STATISTICAL THEORY APPLICATIONS AND ASSOCIATED COMPUTER CODES

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ABSTRACT

A. Theoretical Description

The general format is along the same lines as that used in the O.M. Session, i.e. an introduction to the nature of the physical problems and methods of solution based on the statistical model of the nucleus.

Both binary and higher multiple reactions are considered.

B. Workshop (Problem Session)

The computer codes used in this session are a combination of optical model and statistical theory.

As with the O.M. sessions, the preparation of input and analysis of output are thoroughly examined. Again, comparison with experimental data serves to demonstrate the validity of the results and possible areas for improvement.

A. Introduction

1.0 Purpose and Nature of Physical Problems

The statistical model given here is due to Wolfenstein [1] and Hauser and Feshbach.[2]

The theory is based on the assumption that all states of the compound nucleus which can be excited according to the conservation of energy, angular momentum and parity do take part in the reaction, however, the formation and decay of the compound nucleus takes place in an incoherent manner. The result of this evaporation theory for the cross section $\sigma\langle\alpha|\alpha'\rangle$ integrated over all angles of the outgoing particle pairs and averaged over the resonance structure may be written.

$$\sigma\langle\alpha|\alpha'\rangle = \pi\lambda\!\!\!^{-2} \sum_{J,j,\ell,j',\ell'} \left\{(2J+1)/(2i+1)(2I+1)\right\} \times \left\{T^J_{\alpha' j'\ell'}\, T^J_{\alpha, j\ell} \Big/ \sum_{\alpha'' j'' \ell''} T^J_{\alpha'' j'' \ell''}\right\} \qquad 1.0$$

where ℓ is the orbital angular momentum of the incoming particle, $j = I + i = J - \ell$ its channel spin, while ℓ' and $j' = J - \ell$ are the corresponding values for the emitted particle. The symbols α and α' define a set of values characterizing the entrance and the exit channels, respectively. T stands for transmission coefficients; they are related to the optical-model phase shifts $\delta_{\ell j\alpha}$ by

$$T^J_{\alpha j\ell} \equiv \left\{1 - \left|e^{-2i\delta_{\ell j\alpha}}\right|^2\right\} \qquad 1.1$$

The factor

$$T^J_{\alpha' j'\ell'} \Big/ \sum_{\alpha'' j'' \ell''} T^J_{\alpha'' j'' \ell''} \propto \Gamma' / \sum \Gamma'' \qquad 1.2$$

since the transmission coefficients are proportional to the widths for decay to a given state. The sum in the denominator of (1.2) includes Γ' plus the widths of all possible decay modes competing with Γ'. When the spin-orbit interaction is absent, the transmission coefficients involve only ℓ and are independent of j and J. For incident and emerging particles of spin 1/2, j can have at most only two values $j_{1,2} = I \pm 1/2$ where I is the spin of the target nucleus in its ground state.

Under these circumstances, the cross section may be represented as a sum over ℓ of the contributions from the various J values possible for each ℓ. Thus, Eq. (1.1) now becomes:

$$\sigma\langle\alpha|\alpha'\rangle = \frac{\pi\lambda\!\!\!^{-2}}{2(2I+1)} \sum_{\ell} T_{\alpha\ell} \sum_{J} \frac{\epsilon^J_{j\ell}\,(2J+1) \sum_{\ell'} \epsilon^J_{j'\ell'}\, T_{\alpha'\ell'}}{\sum_{\ell''} \epsilon^J_{j''\ell''}\, T_{\alpha''\ell''}} \qquad 1.3$$

Here $\epsilon^J_{j,\ell}$ is a quantity such that

$$\epsilon^J_{j,\ell} = \begin{cases} 2, \text{ if both values of } j \text{ are included in the range} \\ |J - \ell| \leq j \leq (J + \ell), \\ 1, \text{ if one of the values of } j \text{ is included in the range} \\ |J - \ell| \leq j \leq (J + \ell), \\ 0, \text{ if neither value of } j \text{ is included in the range} \\ |J - \ell| \leq j \leq (J + \ell), \end{cases}$$

The channel designation α includes the energy of the incident particle and excitation state of the target nucleus, while α' is the similar designation for the final system which includes the type and energy of the emergent particle and the state of excitation of the residual nucleus.

