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A UNIFIED CREEP-PLASTICITY MODEL FOR HALITE*

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ABSTRACT

There are two national energy programs which are considering caverns in geological salt (NaCl) as a storage repository. One is the disposal of nuclear wastes and the other is the storage of oil. Both short-time and long-time structural deformations and stresses must be predictable for these applications. At 300K, the nominal initial temperature for both applications, the salt is at 0.28 of the melting temperature and exhibits a significant time dependent behavior. A constitutive model has been developed which describes the behavior observed in an extensive set of triaxial creep tests. Analysis of these tests showed that a single deformation mechanism seems to be operative over the stress and temperature range of interest so that the secondary creep data can be represented by a power of the stress over the entire test range. This simple behavior allowed a new unified creep-plasticity model to be applied with some confidence. The resulting model recognizes no inherent difference between plastic and creep strains yet models the total inelastic strain reasonably well including primary and secondary creep and reverse loadings. A multiaxial formulation is applied with a back stress. A Bauschinger effect is exhibited as a consequence and is present regardless of the time scale over which the loading is applied. The model would be interpreted as kinematic hardening in the sense of classical plasticity. Comparisons are made between test data and model behavior.

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Introduction

Halite, the naturally occurring mineral form of sodium chloride (NaCl) is commonly found around the world. Halite is easily mined and has a moderate creep rate at stresses commonly found in conventional underground room and pillar mining operations. Although salt is mined for its own sake, it is also found in proximity to potash and is removed for ease in mining the potash. Halite has, in the past few years, received attention as a candidate material in which to store oil and to dispose of radioactive wastes. In spite of the vast experience in mining salt, it has not been carefully characterized as a structural material until recently.

Various people have performed tests on pure sodium chloride, but tests on naturally occurring forms are fewer in number. Of particular interest here is rock salt from southeastern New Mexico. The rock salt has impurities which result in lower creep rates than for pure halite. Hansen and Mellegard (1), and Wawersik and Hannum (2,3), are the sources of data for this particular material.

Herrmann, Lauson, and Wawersik (4,5) studied the creep behavior of the rock salt near room temperature and up to 200C using the data from the investigators cited above. The present model is based on that work.

Presentation of Equations

It is a very noble thing to attempt to derive nonlinear constitutive models based on micromechanical mechanisms. This has met with some success in time independent plasticity (6) and metal creep formulations. A review of the theories of Hart, Lagneborg, A. Miller, Ponter and Leckie, Patley and Wells and others with advanced creep models was made by Krieg (7). These models were all shown to have a common basic mathematical structure, differing only in the functional forms used. Lagneborg (8) reviewed more recent work on advanced creep models from the micromechanical point of view. The attempt here is not nearly so lofty; the objective simply to describe externally measurable stress in terms of externally measurable strain. This has not been entirely successful in the past and in fact, is accomplished here only by introducing an additional state variable, the back stress.

The formulation here is a special case of the skeletal model into which most of the hardening/recovery models fall. In terms of the categories of Krieg (7), the present model is a kinematic hardening model. The need for a kinematic model with a back stress is consistent with observations from stress drop tests on metals (8). For briefness and ease of understanding, the model is simplified here to scalar form. The volumetric behavior in the model is approximated as linear elastic although that approximation is not obvious in the form presented here. The model is motivated principally by the experimentally observed mechanical response and partly by micromechanical considerations.

The strainrate (actually the stretching tensor, d) is decomposed into elastic and inelastic parts and rearranged into the form

$$\overset{\nabla}{\sigma} = E\left(d - d^{P}\right) \tag{1}$$

where σ is the stress and E is the elastic Young's modulus. Expressions for the "strainrate" and the evolutionary equation for the back stress are given by

$$d^{P} = A \exp(q|\xi|)\xi \tag{2}$$

$$\overset{\nabla}{\alpha} = B d^{P} \exp(-\xi \alpha \, \mathrm{sgn}\, \xi) - C|\alpha|\alpha \tag{3}$$

where

$$A = A_o \exp(-Q_{A/R\,\theta})$$

$$B = B_o \exp(Q_A/R\theta)$$

$$C = C_o \exp(-Q_C/R\theta)$$

$$\xi = \sigma - \alpha$$

and where q, ζ, A_o, B_o, C_o are simple material constants; Q_A, Q_B, Q_C are activation energies, R is the universal gas constant, and θ is absolute temperature.

