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Nonlinear Transverse Vibrations of Elastic Beams under Tension

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RESEARCH REPORT

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Abstract

Nonlinear transverse vibrations of elastic beams under end-thrust have been examined with full account of the rigorous nonlinear relation of curvature and deformation of elastic beams. When the beams are subject to tension, the derived equation is shown to be reduced to one of the new integrable evolution equations discovered by us.

§ 1. Introduction

Studies of vibrations of elastic beams have attracted much attention in connection with the important technological problems such as vibration set up in aeroplane spars or in whirling shafts. Although vigorous development has been achieved in the investigation of various nonlinear wave phenomena, the classical subjects on vibration of elastic body appear to be left out from interest of the modern researchers.

In the present paper, we examine the nonlinear transverse vibration of elastic beam under the end-thrust. Taking into account the full nonlinear relation of curvature and deformation of elastic beam, we obtain a nonlinear partial differential equation. When the end-thrust acts as tension, this nonlinear partial differential equation is reduced to one of the new integrable equation discovered by us[1)] through generalization of the inverse scattering method.

§ 2. Basic Equation

Following the analysis of Howland[2)], we can write down the equations of motion of the small element AB illustrated in Fig.1 as

$$\rho A \frac{\partial^2}{\partial t^2} y = \frac{\partial}{\partial x} S \qquad 1,a)$$

$$\rho I \frac{\partial^3}{\partial t^2 \partial x} y = \frac{\partial}{\partial x} M + P \frac{\partial}{\partial x} y + S \qquad 1,b)$$

where ρ is density of material, A stands for area of cross section and I for the moment of inertia. As shown in Fig.1, S is the stress resultant parallel to the axis of y, and P, the end-thrust parallel to the axis of x, is assumed to be constant. If effects of

Fig.1

the angular momentum were negligible, we may set $\rho I \to 0$ in eq.(1.b).

For the bending moment M, we have the following relation[3)]

$$M = \frac{EI}{R} = EI\frac{\partial^2 y/\partial x^2}{\{1+(\partial y/\partial x)^2\}^{3/2}} \quad , \qquad 2)$$

where E is Young's modulus, and R represents the radius of curvature of bending beam. Therefore, combining eqs. 1,a), 1,b) and 2), we obtain the following nonlinear partial differential equation

$$\frac{\partial^2}{\partial t^2} y + \frac{P}{\rho A}\frac{\partial^2}{\partial x^2} y + \frac{1}{\rho A}\frac{\partial^2}{\partial x^2}\{EI\frac{\partial^2 y/\partial x^2}{[1+(\partial y/\partial x)^2]^{3/2}}\}$$
$$= \frac{1}{\rho A}\frac{\partial}{\partial x}(\rho I\frac{\partial^3 y}{\partial t^2 \partial x}) \quad , \qquad 3)$$

which describes the nonlinear transverse vibrations of elastic beams under the end-thrust P. If the beam is suject to tension, P is taken to be negative.

§ 3. Reduction to an Intergrable Nonlinear Evolution Equation

Let us consider the uniform elastic beam under the tenstion $P=-T$. We assume also the effect of rotatory inertia is negligible. We treat the deformation effect of the cross section of beam as a small perturbation, of which effect is measured by the small parameter $\varepsilon \sim O(EI/\rho A)$. We introduce the following stretched variables ξ and τ as

$$\xi = x + \lambda t \qquad 4,a)$$

$$\tau = \varepsilon t/\sqrt{2\lambda} \qquad 4,b)$$

where λ is chosen to be

$$\lambda = \sqrt{\frac{T}{\rho A}} \qquad 5)$$

Thus, keeping up to the first order of ε, we can reduce eq.(3) to

$$\frac{\partial}{\partial\tau}\frac{\partial}{\partial\xi} y + \frac{\partial}{\partial\xi^2}\left\{\frac{\partial^2 y/\partial\xi^2}{[1+(\partial y/\partial\xi)^2]^{3/2}}\right\} = 0 \quad . \qquad 6)$$