Eq. (1.3) may also be used to describe fission and capture by ewriting it in simple form as

$$\sigma_r = \frac{\pi ƛ^2}{2(2I+1)} \sum_{\ell=0}^{\infty} T_n(\ell,\epsilon) \frac{\sum_{j=0}^{\infty} \epsilon^J_{j\ell}\,(2J+1)\,T_r(J,\epsilon)}{\sum_r T_r(J,\epsilon) + \sum_{j'\ell'} \epsilon_{j'\ell'}\,T_n(\ell',\epsilon')} \qquad 1.4$$

(r refers to either capture or fission). The transmission coefficients for the fission and radiation channels are given by:

$$T_r(J,\epsilon) = 2\pi \frac{\langle \Gamma_r(J,E) \rangle}{\langle D(J,E) \rangle} \qquad 1.5$$

where $\Gamma_r(J,E)$ is the partial width of a level of spin J formed by a neutron of energy E. D(J,E) is the spacing of levels of spin J.

For radiative capture T_r in the denominator of Eq. (1.4) is the total probability of radiative decay of the compound nucleus, T_γ. This radiation term $T_\gamma(J,E)$ differs from the radiation transmission coefficient $T_c(J,E)$ which is used in the numerator of Eq. (1.4) and gives the neutron radiative capture probability.

For fission the transmission coefficient $T_f(J,\ ,E)$ is interpreted in terms of the Hill-Wheeler model [3] expressed as:

$$T_f(J,\pi,E) = \sum_k N(J,\pi,E-E_{fk})\,P(E-E_{fk}) \qquad 1.6$$

where $N(J,\pi,(E-E_{fk}))$ is the number of transitional states in the saddle point of the fissioning nucleus above the k^{th} fission barrier with energy E_{fk}, and the penetrability $P(E-E_{fk}(J,\pi))$ of the k^{th} fission barrier is given as

$$P\left\{E - E_{fk}(J,\pi)\right\} = \frac{1}{1 + \exp\left\{\frac{2\pi(E_{fk}-E)}{\hbar\omega}\right\}} \qquad 1.7$$

where ω is the circular frequency of the inverted harmonic oscillator.

Recently [4] a more detailed analysis of the fission process has incorporated the concept of a double-hump fission barrier.[5] This treatment allows the transmission coefficient to be expressed in terms of the coefficients for transmission across two peaks instead of one. Thus the fission probability is denoted by

$$P_F = \frac{T_A\,T_B}{T_A\,T_B + T'\,(T_A + T_B)} \qquad 1.8$$

where T' is the summation of all the other decay channels from the compound nucleus. T_A and T_B have the same form of Eq. (1.7) but different values for the barrier heights and curvature.

In Eqs. (1.0) and (1.3) the effect of fluctuations of the compound nucleus-level widths about their average values has been ignored. However, even though the widths in different channels might be independent, the fluctuations in the numerator Eqs. (1.0) and (1.3) are correlated to fluctuations in the denominator. Thus, following Lane and Lynn[6] and Moldauer[7,8], the Hauser-Feshbach equation should be multiplied by a fluctuation correction factor given by

$$R_{\alpha\alpha'} = \left\langle \frac{\Gamma_\alpha \Gamma_{\alpha'}}{\Gamma} \right\rangle \Bigg/ \frac{\langle \Gamma_\alpha \rangle \langle \Gamma_{\alpha'} \rangle}{\langle \Gamma \rangle} \qquad 1.9$$

The particle widths Γ_α are related to the optical model transmission coefficients by

$$T_\alpha^{OM} \simeq 2\pi \frac{\langle \Gamma_\alpha \rangle}{D}$$

Thus Eq. 1.8 may be written as

$$R_{\alpha\alpha'} = \left\langle \frac{T_\alpha T_{\alpha'}}{T} \right\rangle \Bigg/ \frac{\langle T_\alpha \rangle \langle T_{\alpha'} \rangle}{T} \qquad 1.10$$

Recent theoretical developments offer different interpretations of this correction factor where under certain conditions $R_{\alpha\alpha'}$ may become greater than unity for non-elastic processes (i.e. $\sigma_{n\gamma}, \sigma_{nn'}$). (See Refs. 9-11).

This effect may be interpreted in terms of the distribution laws for the various particle decay widths.