The model incorporates a fairly realistic strain hardening behavior used in an earlier unified creep-plasticity model by Krieg (7). The hardening term of Equation (3) is notable in that the rate of hardening depends not only on the inelastic strainrate but also on the component of the back stress which is in the direction of the effective stress. The strain hardening rate decreases as the back stress saturates. The back stress can be micromechanically related (9) to the mobile dislocation density ρ as

$$\alpha = aGb\rho^{0.5}$$

where b is Burgers' vector. The saturation of the back stress in the hardening term can be interpreted then as a saturation of the mobile dislocation density in the forward direction; "forward" meaning the same direction as the effective stress. The hardening rate is enhanced if the back stress is in the direction opposite to that of the effective stress. This behavior was modeled in Gittus' early work (10) by definition of a forward dislocation density and a reverse dislocation density, with "forward" and "reverse" directions being defined with respect to the effective stress. Evolutionary equations were then defined for each component. The present model incorporates this directionality in the hardening term in a continuous manner. Without the recovery term in the evolutionary equation, and with a more Heaviside-like strainrate equation replacing Equation (2), the model mimics time independent plasticity. It is instructive to note that the behavior of this model is very much like the two-surface plasticity model (11). Each of these models has good plastic behavior at small strains and a realistic Bauschinger effect.

The final point worth noting is the recovery term in Equation (3). The $(\sigma_{rs})^2$ dependence in the model is a source of some embarrassment in light of the accepted cubic dependence of Lagneborg (9). For the halite it was found that a cubic recovery term could not mimic the behavior noted in the creep tests. It is felt that since a deformation mechanism change seems evident in the primary creep results (5), and since the simplest mathematical form was needed which would cover the entire range, that the quadratic dependence is probably only an approximation. True recovery may be the sum of a cubic term and a term of low stress dependence with the combined behavior having roughly a quadratic behavior over the range modeled here.

Data Fitting Process

Some two and three stage tests were run by imposing two or three stress levels sequentially, waiting for steady creep before changing stress levels. The transient part of the first stage of these tests was used but only the secondary creep part of the later stages.

Each of the creep tests was independently fit with an expression of the form

$$e = e_o + e_s t + e_a \,(1-\exp(-\lambda t)) \qquad (4)$$

in order to remove irregularities in the data. Three variables, derived from Equation (4) and the specific values of the parameters from each test were used to fit the unified creep-plasticity model. The secondary creep rate, $\dot{e}_s$, is one of them. The second variable is the time t_3 at which the creep rate is three times $\dot{e}_s$. The third variable is the ratio $\dot{e}/\dot{e}_s$ at the time 0.5 t_3. The time and strainrate ratio were found to give a good characterization of the primary creep behavior.

Values for A_{23}, B_{23}, C_{23}, q, and ζ in Equations (1), (2), and (3) were chosen in a regular manner for the temperature 23C and the parameters $\dot{e}_s$, t_3, and $(\dot{e}/\dot{e}_s)^*$ calculated from the model for creep stress levels corresponding to each test. The results of all the creep tests at that fixed temperature were compared with the model predictions. Errors in the logarithms of $\dot{e}_s$ and t_3 were used with errors in the strain ratios to find a root mean square error. The steady state creep error was given four times the weight of the other two parameters to reflect the relative confidence in the experimental data. The constants in the model were varied until a set was found which minimized the least square error. The values found for the room temperature data were:

ζ = 0.310 MPa^{-1}
q = 0.203 MPa^{-1}
A = 2.20 E-9 $(MPa\text{-}sec)^{-1}$
B = 3.38 E4 MPa
C = 7.12 E-9 $(MPa\text{-}sec)^{-1}$

The data and model response for secondary creep are compared in Figure 1. The curvature of the secondary creep rate versus stress on the log-log plot of Figure 1 shows that the model does not strictly follow a straight line characteristic of a power law behavior. The data shows roughly an order of magnitude scatter so that based on this data alone neither the model behavior nor a straight line behavior could be said to have a better secondary creep representation.

