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$4^-$  States of  $^{16}\text{O}$

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Abstract:

A three-state isospin-mixing model is fitted to available data on pion scattering and nucleon pickup relevant to the  $4^-$  states of  $^{16}\text{O}$  at 17.79, 18.98 and 19.80 MeV. An excellent fit to all data is obtained, and the state descriptions are shown to be consistent with inelastic proton scattering measurements. The 18.98 MeV state is predominantly  $1p-1h$   $T=1$ , while the other two states have significant isospin mixing and  $3p-3h$  admixtures.

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## 1. Introduction

Three  $4^-$  states of  $^{16}\text{O}$  at 17.79, 18.98 and 19.80 MeV have been identified in the reactions  $^{17}\text{O}(d,t)^{16}\text{O}$  (Mairle et al. 1978),  $^{16}\text{O}(p,p)^{16}\text{O}$  (Henderson et al. 1979) and  $^{16}\text{O}(\pi^{\pm},\pi^{\pm})^{16}\text{O}$  (Holtkamp et al. 1980). One  $4^-$   $T=1$  state is expected in this energy region as the analogue of the 6.17 MeV state of  $^{16}\text{N}$  observed in  $^{17}\text{O}(d,^3\text{He})^{16}\text{N}$  (Mairle et al. 1978). These  $4^-$  states are of interest for several reasons.

Analysis of inelastic scattering and pickup data becomes simpler when  $1p-1h$  configurations predominate in the excited state. Shell model calculations in the  $1p-1h$  approximation predict two  $4^-$  states of  $^{16}\text{O}$  at about 19 MeV, both of the configuration  $1d_{5/2}^{-1}1p_{3/2}^{-1}$ , one with  $T=0$  and one with  $T=1$  (Elliott and Flowers 1957; Milliner and Kurath 1975). Additional  $4^-$   $T=0$  states are predicted near this energy when  $3p-3h$  configurations are included (Milliner and Kurath 1975). Mairle et al. (1978) used the  $^{17}\text{O}(d,t)$  and  $^{17}\text{O}(d,^3\text{He})$  reactions particularly in order to study the  $1d_{5/2}^{-1}1p_{3/2}^{-1}$  particle-hole strength distribution.

A further simplification occurs in inelastic proton scattering when the excited state has unnatural parity, since the angular momentum transfer and parity selection rules prohibit the direct Wigner contribution and so allow the detailed study of the remaining exchange and tensor  $t$ -matrix reaction contributions. This motivated the study by inelastic proton scattering of the  $4^-$  states in  $^{16}\text{O}$  (Henderson et al. 1979), following a similar study of the  $6^-$  states in  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  (Adams et al. 1977; Amos et al. 1978).

From  $^{16}\text{O}(\pi^{\pm},\pi^{\pm})$  there is evidence of isospin mixing in the  $4^-$  states of  $^{16}\text{O}$ , since the  $\pi^+$  and  $\pi^-$  cross sections are unequal for two of the three states observed (Holtkamp et al. 1980). There is appreciable isospin mixing in other  $^{16}\text{O}$  states, namely  $T=0$  and  $T=1$  pairs of  $0^-$ ,  $1^-$ ,  $2^-$  and  $3^-$  states, which appear to have dominant  $1p-1h$

components (Barker 1978). Previous analyses of the  $^{17}\text{O}(d,t)$  and  $^{16}\text{O}(p,p)$  data assumed  $4^-$  states of pure isospin (Mairle et al. 1978; Henderson et al. 1979). Holtkamp et al. (1980) included isospin mixing in their analysis of their  $^{16}\text{O}(\pi^{\pm},\pi^{\pm})$  data and claimed good agreement with the  $^{17}\text{O}(d,t)$  data, but made no comparison with the  $^{16}\text{O}(p,p)$  data.

In this work, we make somewhat different assumptions from those of Holtkamp et al. (1980). Also we fit the observed  $(\pi^{\pm},\pi^{\pm})$ ,  $(d,t)$  and  $(d,^3\text{He})$  data simultaneously, and use the resultant  $4^-$  wave functions to predict the  $(p,p)$  cross sections for comparison with the measured values.