Programs were written[12] to [14] to be used in computer experiments that produced synthetic cross sections analagous to those encountered in experimental measurements. The results were interpreted so as to determine the limitations imposed on the average values of the various quantities (e.g. matrix elements) used to generate these synthetic cross sections.

The values of these variables as obtained from the statistics were compared with those resulting from the conventional Hauser-Feshbach theory.

These numerical experiments have shown that in the presence of direct reactions, it is necessary that the fluctuation corrections to the cross sections and polarization data be analyzed in a much more sophisticated and generalized form.

A recent paper by Gruppelaar and Reffo[12] reviews some of various properties of the width fluctuation factor and the effect on decay channels.

These various treatments for the reinterpretation of the Hauser-Feshbach formula differ in certain respects and are still open to question; nonetheless, the results are encouraging and upon application to a real situation the validity of the assumptions will be further tested.

1.2 Multiple Reactions

The foregoing treatment of the Hauser-Feshbach method was devoted to binary reactions only, however, it can be extended to include many particle reactions.

Of special interest to dosimetry and CTR application are the so called "rare" nuclear reactions such as (n,nγ), (n,γn) or (n,np) and (n,pn) which necessitate a three-particle analysis.

This tertiary reaction is assumed to proceed as

$$\alpha + A \longrightarrow A^* \longrightarrow B^* + \beta, \qquad B^* \longrightarrow C^* + \gamma$$

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TAB(50);

where β and γ are the emitted particles (n,p,d,t,He^3,He^4) and α is the particle incident on nucleus A leading to the compound nuclei A^* and B^*. Thus

$$\sigma^J_{\alpha,\beta\gamma} = \pi \lambda^2_\alpha \frac{(2J+1)}{(2i_\alpha+1)(2I_\alpha+1)} T^J_{J_\alpha \ell_\alpha}(\epsilon_\alpha) \times \omega^J_{J_\beta \ell_\beta I_\beta}(E_0; E_\beta, \epsilon_\beta)\, \omega^{I_\beta}_{J_\gamma \ell_\gamma J_\gamma}(E_0; E_\gamma, \epsilon_\gamma) \qquad 1.11$$

where $T^J_{J_\alpha \ell_\alpha}(\epsilon_\alpha)$ represents the transmission coefficient of particle α (e.g. a neutron) which has kinetic energy ϵ_α, total angular momentum J_α, orbital angular momentum ℓ_α, and total angular momentum J that is produced jointly with the target nucleus; $W^J_{J_\beta \ell_\beta I_\beta}(E_0; E_\beta \epsilon_\beta)$ is the probability of disintegration of the compound nucleus with spin J and excitation energy E_0 into a residual nucleus with spin I_β and excitation energy E_β and a particle with kinetic energy ϵ_β, total angular momentum J_β, and orbital angular momentum ℓ_β; $W^{I\beta}$ is the probability of disintegration of the compound nucleus following the emission of particle β.

An exact calculation of the cross section given in Eq.(1.11) reduces to the determination of the probability of disintegration of the compound nuclei.

B, assuming the lifetime of all the compound nuclei to be sufficiently long and by writing the set of detailed balance equations for the series decay, one obtains the following equation for the three-particle reaction:

$$\sigma_{\alpha,\beta\gamma} = \pi \lambda^2_\alpha \sum_{J,J_\alpha \ell_\alpha} \frac{(2J+1)}{(2i_\alpha+1)(2I_\alpha+1)} T^J_{J_\alpha \ell_\alpha}(\epsilon_\alpha) \times \left\{ \sum_{J_\beta \ell_\beta} T^J_{J_\beta \ell_\beta}(\epsilon_\beta) \Big/ \sum_{\beta'} \sum_{E_{\beta'}=0}^{E_0 - Q_{\beta'}} \sum_{J_{\beta'} \ell_{\beta'} I_{\beta'}} T^J_{J_{\beta'} \ell_{\beta'}}(\epsilon_{\beta'}) \right\} \times \left\{ \sum_{J_\gamma \ell_\gamma} T^J_{J_\gamma \ell_\gamma}(\epsilon_\gamma) \Big/ \sum_{\gamma'} \sum_{E_{\gamma'}=0}^{E_0 - Q_{\gamma'}} \sum_{J_{\gamma'} \ell_{\gamma'} I_{\gamma'}} T^{I_\beta}_{J_{\gamma'} I_{\gamma'}}(\epsilon_{\gamma'}) \right\} \qquad 1.12$$

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The expressions for the n-particle reaction (n > 3) are similar.