The data at 100C were also fitted with the same creep model but the exponential factors were fixed at the values found from reduction of the 23C data. This is in accordance with the assumed temperature variation given in Equations (1) through (3). The least square error was again

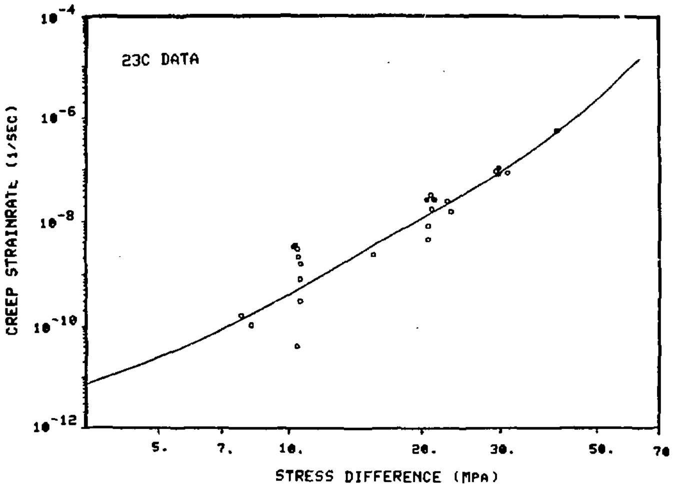

Fig. 1. Secondary Creep Rate at Room Temperature Fitted by the Model and Compared with Experimental Data.

used to define A_{100}, B_{100}, and C_{100} for these data. These two sets of (A, B, C) were used to find Q_A = 8320 cal/mole, Q_B = 7600 cal/mole, Q_C = 5110 cal/mole for the activation energies. Note in particular that the sign of the exponential for A, B, and C are chosen such as to increase inelastic strainrate (through A), decrease the rate of hardening (through B), and increase the recovery (through C) as the temperature is raised.

Behavior of the Model

At this point, it can be said that the model mimics the primary and secondary creep behavior of southeastern New Mexico halite over the stress and temperature range observed in the tests. This is all that is ordinarily expected of a creep model for stress analyses. But if the formulation is correct, the behavior of the equations should be reasonable for any stress or strain history as long as new micromechanisms do not become dominant. Unfortunately, proof tests are difficult to interpret due to the statistical spread in data from creep-like tests, particularly on a naturally obtained mineral. At this point we will examine the qualitative behavior of the model to see that it follows the general behavior observed in tests.

The most definitive tests are the multistage creep tests which, as stated earlier, were discarded for use in data fitting for the unified creep-plasticity model. But for proof tests, the transient behaviors of the second and third stages are very useful. Figure 2 is a typical strain versus time plot for one of these multistage tests as compared with the unified creep-plasticity model. The agreement is generally good, particularly in light of the scatter in the data of a factor of

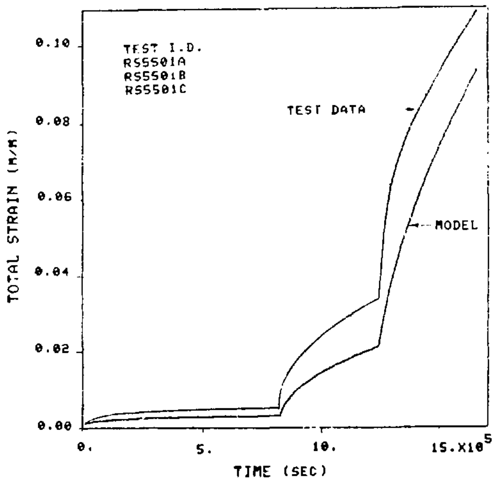

Fig. 2. Multistage Creep Strain Data at 10.5 Mpa, 20.8 Mpa, and 29.7 Mpa Compared to the Predictions of the Model.

four above and below the modeled mean of the data, as seen in Fig. 1. Since the verified creep-plasticity model can account for prior work hardening it appears that multistage testing is a very useful proof test procedure for this constitutive model.