Defining

$$q(\xi, \tau) = \partial y/\partial\xi \quad , \qquad 7)$$

we can see immediatedly that eq.(6) is nothing but the equation of the second kind of our new integrable nonlinear evolution equations with the choice of $r=-q$, that is

$$\frac{\partial}{\partial\tau} q + \frac{\partial^2}{\partial\xi^2}\left\{\frac{\partial q/\partial\xi}{[1+q^2]^{3/2}}\right\} = 0 \quad . \qquad 8)$$

§ 4. Concluding Remarks

In the preceeding sections, we have shown explicitly the second kind of the new integrable evolution equation discovered by us through generalization of the inverse scattering method describes the nonlinear transverse vibrations of the elastic beams under the tension.
It would be worthwhile to note that Shimizu and Wadati[4] have carried out the inverse scattering analysis of the first kind of the new nonlinear evolution equation discovered by us. We will present in a seperate paper the inverse scattering solution of the second kind of the new nonlinear evolution equation expressed as eq.8). The series of our analysis have demonstrated that the systematic generalization of the inverse scattering trasformation is not providing mere mathematical games, but serving to extend our understanding of the physical reality of the nature.

References

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3) I.S. Sokolnikoff, Mathematical Theory of Elasticity (McGraw-Hill, New York) (1956), 106.

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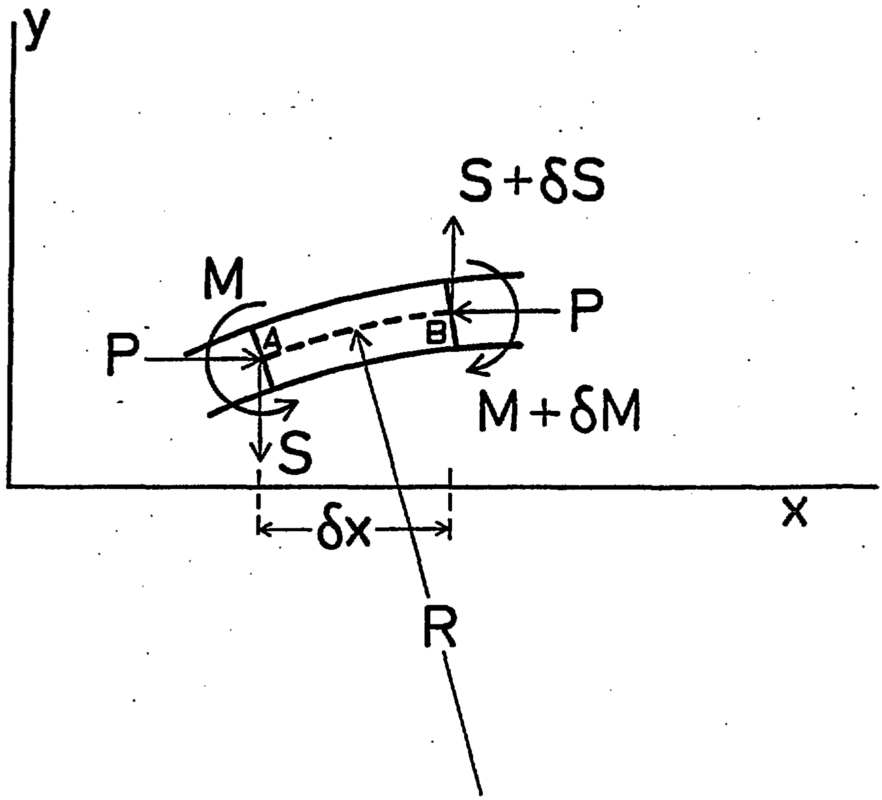
Caption of Figure

Fig. 1 Transverse displacement of elastic beam under the end-thrust.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