If the density of final states is a continuous function of energy, then by summing or integrating over a definite range of energy, Eq.(1.0) becomes

$$\sigma_{a,\upsilon}(\epsilon_0,\epsilon)\,d\epsilon = \pi\lambda\!\!\!^{-2} \sum_{J,j,\ell} \frac{(2J+1)}{(2i+1)(2I+1)} \times \frac{\left\{ T^{J}_{a,\ell}(\epsilon_0) \sum_{j',\ell',I'} T^{J}_{\upsilon j'\ell'}(\epsilon)\,\rho_\upsilon(I',\epsilon)\,d\epsilon \right\}}{\sum_{\upsilon'' j'' I''} \int_0^{\epsilon_0 - Q_{\upsilon''}} d\epsilon\; T^{J}_{\upsilon'' j'' \ell''}(\epsilon)\,\rho_{\upsilon''}(I'',\epsilon'')\,d\epsilon''} \qquad 1.13$$

where the symbols α and α' for these incoming and outgoing channels have been replaced by a and υ, respectively.

In Eq.(1,13) the sum in the denominator is taken over all energetically possible emitted particles υ'' (usually taken as n, P, d, t, He^3 and α) and ρ_υ (I;E) is the density of the residual nucleus at an excitation energy

$$E = \epsilon_0 - \epsilon - Q \qquad 1.14$$

where Q is the Q value of the reaction.

It should also be noted that for simplicity the competing channels υ' leading to fission or radiactive capture have been neglected.

2.0 Parametrization Methods

In analyzing nuclear reactions using the statistical model, one of the most important aspects is a knowledge of the level density.

Empirical information on the level density is usually obtained by analyzing

a. levels of residual nuclei from reactions such as (n,n'), (p,p'), (d,p), (d,α) etc.,
b. spectral shape of emitted particles,
c. slow neutron resonances and associated widths.

Some of these experimental data provide direct information on the level density while in others, it is provided indirectly. The latter method,

in many instances, provides only rough estimates since the results are influenced by estimation of other nuclear quantities.

The most commonly used expressions for the level densities are due to Ericson[15] and Lang and LeCouteur.[16]

$$\rho(E,J) = \text{const.}\,(2J+1)\exp\left(-\frac{J(J+1)}{2\sigma^2}\right)\frac{a^{1/2}\exp(2\sqrt{aU})}{\mathcal{I}^{3/2}U^2} \qquad 2.1$$

or

$$\rho(E,J) = \text{const.}\,(2J+1)\exp\left(-\frac{J(J+1)}{2\sigma^2}\right)\frac{a^{1/2}\exp(2\sqrt{aU})}{\mathcal{I}^{3/2}(U+t)^2} \qquad 2.2$$

which give a density of levels of spin J of a nucleus excited to an energy U.

Eqs. 2.1, 2.2 are derived on the basis of the Fermi gas model of the nucleus where σ is the so called spin cut-off parameter and is related to the nuclear moment of inertia by

$$\sigma^2 = \frac{\mathcal{I}t}{\hbar^2} \qquad 2.3$$

The nuclear temperature T is related to the nuclear thermodynamic temperature t by

$$\frac{1}{T} = \frac{1}{t} - \frac{2}{U} \qquad 2.4$$

$$\frac{1}{T} = \frac{1}{t} - \frac{2}{U+t} \qquad 2.5$$

corresponding to either 2.1 or 2.2, respectively.

The quantity a is a characteristic parameter related to the spacing of single particle nucleon states near the top of the Fermi sea and is related to the thermodynamic temperature by

$$at^2 - t = U \qquad 2.6$$

where U is the effective excitation energy, given by $U = (E + \Delta)$; E being the excitation energy and Δ is a negative term representing the pairing energy of the last two protons when Z is even; of the last two neutrons when N is even; and the sum of both pairing energies for even-even A; $\Delta = 0$ for odd-odd nuclei.

The parametrization has been carried out by several investigators using experimental neutron resonance data. (Refs. 17-20).

This produced meaningful results only for a narrow range in the region where the fit was carried out. The a-parameters determined at the neutron binding energies predicted level densities which were too high at excitations near the ground state and much too large level densities for energies greater than 15 MeV.

