CHROMOMAGNETIC VACUUM FIELDS IN A SPHERICAL BAG

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ABSTRACT

Vacuum expectation values of the gauge fields within a spherical bag is calculated. The colourmagnetic contribution is dominant and the energy density is everywhere negative.

In a non-perturbative attempt to incorporate confinement in the hadronic ground state wave functional, Johnson [1] has recently proposed a model where the QCD vacuum is filled with empty bags. The boundary conditions on the bag surfaces are made to confine colour. When the sizes of the bags are sufficiently small, one can use asymptotically free perturbation theory within each of them.

The empty bags are supposed to be stabilized by vacuum field fluctuations. To lowest order in the coupling constant, i.e. free field theory, these give rise to a repulsive Casimir force. In the next order there will be an attractive force due to the spin interactions between the elementary quanta of QCD, quarks and gluons. These two effects give rise to a minimum energy stable state for a certain size of the bag when one uses the running coupling constant depending on the bag radius. The size of the stable, empty bag will therefore be given directly in terms of the fundamental QCD parameter Λ. Masses of hadronic states where there are quark or gluon excitations within the bag, are then also given in terms of this parameter.

This is obviously a crude, but physically attractive picture of the hadronic ground state $|\Omega\rangle$. Phenomenologically, we know [2] that it is dominantly chromomagnetic, i.e. $\langle\Omega|F_{\mu\nu}F^{\mu\nu}|\Omega\rangle > 0$ and with broken chiral symmetry, $\langle\Omega|\bar{q}q|\Omega\rangle < 0$. It is a priori not obvious if the empty bag vacuum model has these properties. Here we will present some results of a calculation [3] which supports the chromomagnetic nature of this vacuum state.

We will consider a spherical bag with a radius a so small that one can set the QCD coupling constant equal to zero within the bag. The theory is then Abelian like QED. For simplicity, we will consider the contributions to the vacuum fluctuations from one colour only. The results obtained must then be multiplied by a factor eight for the case of a SU(3) gauge theory.

In terms of the colourelectric field $\underline{E}$ and the colourmagnetic field $\underline{B}$ the colour confining boundary condition $n_\mu F^{\mu\nu} = 0$ becomes $\underline{n}\cdot\underline{E} = \underline{n}\times\underline{B} = 0$ on the surface of the sphere with normal vector $\underline{n}$. For a QED cavity with a perfectly conducting boundary the roles of $\underline{E}$ and $\underline{B}$ are interchanged.

Until now only the total Casimir energy for spherical shells has been calculated [4-6, 11]. Bender and Hays [7] have investigated the Casimir energy within a spherical bag. This has more recently also been done by Milton [8]. Our calculation is a local extension of these previous contributions.

The gauge field within the bag is a superposition of TE and TM eigenmodes labelled by the angular momentum ℓ and a radial quantum number n. Mathematically these can be described within the standard formulation as used for instance by Milton et al. [6]. After quantization we then find that the vacuum expectation values at radius r can be written as:

$$\langle\Omega|\underline{B}^2(r)|\Omega\rangle = \frac{1}{8\pi}\sum_{\ell=1}^{\infty}\sum_n \Big\{N_E^2\,\omega_{\ell n}(2\ell+1)\,j_\ell^2(\omega_{\ell n}r)\,e^{-\omega_{\ell n}\tau} + N_M^2\,\bar\omega_{\ell n}\Big[(\ell+1)\,j_{\ell-1}^2(\bar\omega_{\ell n}r) + \ell j_{\ell+1}^2(\bar\omega_{\ell n}r)\Big]\,e^{-\bar\omega_{\ell n}\tau}\Big\} \tag{1}$$

$$\langle\Omega|\underline{E}^2(r)|\Omega\rangle = \frac{1}{8\pi}\sum_{\ell=1}^{\infty}\sum_n \Big\{N_M^2\,\bar\omega_{\ell n}(2\ell+1)\,j_\ell^2(\bar\omega_{\ell n}r)\,e^{-\bar\omega_{\ell n}\tau} + N_E^2\,\omega_{\ell n}\Big[(\ell+1)\,j_{\ell-1}^2(\omega_{\ell n}r) + \ell j_{\ell+1}^2(\omega_{\ell n}r)\Big]\,e^{-\omega_{\ell n}\tau}\Big\} \tag{2}$$