Another test which was performed was a stress drop test. In Figure 3 is a stress drop test result. Unfortunately, it is on salt from a different depth than that for which the data was reduced. It has generally the same behavior except the creep rates are lower. For this reason, the entire curve of Fig. 3 has been scaled up to coincide with the model prediction before the stress drop. The drop follows a multistage test of 20.14 Mpa (2920 psi) for 3 92E6 sec and 30.60 Mpa (4437 psi) for 4.52E6 sec. The drop is to 20.33 Mpa (2948 psi). Note the time immediately following the drop is characterized by a period of almost zero stress rate.

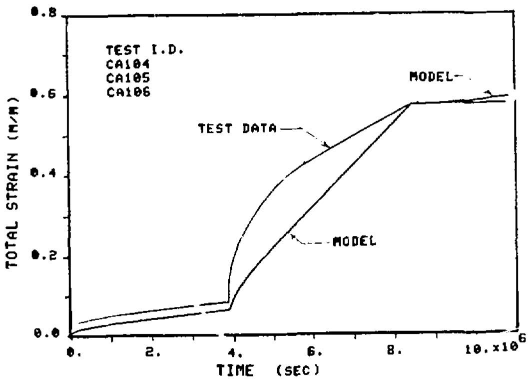

Fig. 3. Model Predictions for Multistage Creep Test at 20.1 Mpa, 30.6 Mpa, and 20.3 Mpa Compared to Data on a Different Salt where the Strain has been Scaled to Coincide Before the Drop.

Another set of tests which were run by Wawersik and Hannum (2) is a set at the moderately fast loading rate of 1.5 kpa/sec. In terms of creep tests, this type of test is a severe case where the entire behavior is in the transient creep range. In Figure 4 is plotted experimental and modeled strain. The results show a model behavior which is slightly too stiff. One point worth mentioning, however, is that these tests, in contrast to the creep tests, did show pressure dependent behavior and dilation.

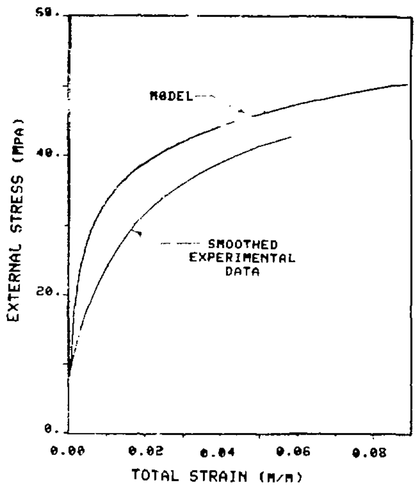

Fig. 4. Constant Stress Rate Test Results for 1.5 kpa/sec Compared with Model Predictions.

Conclusions

A unified creep-plasticity model is presented for southeastern New Mexico halite. The model is not claimed to be micromechanically based, however, some ties can be made to other micromechanically based models. The behavior is postulated to be only kinematically hardening with postulated functional forms in the inelastic strainrate equation and the evolutionary equation for the back stress. Arbitrary constants in the model are evaluated only based on a set of single stage creep tests at two temperatures. Experimental verification for the model is based on comparison with four multistage creep tests, two stress drop tests, and a set of constant stress rate tests. The behavior of the model was generally satisfactory. With the variability inherent in creep behavior generally, and particularly so with the variability in rock salt it is difficult to ascribe causes to the observed differences between the model and the test data. It is felt at this time that no serious deficiency in the model has been found unless very short time behavior is more important.

The model presented here was implemented in a one-dimensional form in order to calculate the results presented here. Implementation in a multidimensional finite element structural computer program is the next step which must be taken. Only then will a model such as this be useful to the community outside material science and applied mechanics.

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