## 2. Experimental Data

From their measured cross sections for the reactions  $^{17}\text{O}(d,t)^{16}\text{O}$  and  $^{17}\text{O}(d,^3\text{He})^{16}\text{N}$ , Mairle et al. (1978) extracted spectroscopic factors  $C^2S$  for the  $4^-$  states of  $^{16}\text{O}$  and  $^{16}\text{N}$ . These are given in Table 1, where the three  $^{16}\text{O}$  states are labelled A, B and C and the  $^{16}\text{N}$  state D. The uncertainties are taken as  $\pm 15\%$ , except for the state A for which Mairle et al. give the uncertainty explicitly.

In large angle electron scattering, a pronounced peak near 19 MeV excitation may be attributed to the  $4^- T=1$  state (Sick et al. 1969), while the weakness of possible peaks near 18 and 20 MeV indicates that there is little  $T=1$  strength in the states A and C.

Henderson et al. (1979) have measured differential cross sections for excitation of the  $4^-$  states in  $^{16}\text{O}(p,p)$  using 135 MeV protons. Their results are given in Fig.1 below.

Holtkamp et al. (1980) used 164 MeV  $\pi^+$  and  $\pi^-$  inelastic scattering to excite the  $4^-$  states of  $^{16}\text{O}$ , and measured the differential

cross sections over the angular range  $53^\circ$  to  $89^\circ$  (laboratory). Since their six angular distributions are all similar, they represented these data by cross sections summed over the angular range. Values of the summed cross sections  $\sigma^\pm$  are given in Table 1, with estimated uncertainties chosen to fit the uncertainties in the ratios  $R(\pi^+/\pi^-)$  quoted by Holtkamp et al.

### 3. Three-State Isospin-Mixing Model

Since the experimental evidence suggests only three  $4^-$  states of  $^{16}_0$  in the region near 19 MeV, and since the unequal yields for  $\pi^+$  and  $\pi^-$  scattering suggest that the states do not have pure isospin, we assume a three-state isospin-mixing model<sup>†</sup>. The three basis states are

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<sup>†</sup> Such a model has been used by Castel et al. (1980) to describe properties of the three  $2^-$  states of  $^{16}_0$  at 8.88, 12.53 and 12.97 MeV.

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taken to be the expected  $1p-1h$   $T=0$  and  $T=1$  states, and a  $3p-3h$   $T=0$  state (or at least a state with more complicated configuration than  $1p-1h$ ). Thus the orthonormal basis states are

$$\begin{aligned}
 |a\rangle &= |1d_{5/2} \ 1p_{3/2}^{-1} \ 4^-, \ T=1, \ M_T=0\rangle, \\
 |b\rangle &= |1d_{5/2} \ 1p_{3/2}^{-1} \ 4^-, \ T=0, \ M_T=0\rangle, \\
 |c\rangle &= |3p-3h \ 4^-, \ T=0, \ M_T=0\rangle.
 \end{aligned}
 \tag{1}$$

The configuration mixing and isospin mixing of these basis states to

form the eigenstates can be described conveniently by three Euler angles, giving

$$\begin{aligned}
 |A\rangle &= \cos\beta \cos\gamma |a\rangle + \sin\beta \cos\gamma |b\rangle + \sin\gamma |c\rangle \quad , \\
 |B\rangle &= (\cos\alpha \sin\beta + \sin\alpha \cos\beta \sin\gamma) |a\rangle + (-\cos\alpha \cos\beta + \sin\alpha \sin\beta \sin\gamma) |b\rangle \\
 &\quad - \sin\alpha \cos\gamma |c\rangle \quad , \quad (2) \\
 |C\rangle &= (\sin\alpha \sin\beta - \cos\alpha \cos\beta \sin\gamma) |a\rangle + (-\sin\alpha \cos\beta - \cos\alpha \sin\beta \sin\gamma) |b\rangle \\
 &\quad + \cos\alpha \cos\gamma |c\rangle \quad .
 \end{aligned}$$

We note that the values  $\alpha=0$  and  $\beta=\pi/2$  give no isospin mixing, with the state  $|B\rangle$  being pure  $T=1$ .