Gilbert and Cameron[21] introduced a four-parameter formula in which they used a shifted Fermi gas[22] formula at higher excitations which was smoothly joined to a constant temperature formula at lower energies. The fictive ground state was obtained from experimental mass differences, thus only the level density parameter, a, was left as the adjustable constant.

Carrying out a fit to the four constants in both the high and low regions produced fairly good results.

Another approach has employed the so-called "back-shifted" Fermi gas model[23-25] where the Fermi gas formula was used with both a and the ground state shift as adjustable parameters.

Dilg et al.[26] show that using the "back-shifted" Fermi gas produces rather large negative Δ values for odd mass nuclei and moderately positive Δ values for even nuclei.

The a values while not showing any drastic odd-even effects, do show strong shell effects similar to the one parameter fits.

While the use of the four-parameter (e.g. Gilbert and Cameron) or the two-parameter (e.g. Dilg et al.) formula has produced fair to excellent fits to experimental data, one must still be cognizant of the fact that the semi-empirical formulas do not necessarily justify the adopted energy dependence of the level density.

Also for nuclei near closed shells the values of a and Δ resulting from the parametrization can only be a coarse approximation and extrapolation beyond the range of validity can produce absurd results.

References

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B. Workshop (Problem Session)

The following codes have been chosen for calculating reaction cross sessions using a combined optical model and statistical theory.

a) ABACUS (See my lecture on optical model codes)
b) HAUSER-5; F. M. Mann, HEDL-TME 76-80 (1976)
c) CERBERO 2 (See my lecture on optical model codes)
d) ERRINI, F. Fabbri and G. Reffo RT/FI (77)4 (1977)

1. HAUSER – 5

Name of Code: HAUSER

Author: F. M. Mann

Establishment: Hanford Engineering Development Laboratory, Richland, WA., USA.

Nature of Problem Solved: Program HAUSER calculates the total reaction cross section for T(a,bc)F where T is the target nucleus, a is the projectile (any particle-charged or uncharged), b and c are emitted particles or gamma rays, and F is the final nucleus. The statistical model is employed with allowance for angular momentum and parity effects. The transmission coefficients can either be calculated (without spin-orbit interaction) or read-in. Width fluctuation corrections can be included through the method of Tepel, Hofmann, and Weidenmüller. Cross sections can be printed for discrete states, two-body or three-body reactions.

Program Language: FORTRAN IV

Size: 170 K for 7 values of bc, 6 values of b, 100 discrete states. Can be considerably reduced by reducing size of tables.

Status: Has run on IBM 370 and CDC 6600 and 7600.

Introduction

HAUSER-5 is a recent statistical model code which handles both binary and tertiary processes along with capture and fission.

Two basic equations are solved given as

$$\sigma_{\alpha\beta} = \frac{\pi \lambda_\alpha^2}{(2I_{\alpha_1}+1)(2I_{\alpha_2}+1)} \sum_{J^\pi} \frac{\sum_{\ell s} (T_\alpha)^{J^\pi}_{\ell s} \sum_{\ell' s'} (T_\beta)^{J^\pi}_{\ell' s'}}{T^{J^\pi}} W^{J^\pi}_{\alpha\beta} \quad 1.1$$

for conventional two-body reactions and

$$\sigma_{\alpha(\beta\gamma)} = \sum_{J^\pi} \left| \frac{\pi \lambda_\alpha^2}{(2I_{\alpha_1}+1)(2I_{\alpha_2}+1)} \sum_{\ell s} (T_\alpha)^{J^\pi}_{\ell s} \right|$$

$$* \left| \frac{\sum_{\ell' s'} (T_\beta)^{J^\pi}_{\ell' s'}}{T_1^{J^\pi}} W^{J^\pi}_{\alpha\beta} \right| * \left| \sum_{\ell'' s''} \frac{(T_\gamma)^{J^\pi}_{\ell'' s''} W^{J^\pi}_{S\gamma}}{T_2^{J^\pi}} \right| \quad 1.2$$

for three body (e.g., n,2n, n, n etc.) ($\omega^{J^\pi}_{\alpha\beta}$ is the fluctuation correction factor). The particle transmission coefficients $T^{J\pi}_{\ell s}$ are obtained from the optical model.