Here N_E and N_M are field normalization constants:

$$N_E^2 = \frac{2}{a^3}\,\frac{1}{[j_\ell(z)]'^2}\Bigg|_{z=\omega_{\ell n}a}$$

$$N_M^2 = \frac{2}{a^3}\,\frac{z^2-\ell(\ell+1)}{[zj_\ell(z)]''^2}\Bigg|_{z=\bar\omega_{\ell n}a}$$

The eigenfrequencies $\omega_{\ell n}$ and $\bar\omega_{\ell n}$ are given by the TE and TM boundary conditions on the surface of the sphere:

$$j_\ell(\omega_{\ell n} r)\Big|_{r=a} = 0 \tag{3}$$

$$\frac{d}{dr}\, r j_\ell(\bar{\omega}_{\ell n} r)\Big|_{r=a} = 0 \tag{4}$$

Since the field expectation values are divergent we have introduced a cut-off τ in (1) and (2). This should be taken to zero at the end of the summations. It arises formally from letting the time difference between the two coincident field points become imaginary.

We see that the field expectation values at the center $r = 0$ of the sphere only get contributions from the $\ell = 1$ modes. The corresponding eigenvalues are then found from (3) and (4) as asymptotic expansions in n^{-1}. These turn out to be quite accurate all the way down to small n. By similarly calculating the derivatives of the expectation values in the same point $r = 0$, we find that they can be written as

$$\langle\Omega|\underline{B}^2(r)|\Omega\rangle = \frac{3}{\pi^2\tau^4} + \frac{1}{a^4}\sum_{s=0} A_s^B \left(\frac{r}{a}\right)^{2s} \tag{5}$$

$$\langle\Omega|\underline{E}^2(r)|\Omega\rangle = \frac{3}{\pi^2\tau^4} + \frac{1}{a^4}\sum_{s=0} A_s^E \left(\frac{r}{a}\right)^{2s} \tag{6}$$

where A_s^B and A_s^E are finite numerical coefficients independent of the cut-off τ. The first terms in these two expansions are the free field expectation values when there are no physical boundaries in space. These are subtracted away when we later refer to the fields within the spherical bag.

For the lowest expansion coefficients in (5) and (6) we have obtained the following numerical values:

$$A_o^B \simeq +0.14 \quad , \quad A_1^B \simeq +0.61$$

$$A_o^E \simeq -0.24 \quad , \quad A_1^E \simeq -1.00$$

At least in the center of the bag the magnetic field is seen to dominate. The accuracy of these results are related to the number of terms we keep in the asymptotic expansions of the eigenfrequencies. For the problem at hand there is no real need for better numerical accuracy.

In order to find the field expectation values near the surface of the bag where $r \to a$, we would need many more higher coefficients in the expansions (5) and (6) since in this region modes with large ℓ contribute. Instead of undertaking such a hopelessly lengthy and boring calculation, we have rewritten the summation over the radial quantum number in (1) and (2) as a complex integration over eigenfrequencies ω. The integrands are given factors containing Bessel functions and their derivatives which produce discrete poles corresponding to the eigenvalue equations (3) and (4). The integration contour around the real, positive ω-axis can then be rotated into the imaginary axis. This procedure gives integrals over Bessel functions with complex arguments. These can be identified with individual terms of formula (3.9) in Milton et al. [6] or (2) in Milton [8]. As also pointed out by him these do not agree completely with the expressions of Bender and Hays [7].