Similarly the  $4^-$   $T=1$  state in  $^{16}\text{N}$  is written as

$$|D\rangle = |d\rangle = |1d_{5/2} \ 1p_{3/2}^{-1} \ 4^- \ , \ T=1 \ , \ M_T=1) \quad . \quad (3)$$

Then, if the spectroscopic amplitudes for the basis states in the reactions  $^{17}\text{O}(d,t)$  and  $^{17}\text{O}(d,^3\text{He})$  are written  $f_i$  ( $i=a,b,c,d$ ), one has

$$(C^2S)_A = |\cos\beta \cos\gamma f_a + \sin\beta \cos\gamma f_b + \sin\gamma f_c|^2 \quad , \quad (4)$$

and similar equations for states B, C and D. With the usual assumption that these reactions are direct and populate only the  $1p-1h$  components of the states, one has

$$f_i \propto (2T+1)^{1/2} (T \ 1/2 \ M_T \ 1/2 - M_T | 1/2 \ 1/2) \quad , \quad (5)$$

giving

$$f_b = -f_a, \quad f_c = 0, \quad f_d = -\sqrt{2} f_a. \quad (6)$$

It is easy to see that this model requires

$$(C^2S)_A + (C^2S)_B + (C^2S)_C = (C^2S)_D, \quad (7)$$

a relation that is satisfied approximately by the experimental values. For no isospin mixing,  $(C^2S)_B = \frac{1}{2} (C^2S)_D$ , which is also satisfied approximately, so that these experimental  $C^2S$  values by themselves do not necessitate isospin mixing.

Similarly for the  $\pi^\pm$  inelastic scattering, one has amplitudes  $g_i^\pm$  ( $i = a, b, c$ ) such that

$$\sigma_A^\pm = |\cos\beta \cos\gamma g_a^\pm + \sin\beta \cos\gamma g_b^\pm + \sin\gamma g_c^\pm|^2, \quad (8)$$

and similar equations for states B and C. These amplitudes satisfy

$$g_a^+ = -g_a^-, \quad g_b^+ = g_b^-, \quad g_c^+ = g_c^-. \quad (9)$$

Thus no isospin mixing implies  $\sigma^+ = \sigma^-$  for each eigenstate. On the assumption that only  $1p-1h$  components of the states are populated,

$$g_c^+ = 0. \quad (10)$$

If in addition only the  $T = \frac{3}{2}$  pion-nucleon channel contributes, due to (3,3) dominance, then (Holtkamp et al. 1980)

$$g_b^+ = 2 g_a^+. \quad (11)$$

#### 4. Inelastic Proton Scattering Formalism

The transition amplitudes for direct reaction inelastic scattering of protons from nuclei can be written (Geramb and Amos 1971)

$$T_{if} = \sum_{j_1 j_2 I N \tau} S(j_1 j_2; J_i J_f; I)_{\tau} (J_i \ I \ M_i \ N | J_f \ M_f) M_{j_1 j_2}^{SP}(\tau) \quad (12)$$

The spectroscopic amplitude  $S$  is defined by

$$S(j_1 j_2; J_i J_f; I)_{\tau} = \langle J_f || (a_{j_2}^{\dagger} \times a_{j_1})^I || J_i \rangle_{\tau} \quad (13)$$

where  $\tau$  is the target nucleon isospin projection ( $\tau = \pi$  (proton),  $\nu$  (neutron)). For scattering to the basis states |a) and |b) defined in (1), it is easily seen that

$$S(\frac{3}{2} \ \frac{5}{2}; 04; 4)_{\tau} = \sqrt{2} (\delta_{\tau\pi} + (-)^T \delta_{\tau\nu}) \quad (14)$$

where  $T$  is the final state isospin.

