering anisotropy and $S_9$ angular quadrature are used for the solution of the Tokamak problem. The 22 group constants were generated by collapsing, with ANISN, the 100 group NLC-2 library using ANISN calculated flux (for a representative 1-D mockup of the problem) for the weighting spectrum. The boundaries of the angular quadratures used are $\cos = 0.0, +.41555, +.69105, +.8934$ and $+1.0$.

For the purpose of the adjoint calculations (in the RMC procedure) we neglected the curvature around the $X$-axis. This approximation enables using geometrical imaging (and thus improve the efficiency of the RMC procedure) and is not expected to significantly distort the importance function distribution* (due to the relatively large radius of curvature of the problem). The division of the system into "small" regions poses a more difficult problem than in the previous example (Sec. 4) not only because of the heterogeneous structure of the blanket/shield but also due to the large spread in the value of the mfp pertaining to the different compositions and energy groups used (see Table 3); the smallest mfp (0.004 cm) is less than one thousandths of the largest mfp (5.8 cm). Anticipating that most of the contribution to the detectors reading will come from the MeV and upper KeV energy range (the first 13 energy groups), we divided the blanket/shield (by planes parallel to the first wall) into 2.5 cm thick regions (or 1/2 to 1 mfp in thickness for the first 13 energy groups). No buffer zones were used in this problem.†

Each surface (used for the RMC calculations) is divided into three sections (similar to the division in Sec. 4, except that now we do not have a distinction between left and right): *There is no difficulty in applying the RMC method to curved geometry. †An indication which we also got from the 1-D ANISN run. #The buffer zones appeared to have only very small effect on the results of this problem. The buffer zone issue deserves additional examination.
The adjoint calculations proceed as follows: First, the collision tapes pertaining to the three regions which can represent the system - a stainless steel region (Zones 4, 6 etc., including the portion of the vacuum duct in these zones, see Fig. 8), a B$_4$C region (Zones 9, 11, etc.) and vacuum space (Zone 17), are generated (using the RMC approach). Fifteen hundred source neutrons per surface section, energy group and angular bin are used for these calculations. Then the importance function pertaining to the end-cap detector is found on the first reference surface, taken to be the plane passing through the basis of this cap (parallel to the first wall). Geometrical imaging is then used to find, recursively, the average importance function (per group and angular bin) on each of the reference surfaces down to the fusion neutron source plane. The total computation (CPU) time until this phase was 80 min. Starting with the first reference surface for the TFC detector (regions 18 and 19, see Fig. 8), located at the outer surface of Zones 18 and 19, the importance function pertaining to this detector was then calculated with the RMC method, using the collision tapes already available. The CPU time required for this phase of the work was 48 min.

The importance functions thus obtained are used, in the detectors calculation (using MORSE), for biasing of the external neutron source, Russian roulette, splitting and for the generalized exponential transform. The detectors response (i.e., total neutron flux in the detector region) were calculated using a track length estimator. Six thousand source neutrons were used for the calculation of each detector's response, requiring 53 min and 75 min of CPU time for, respectively, the end-cap and TFC detectors.

Table 4 summarizes the results for the detectors response, as obtained using the RMC procedure described above, and compares them with the corresponding results obtained by Jung and Abdou$^5$ (J & A), using 50,000 source neutrons for each detector (or region). It is seen that even

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<thead>
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<th>Group Number</th>
<th>Upper Boundary (eV)</th>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.4918 + 7</td>
<td>5.38</td>
<td>4.59</td>
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</tr>
<tr>
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<td>1.3499 + 7</td>
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<tr>
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<td>3.20</td>
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<tr>
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<td>3.14</td>
</tr>
<tr>
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<tr>
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<td>3.36</td>
<td>3.42</td>
<td>3.55</td>
</tr>
<tr>
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<td>3.72</td>
</tr>
<tr>
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<td>2.2313 + 6</td>
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<td>2.22</td>
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<tr>
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<td>0.23</td>
<td>0.76</td>
<td>0.85</td>
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<tr>
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<td>0.11</td>
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</table>
though J & A followed about one order of magnitude more source neutrons per detector than used for the RMC calculations, the results from the latter method exhibit substantially lower standard deviation. Even when the time required for the adjoint calculations (which is of the same order as the time for the direct calculations) is taken into account, the efficiency of the RMC procedure is found to be significantly higher than that of a "brute-force" M.C. calculation; for the J & A method of calculation to provide a standard deviation of 7% for region 25, for example, it had to process about 1600 times more source neutrons than in the RMC procedure (the detector calculation phase).

The differences in the average value of the detectors response obtained between the RMC and J & A methods, even though within the statistical uncertainty, may be due, in part, to some differences in the representation of the problem. The isotropic shell source used for the RMC calculations, for example, ought to be replaced by a more realistic angular distribution. In addition, the presence of the TFC (and neutron interactions with them) should also be taken into account for the RMC calculations. These and other (such as accounting for the curvature in the poloidal direction) refinements are not expected, however, to affect substantially the applicability and accuracy of the RMC method.

6. CONCLUDING REMARKS

The experience gained so far with the RMC method confirms the need for importance sampling for efficient solution, using M.C. techniques, of deep penetration problems (see, for example, Ref. 10). It also confirms the need for accurate enough knowledge of the importance function distributions used for importance sampling; using an importance function from two-dimensional calculations for importance sampling in a three-dimensional problem (different from the 2-D one only in the location of the detector) led to a reduction in the efficiency of the M.C. solution of the concrete shield problem (considered in Sec. 4) by about two orders of magnitude.

The RMC method appears practical and efficient for the estimation of the importance function distributions in realistic deep-penetration problems with an accuracy suitable for importance sampling applications (and it ought to be realized that the solutions, with the RMC procedure,

<table>
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<tr>
<th>Region</th>
<th>Total Neutron Flux* (n/cm²·sec) RMC Procedure</th>
<th>Ref. 5</th>
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<tr>
<td>25</td>
<td>10.87 x 10¹¹ (+7%)</td>
<td>5.35 x 10¹¹ (+100%)</td>
</tr>
<tr>
<td>18</td>
<td>5.60 x 10⁹ (+13%)</td>
<td>1.10 x 10¹⁰ (+66%)</td>
</tr>
<tr>
<td>19</td>
<td>3.52 x 10⁹ (+15%)</td>
<td>8.04 x 10⁹ (+70%)</td>
</tr>
</tbody>
</table>

*normalized to a neutron wall loading of 1 MW/m²
of the deep-penetration problems described in Secs. 4 and 5 were not optimized. It thus might enable applying the M.C. technique to the solution of realistic complicated deep-penetration problems otherwise found very difficult to solve (see, for example, Refs. 5 and 6).

Many more numerical experimentations are, nevertheless, required before the practicality and efficiency of the RMC procedure could reliably be assessed, and in order to devise recipes for the optimal application of this procedure to a wide range of problems.

The recursive approach developed for the RMC calculations could, in principle, also be applied with deterministic methods for the solution of the transport equation. The deterministic methods are expected to have not only a limited range of applicability (usually to 2-D regions) but may also be less efficient than the RMC method; they require a solution of \( G \times S \times A \) equations per region (i.e., recursive step) where \( G \) is the number of energy groups, \( S \) is the number of sections a reference surface is divided into and \( A \) is the number of angular bins used.

The RMC procedure has been incorporated within the MORSE Monte Carlo system. The resulting code, named REMOP, is to become available through the Radiation Shielding Information Center of the Oak Ridge National Laboratory.

REFERENCES