Transmission coefficients for *gamma*-ray channels are calculated by assuming E and M transitions exist between all possible states consistent with the appropriate energy and selection rules,

$$T^{J^\pi}_{E\ell \text{ or } M\ell}(E) = 2\pi\, \Gamma^{J^\pi}/D^{J^\pi}_\gamma = N_\ell \int_0^E \rho(E')(E-E')^n\, S(E')\, dE' \quad 1.3$$

where N_ℓ is a normalization varying smoothly with A,

ρ is the level density, $n=\ell+2$, and
S is the form factor (usually taken to be unity or of a Lorentzian shape to represent giant resonances).

Because higher multipolarity transitions are unlikely, usually only E1 and M1 transitions are included in the calculations.

Transmission coefficients for fission channels are given in the Hill-

The Tepel et al.[1] approximation is used for determining the width

Transmission coefficients for fission channels are given in the Hill-Wheeler formalism as

$$(T_\ell)^{J^\pi} = \frac{1}{\exp\left[(E-E_1)/\hbar\omega\right]+1} \qquad 1.4$$

where E_1 is the barrier height and $\hbar\omega$ is the width. With the advent of fission isomers, the potential is often taken to be doubly humped with an intermediate well. No simple equation like Eq. 1.4 exists for such a case. However, a closed form can be obtained if the two barriers and intermediate well are taken to be harmonic oscillator potentials.

In order to handle both discrete and continuum regions, the total transmission coefficient is given by

$$T^{J^\pi} = \sum_{\substack{i \\ \text{discrete}}} T_i^{J^\pi} + \int_{E_1}^{E_2} T^{J^\pi}(E)\,\rho^{J^\pi}(E)\,dE\,. \qquad 1.5$$

Two forms of the level density are often used, the constant temperature formula

$$\rho(E) = \exp\left[(E-E')/T\right]/T \qquad 1.6$$

and the Fermi gas formula

$$\rho(E) = \exp\left[2\sqrt{a(E-\Delta)}\right]/\left[12\sqrt{2}\,\sigma\, a^{1/4}\,(E-\Delta)^{5/4}\right]. \qquad 1.7$$

Spin and parity dependence is included by a multiplication factor

$$\rho(E)^J = \rho(E)\,(2J+1)\,\exp\left[-(J+1/2)^2/2\sigma^2\right]/(2\sigma^2). \qquad 1.8$$

The width fluctuation factor $W^{J\pi}$ for channels other than capture is

$$W_{\alpha\beta}^{J^\pi} = (1+2\delta_{\alpha\beta}/\nu_\beta)\int_0^\infty \frac{\exp(-T_\gamma^{J^\pi}t)\,dt}{\left(1+2t\frac{T_\alpha^{J^\pi}}{\nu_\alpha}\right)\left(1+2t\frac{T_\beta^{J^\pi}}{\nu_\beta}\right)\prod_{\varepsilon\neq\gamma}\left(1+2t\frac{T_\varepsilon^{J^\pi}}{\nu_\varepsilon}\right)^{\nu_\varepsilon/2}} \qquad 1.9$$

and for capture

$$W_{\alpha\gamma}^{J^\pi} = \int_0^\infty \frac{\exp(-T_\gamma^{J^\pi}t)\,dt}{\left(1+2t\frac{T_\alpha^{J^\pi}}{\nu_\alpha}\right)\prod_{\varepsilon\neq\gamma}\left(1+2t\frac{T_\varepsilon^{J^\pi}}{\nu_\varepsilon}\right)^{\nu_\varepsilon/2}} \qquad 1.10$$

The Tepel et al.[7] approximation is used for determining the width fluctuation factors. Transmission coefficients are interpolated from a table of previously calculated transmission coefficients, which can either be read as input or calculated internally.

The real spherical potential is taken to be the Woods-Saxon shape, and the imaginary potential can be a sum of potentials of the Woods-Saxon shape and the derivative Woods-Saxon shape, or can be a Gaussian shape. If the user desires, default potentials (shown in Table 1) can be used. However, (j,ℓ,s) dependent transmission coefficients $(T_{\ell j}^{s})$ must first be averaged

$$T = \sum_{j=\ell-s}^{\ell+s} (2j+1)\,T_{\ell,j}^{s}/(2\ell+1)(2s+1) \qquad 1.11$$

over j before they can be input.