In DWBA the single-particle transition amplitudes have the form

$$M_{j_1 j_2}^{SP}(\tau) = (2J_f + 1)^{-1/2} \sum_{m_1 m_2} (-)^{j_1 - m_1} (j_1 \ j_2 \ m_1 \ -m_2 | I \ -N) \\ \langle \chi_f^{(-)}(1) \phi_{j_2 m_2 \tau}(2) | t(12) | \chi_i^{(+)}(1) \phi_{j_1 m_1 \tau}(2) - \chi_i^{(+)}(2) \phi_{j_1 m_1 \tau}(1) \rangle \quad (15)$$

The bound state wave functions  $\phi_{j m \tau}$  are assumed to be harmonic oscillator (with  $\hbar\omega = 41 A^{-1/3}$  MeV). The continuum wave functions  $\chi_i$  and  $\chi_f$  are assumed to be distorted waves generated in an optical potential well chosen to describe the appropriate elastic proton scattering. Two possible parameter sets for the optical potentials are given in Table 2. Set 1 provides a "best" fit to data from the elastic

scattering of 135 MeV protons on  $^{16}\text{O}$  (Norum et al. 1978; Bertozzi, private communication). This is strongly preferred over set 2, which is an "average" parameter set similar to those used in analyses of data from the elastic scattering of 100 and 155 MeV protons on  $^{14}\text{N}$ ,  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  (Willis et al. 1968; Geoffrion et al. 1968; Horowitz 1972), but which overestimates the forward angle elastic cross section for  $^{16}\text{O}$  by a factor of two.

For the two-nucleon t-matrix  $t(12)$  we use a central even-state plus tensor form of potential (Smith and Amos 1979), which has been used extensively in inelastic nucleon scattering analyses (Geramb et al. 1975; Smith and Amos 1979) and which has been shown to be comparable with forms recently derived from the Reid, Elliott and Hamada-Johnston potentials (Smith and Amos 1979). The tensor strength we use is 70% of that obtained originally by Eikemeier and Hackenbroich (1971) from fitting nucleon-nucleon scattering data. The full tensor strength was used in the earlier analyses (Geramb et al. 1975; Smith and Amos 1979); however, these were relatively insensitive to the tensor strength, and subsequent analyses of scattering to  $6^-$  particle-hole states in  $^{24}\text{Mg}$  and  $^{28}\text{Si}$  (Amos et al. 1978), which are sensitive to the tensor strength, show that the above strength reduction is necessary.

## 5. Fits to Data

We adjust the mixing parameters (Euler angles) and amplitudes  $(f_i, g_i^\pm)$  to give a least squares fit to the  $^{17}\text{O}(d,t)$ ,  $^{17}\text{O}(d,^3\text{He})$  and  $^{16}\text{O}(\pi^\pm, \pi^\pm)$  experimental data given in Table 1. We then test the resultant wave functions by comparing the  $^{16}\text{O}(p,p)$  cross sections that



they predict with the measured values.

In the least squares fit, we assume that the  $f_i$  satisfy (6) and the  $g_i^\pm$  satisfy (9), and moreover that the  $g_i^\pm$  are all real. Fits are made with and without the additional restrictions (10) and/or (11). The results are given in Table 3.

The  $\chi^2/(\text{degree of freedom})$  values in Table 3 show that the data are well fitted when neither of the additional restrictions (10) and (11) is imposed (fit I). Imposition of (10) (fit II) slightly improves the fit, so that these data give no evidence for feeding of the  $3p-3h$  component in  $\pi^\pm$  inelastic scattering, which might have been expected from multiple scattering. On the other hand, the additional imposition of (11) (fit III) does seriously worsen the fit, suggesting that contributions from  $T=\frac{1}{2}$  pion-nucleon channels are significant. The value  $g_b^+/g_a^+ = 1.44$  implies a  $T=\frac{1}{2}$  amplitude about 20% of the  $T=\frac{3}{2}$  amplitude; such a ratio does not seem to be unreasonable since the s-wave  $T=\frac{1}{2}$   $J=\frac{1}{2}$  phase shift for  $\pi$ -N scattering is about  $10^\circ$  in the region of the (3,3) resonance (Koch and Pietarinen 1980; Zidell et al. 1980).