Near the surface the dominant contributions come from modes with large $\sqrt{\ell^2 + n^2}$. These terms can therfore be obtained after a Debye expansion of the Bessel functions. Details of these calculations will be published elsewhere [3]. We find

$$\langle\Omega|\underline{B}^2(r)|\Omega\rangle = \frac{3}{\pi^2\tau^4} + \frac{1}{\pi^2 a^4}\left(\frac{3}{16\varepsilon^4} + \frac{13}{60\varepsilon^2} + \cdots\right) \tag{7}$$

$$\langle\Omega|\underline{E}^2(r)|\Omega\rangle = \frac{3}{\pi^2\tau^4} - \frac{1}{\pi^2 a^4}\left(\frac{3}{16\varepsilon^4} + \frac{17}{60\varepsilon^3} + \cdots\right) \tag{8}$$

where $\varepsilon = 1 - r/a$. Again we have isolated a divergent, free term which can be subtracted away. The remaining terms are independent of the cut-off τ. They are seen to diverge near the surface where $r \to a$. We are now in the process of calculating the coefficients of the ε^{-2} terms in these expansions.

These field expectation values are plotted in Fig. 1 together with the previously calculated values around $r = 0$. Expecting a smooth transition between these two regions, they are joined by dotted lines.

The vacuum expectation value of $F^2_{\mu\nu} = 2(\underline{B}^2 - \underline{E}^2)$ near the surface becomes:

$$\langle\Omega|F^2_{\mu\nu}(r)|\Omega\rangle = \frac{1}{\pi^2 a^4}\left(\frac{3}{4\varepsilon^4} + \frac{1}{\varepsilon^3} + \cdots\right) \tag{9}$$

It is positive which shows that also here the vacuum is dominated by magnetic fields as conjectured by Johnson [9]. This is in agreement with what is known about the physical QCD vacuum [2]. Due to the rapid variations of the field within the bag, there is no purpose in trying to compare its absolute size with phenomenologically established values. All we can say is that it is as expected set by the bag radius.

It is also of interest to find the vacuum expectation value of the energy density $u = \frac{1}{2}(\underline{E}^2 + \underline{B}^2)$:

$$\langle\Omega|u(r)|\Omega\rangle = \frac{3}{\pi^2 \tau^4} - \frac{1}{\pi^2 a^4}\left(\frac{1}{30\varepsilon^3} + \cdots\right) \tag{10}$$

We have plotted this result minus the free term in Fig. 1 together with the corresponding values found around $r = 0$. The vacuum energy density is seen to be negative within the bag.

One notices that the term going like ε^{-4} in (10) has cancelled out. This is in agreement with more general conclusions of Deutsch and Candelas [10]. They have shown that the most divergent term in the vacuum energy density near a boundary should go like ε^{-3} for all conformally invariant theories. The absolute size we have found for this term is also in agreement with their numerical results and with a similar calculation of Balian and Duplantier [11].

The energy density (10) is smaller than the field density (9). This can be understood as an attempt of the bag vacuum to be Lorentz invariant [12]. With a Lorentz invariant vacuum $|0\rangle$ we would have had

$$\langle 0|F_{\mu\nu}F_{\rho\sigma}|0\rangle = A(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$

This gives $\langle 0|\underline{B}^2|0\rangle = 3A$ and $\langle 0|\underline{E}^2|0\rangle = -3A$ so that $\langle 0|u|0\rangle = 0$. In the center of the bag these expressions of Lorentz invariance are roughly satisfied.

The total vacuum field energy within the bag can be obtained by integrating up the energy density. This will give a negative, total energy which will diverge when the upper integration limit approaches the surface. Very recently, Milton [8] has been able to calculate this quantity without having to calculate the energy density first. He also finds a negative, divergent answer when the cut-off is taken to zero. But still he manages to isolate a finite, positive piece which he identifies with the inside Casimir energy of the bag. It is not clear how meaningful it is to talk about a positive energy of the empty bag when the energy density within the bag is everywhere negative.

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References

1) K. Johnson, preprint SLAC-PUB-2436 (1979).

2) M. Shifman, A. Vainshtein and V. Zakharov. Nucl. Phys. B147 (1979) 385.

3) K. Olaussen and F. Ravndal, in preparation.

4) T.H. Boyer, Phys. Rev. 174 (1968) 1764.

5) B. Davies, J. Math. Phys. 13 (1972) 1324.

6) K.A. Milton, L.L. DeRaad, Jr. and J. Schwinger, Ann. Phys. (N.Y.) 115 (1978) 388.

7) C.M. Bender and P. Hays, Phys. Rev. D14 (1976) 2622.