TABLE 1

DEFAULT OPTICAL PARAMETERS

(Real potential is Woods-Saxon shape, imaginary potential is sum of Woods-Saxon and derivative Woods-Saxon shapes)

	Neutron[a]	Proton[b]	Deuteron[c]	Triton[d]	^{3}He[d]	Alpha[e]
V_{WS}(MeV)	47.01-0.267E -0.0018E^2	54+24*(N-Z)/A +0.4*Z/A$^{1/3}$-0.32E	81+2*Z/A$^{1/3}$	136.4+55*(N-Z)/A -0.17E	165-7*(N-Z)/A -0.17E	185.00
W_{WS}(MeV)	0.00	-2.7+0.22E	0.00	41.3+63*(N-Z)/A -0.33E	56-110*(N-Z)/A -0.33E	25.00
W_{der}(MeV)	9.52-0.053E	11.8+12*(N-Z)/A -0.25E	14.40	0.00	0.00	0.00
R_R(fm)/A$^{1/3}$	1.322-0.0076*A (1-0.005*A)	1.17	1.15	1.20	1.20	1.40
R_I(fm)/A$^{1/3}$	1.266-0.0037*A (1-0.005*A)	1.26	1.34	1.40	1.40	1.40
a_R(fm)	0.66	0.75	0.81	0.72	0.72	0.52
a_I(fm)	0.48	0.58	0.68	0.68	0.86	0.52

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2. ERRINI

Name of Code: ERRINI (An Optical Model Fortran IV Code for the calculation of multiple cascading particle emissions) Rpt. CNEN RT/FI (77) 4

Authors: F. Fabbri and G. Reffo

Establishment: CNEN, Bologna, Italy

Program Language: Fortran IV

Size: 240 K - bytes (overlay structure)

Status: IBM 375, modified for CDC

Introduction

ERRINI is a multiple emission code based on Hauser-Feshbach analysis and is primarily for high-energy condensing particles.

The transmission coefficients are calculated similar to those in CERBERO II, except no width fluctuation corrections are applied.

a) First step decay calculations

The total transmission coefficients $T^{J\pi}_a$ are first calculated for all J and π, then the various binary cross sections $\sum_{J\pi} \sigma^{J\pi}_{x,a}$ are calculated, where

$$\sigma^{x,a}_{cc'} \propto \frac{T^x_c \, T^a_{c'}}{T^a} \qquad 2.1$$

and

$$T_a = \sum_{c''} T^a_{c''} \qquad 2.2$$

For each channel c" and c' leading to excitation of levels in a continuum band, the corresponding transmission coefficients are weighted according to the number $\rho \, dU$ of levels in the band:

$$T_{c''} \rightarrow T_{c''} \, \rho^a_{c''} \, dU^a_{c''} \qquad 2.3$$

$\rho^a_{c''}$ being the density of levels available to channels c" in the residual nucleus corresponding to emission of particle a.

This part of the code calculates the same quantities as CERBERO *II* does, from which it is taken. As ERRINI was devised for use at high energies, no width fluctuation correction is introduced.

For any binary process, for which the cross section $\sigma^{x,a}$ is greater than 10^{-5}b.

b) Second step decay processes are accounted for by starting with the calculations of total transmission coefficients $T^{J\pi}_b$ for each J and π, then working out the branching ratios P^J_b for each J and π.

Formula (1) is generalized to describe a two step cascade as follows:

$$\left[d^2\sigma^{x,ab}_{c,c'} \propto \frac{T^x_c T^a_{c'} \, \rho^a_{c'} \, dU^a_{c'} \sum_{J'\pi'} \left(P_b(U^a_{c'}) \, dU^b_{c''}\right)_{J'\pi'}}{T^a} \right]_J \qquad 2.4$$

where

$$\left[P_b(U^a_{c'}) \, dU^b_{c''} = \frac{\sum_{c''} T^b_{c''} \rho^b_{c''} \, dU^b_{c''}}{T_b} \right]_{J'\pi'} \qquad 2.5$$

When channels c" lead to excitation of an isolated level, the code assumes $\rho^b_{c''} \, dU^b_{c''} = 1$.

Whenever second emission leaves the residual nucleus excited above the neutron binding, a neutron is assumed to be emitted.