In the remainder of this paper, we consider only the results of fit II. The calculated values of the fitted quantities are given in Table 4. The eigenstates are

$$\begin{aligned}
 |A\rangle &= 0.130 |a\rangle + 0.683 |b\rangle + 0.718 |c\rangle \quad , \\
 |B\rangle &= 0.984 |a\rangle + 0.000 |b\rangle - 0.178 |c\rangle \quad , \\
 |C\rangle &= 0.122 |a\rangle - 0.730 |b\rangle + 0.673 |c\rangle \quad .
 \end{aligned}
 \tag{16}$$

It is seen that the state |B) is predominantly  $T=1$ , which is consistent with the inelastic electron scattering results of Sick et al. (1969). It then follows from the model that

$$R \equiv (\sigma_A^+ + \sigma_A^- + \sigma_C^+ + \sigma_C^-) / (\sigma_B^+ + \sigma_B^-) = (\xi_b^+ / \xi_a^+)^2 \quad (17)$$

Experimentally  $R = 2.2$ , explaining why improved fits can be obtained by reducing  $\xi_b^+ / \xi_a^+$  below the value of 2 given in (11).

The  $^{16}_0(p,p)$  differential cross sections for excitation of the  $4^-$  states of  $^{16}_0$  by 135 MeV incident protons, calculated using the preferred optical potential parameter set 1, are compared with the experimental values of Henderson et al. (1979) in Fig.1. The predictions of the central and tensor forces alone are also shown, from which it can be seen that the tensor contribution dominates in each case, although the central contribution is significant for excitation of the mainly  $T=1$  state at 18.98 MeV. Use of the parameter set 2 increases the magnitudes of all cross sections by approximately 20%, without changing their shapes appreciably.

Although the spectroscopic amplitudes  $S$  used in the calculation of the  $^{16}_0(p,p)$  cross section are sensitive to isospin mixing, the cross sections themselves are not in this case. Table 5 compares values of  $S$  for the wave functions (16) of fit II with those for no isospin mixing ( $\alpha=0^\circ$ ,  $\beta=90^\circ$ ,  $\gamma=45.92^\circ$ ). In spite of the very different proton and neutron values of  $S$  for the 17.79 and 19.80 MeV states, due to isospin mixing, the cross sections for no isospin mixing differ from the curves in Fig.1 by less than 10%.

## 6. Discussion

Descriptions of the  $4^-$  states of  $^{16}_0$  at 17.79, 18.98 and 19.80 MeV, as given by equations (16) and (1), have been found to provide

calculated values in good agreement with experimental data obtained from the  $^{17}\text{O}(d,t)$ ,  $^{16}\text{O}(\pi^{\pm},\pi^{\pm})$  and  $^{16}\text{O}(p,p)$  reactions. These three states exhaust all the  $1p-1h$   $4^{-}$  strength in  $^{16}\text{O}$  for both  $T=0$  and  $T=1$  states. The  $T=1$  strength lies principally in the 18.98 MeV state, although the  $T=1$  admixtures in the 17.79 and 19.80 MeV states are obviously essential to fit the  $(\pi^{\pm},\pi^{\pm})$  data. Such admixtures are also required by the  $(p,p)$  and  $(d,t)$  data together, as the following argument shows. The mainly  $T=0$  states at 17.79 and 19.80 MeV each have approximately equal amounts of  $1p-1h$  and  $3p-3h$  configurations; this follows in particular from the approximate equality of the  $(p,p)$  cross sections for these states, which are sensitive to configuration mixing but not to isospin mixing. Then the spectroscopic factors for these states observed in the  $(d,t)$  reaction would be equal if they had no  $T=1$  admixtures. The observed difference in  $C^2S$  values therefore indicates isospin mixing. The sign of this difference is such that the lower state is due preferentially to a proton excitation, the upper state to a neutron excitation; this is similar to but less extreme than the case of almost complete isospin mixing in the  $2^{+}$  states of  $^8\text{Be}$  at 16.63 and 16.92 MeV, where the  $C^2S$  values for the  $^9\text{Be}(d,t)$  reaction are in the ratio 1:20 (Oothoudt and Garvey 1977).