8) K.A. Milton, Phys. Rev. D22 (1980) 1441.

9) K. Johnson, private communication.

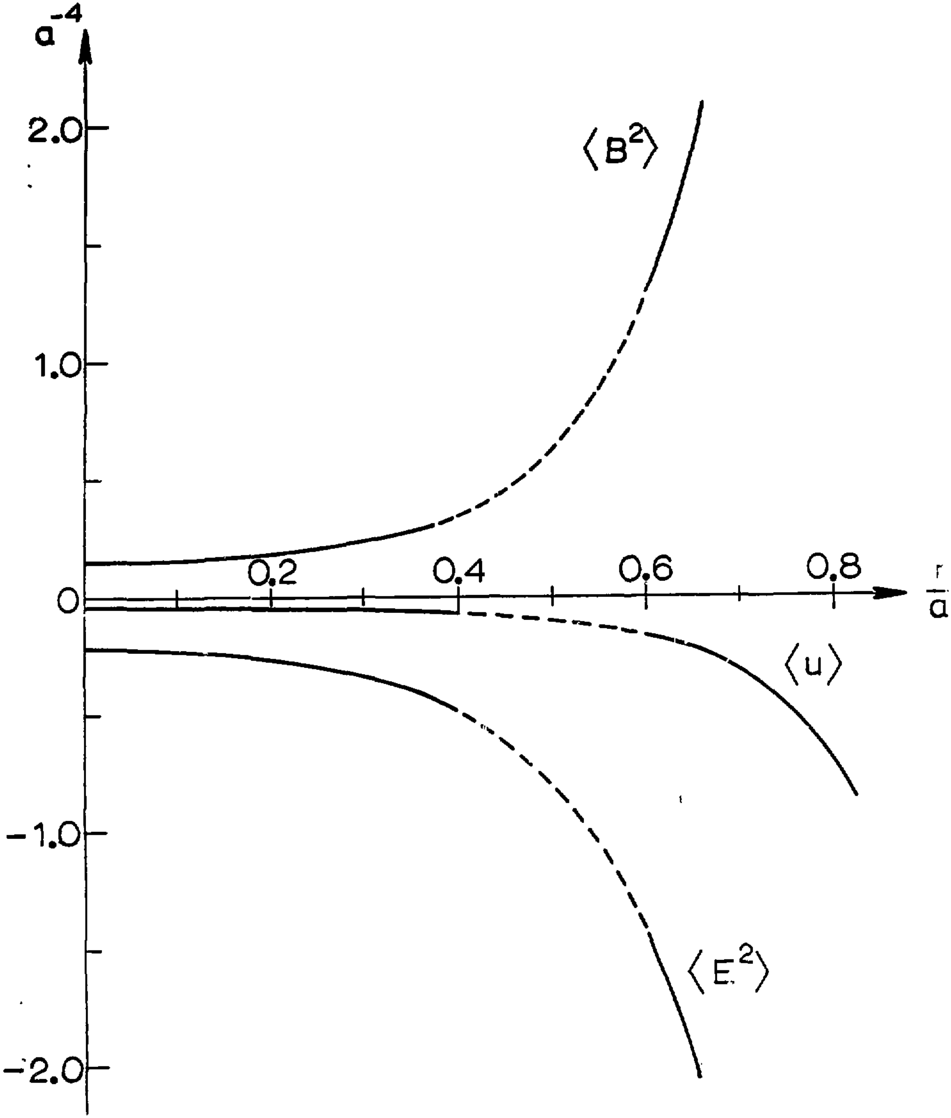
10) D. Deutsch and P. Candelas, Phys. Rev. D20 (1979) 3063.

11) R. Balian and B. Duplantier, Ann. Phys. (N.Y.) 112.

12) P. Minkowski, private communication. See also H.Leutwyler, Vacuum fluctuations surrounding soft gluon fields, University of Bern, preprint (1980).

Figure Captions

Fig. 1: Vacuum fields and energy density expectation values in a spherical bag.



a^-4
2.0
1.0
0
-1.0
-2.0
0.2
0.4
0.6
0.8
r/a
⟨B²⟩
⟨u⟩
⟨E²⟩