With our three-state model, we found that a good fit to all the data could be obtained only if we relaxed the commonly made assumption, based on  $(3,3)$  dominance, that the  $(\pi^{\pm},\pi^{\pm})$  cross sections are due entirely to contributions from the  $T=3/2$  pion-nucleon channel. Holtkamp et al. (1980) made this assumption and were still able to fit the  $(\pi^{\pm},\pi^{\pm})$  cross sections. The reason for this is that they did not restrict themselves to a model with three basis states. The three eigenstates of their model exhaust at most 54% of the  $1p-1h$   $T=0$  strength. Consequently at least 46% of this strength must be located elsewhere, and one could

have reasonably expected such strength to be evident in the  $\pi^+$  and  $p$  inelastic scattering. For the  $^{17}\text{O}(d,t)$  and  $^{17}\text{O}(d,^3\text{He})$  reactions, Holtkamp's model would modify the relation (7) to  $(C^2S)_A + (C^2S)_B + (C^2S)_C \leq 0.77 (C^2S)_D$ , which is not consistent with the experimental values. Also the smaller  $1p-1h$   $T=0$  strength in Holtkamp's states would result in a reduction of the calculated  $^{16}\text{O}(p,p)$  cross sections to the 17.79 and 19.80 MeV states by 46% or more compared with the curves in Fig.1, which would clearly be in disagreement with the experimental values.

From the description (16) of the eigenstates and their observed energies, the matrix elements of the Hamiltonian coupling the basis states are found to be (in MeV)

$$\begin{aligned} H_{aa} &= 18.972 & , & & H_{bb} &= 18.861 & , & & H_{cc} &= 18.737 & , \\ H_{ab} &= -0.179 & , & & H_{ac} &= -0.044 & , & & H_{bc} &= -0.987 & . \end{aligned}$$

The quantities of most significance are probably  $H_{aa} - H_{bb}$ , the separation of the  $1p-1h$   $T=0$  and  $T=1$  states, and  $H_{ab}$ , the isospin-mixing matrix element coupling these states. Shell model values of  $H_{aa} - H_{bb}$  vary considerably, for example Elliott and Flowers (1957) obtained about -0.9 MeV and Milliner and Kurath (1975) about +0.5 MeV; these may be compared with our deduced value of 0.11 MeV. Our value of -179 keV for  $H_{ab}$  differs somewhat from the  $-240 \pm 40$  keV obtained by Holtkamp et al. (1980). Values of  $H_{ab}$  may be calculated using the method described in Barker (1978). Since there is only the one  $1p-1h$  configuration giving a  $4^-$  state of  $^{16}\text{O}$ , the internal contribution to  $H_{ab}$  is unique<sup>†</sup> and equals -190 keV (for  $b = 1.83$  fm). The surface

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<sup>†</sup> It should be noted that  $H_{ab}$  is a matrix element between pure  $1p-1h$

states, whereas the matrix elements discussed in Barker (1978) were between states that possibly contained many particle-many hole admixtures.

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contribution depends on the assumed channel radius  $a$ . Values of  $R_{ab}$  are given in Table 6 for various values of  $a$ . In many other cases, agreement between calculated and experimental values of the isospin-mixing matrix element has been obtained with reasonable values of the channel radius of about 4 to 6 fm (Barker 1978; Barker and Ferdous 1978). Here the calculated values for such channel radii are more negative than the value we have deduced from experiment, and are more consistent with the value obtained by Holtkamp et al.

## 7. Conclusion

A model of the three  $4^-$  states of  $^{16}\text{O}$  near 19 MeV, based on the  $1p-1h$   $T=0$  and  $T=1$  states and a  $3p-3h$   $T=0$  state, gives calculated values in good agreement with recent data from nucleon pickup reactions and inelastic electron, proton and pion scattering. The 18.98 MeV state is predominantly  $T=1$ , while the 17.79 and 19.80 MeV states have approximately equal  $1p-1h$  and  $3p-3h$  strengths, with significant  $T=1$  admixtures. The assumption that only the  $T=\frac{3}{2}$   $J=\frac{3}{2}$  pion-nucleon channel is important has to be relaxed in order to obtain a good fit to all the data, but the required  $T=\frac{1}{2}$  contribution is not unreasonable. The isospin-mixing matrix element between the  $1p-1h$  states is found to be -179 keV, in fair agreement with calculations.

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Table 1. Experimental values for  $4^-$  states of  $^{16}\text{O}$  and  $^{16}\text{N}$

	State	A	B	C	D	
	$E_x$ (MeV)	17.79	18.98	19.80	6.17	
Reaction	Quantity					Ref.
$^{17}\text{O}(d,t)^{16}\text{O}$	$C^2S$	$0.17 \pm 0.05$	$0.73 \pm 0.11$	$0.52 \pm 0.08$		a
$^{17}\text{O}(d,^3\text{He})^{16}\text{N}$	$C^2S$				$1.20 \pm 0.18$	a
$^{16}\text{O}(\pi^+, \pi^+)^{16}\text{O}$	$\sigma^+$ (mb/sr)	$0.334 \pm 0.015$	$0.261 \pm 0.015$	$0.231 \pm 0.015$		b
$^{16}\text{O}(\pi^-, \pi^-)^{16}\text{O}$	$\sigma^-$ (mb/sr)	$0.210 \pm 0.015$	$0.271 \pm 0.015$	$0.382 \pm 0.015$		b

a Mairle et al. (1978)

b Holtkamp et al. (1980)



Table 2. Optical potential parameters used in  $^{16}\text{O}(p,p)$  analyses for 135 MeV incident protons

Set	$V_0$ (MeV)	$r_0$ (fm)	$a_0$ (fm)	$W$ (MeV)	$r_I$ (fm)	$a_I$ (fm)	$V_s$ (MeV)	$r_s$ (fm)	$a_s$ (fm)	$r_C$ (fm)
1	11.35	1.24	0.75	10.56	1.52	0.50	4.70	1.04	0.48	1.25
2	20.0	1.25	0.70	8.0	1.30	0.60	7.0	1.08	0.61	1.25

Table 3. Values of fitting parameters for  $4^-$  states of  $^{16}\text{O}$

Fit	I	II	III
$g_c^+/g_a^+$	0.079	0*	0*
$g_b^+/g_a^+$	1.440	1.441	2*
$\alpha$ (deg)	15.34	14.83	15.34
$\beta$ (deg)	78.87	79.23	78.92
$\gamma$ (deg)	48.95	45.92	45.65
$\chi^2$ /(deg. of freedom)	0.79	0.73	27

\* Value kept fixed in fit.

Table 4. Calculated values for  $4^-$  states of  $^{16}\text{O}$  and  $^{16}\text{N}$  from fit II

Quantity	State	A	B	C	D
$c^2s$		0.203	0.641	0.480	1.324
$\sigma^+$ (mb/sr)		0.341	0.266	0.237	
$\sigma^-$ (mb/sr)		0.200	0.266	0.378	

Table 5. Spectroscopic amplitudes for  $^{16}\text{O}(p,p)$  to  $4^-$  states

$4^-$ state	Nucleon	$S(\frac{3}{2} \frac{5}{2}; 04; 4)/\sqrt{2}$	
		Fit II	No isospin mixing
17.79	$\pi$	0.813	0.696
	$\nu$	0.553	0.696
18.98	$\pi$	0.984	1.000
	$\nu$	-0.984	-1.000
19.80	$\pi$	-0.608	-0.718
	$\nu$	-0.852	-0.718

Table 6. Calculated values of  $H_{ab}$

a (fm)	3	4	5	6	7
$H_{ab}$ (keV)	-532	-454	-328	-260	-227

Figure caption

Fig. 1. Differential cross sections for excitation of  $4^-$  states of  $^{16}\text{O}$  by inelastic scattering of 135 MeV incident protons. The experimental points are from Henderson et al. (1979). The full curves are calculated values using the optical potential parameters set 1 of Table 2 and the wave functions (16). The predictions of the central and tensor forces alone are shown by the dashed and dot-dashed curves respectively.

