NON-LINEAR CALCULATION OF PCRV USING DYNAMIC RELAXATION

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1. Introduction

The development of prestressed concrete reactor pressure vessels began in the end of the fifties and expanded parallelly to the fast development of big computers in the sixties. The difficulties of calculating the state of stress and deformation of three-dimensional structures like PCRVs are well known. But the computers enabled engineers to solve these difficult tasks. Mainly two different methods are used for solving the problem of calculating these complex structures taking into account arbitrary boundary conditions and non-linear behaviour of materials: the finite element and the dynamic relaxation method.

Regarding the concrete structures only the physical non-linearity is of interest for PCRVs. The influence of geometrical non-linearities is neglectable for this; but it is interesting for the calculation of the liner and its interaction with the concrete structure. Solutions for these problems are discussed in papers presented by us at former SMiRT-Conferences or a paper being in preparation for the next SMiRT-Conference. This non-linearity is not the subject of this paper.

The examples given in this paper have been calculated with the dynamic relaxation method. Day and Otter /1/ proposed to solve the problem of calculating concrete pressure vessels with a special difference method, named "Dynamic Relaxation". The main idea of this method is to consider the dynamic equilibrium instead of the static equilibrium. This means, the three-dimensional elliptic differential equations of the continuum are changed into the four-dimensional hyperbolic differential equations, known as wave equations. They describe spreading and oscillating processes in elastic bodies. The boundary value problem of the static system is changed into an initial and boundary value problem for which a solution is existing, if the physical system is defined at the time $t = 0$.

These hyperbolic equations can be solved numerically by the way of difference equations avoiding the known problems of other difference methods concerning the fullfilling of the boundary conditions. The calculation then starts from a body free of stresses and only loaded at the surfaces or by volume forces. It decribes the temporal spreading of the stresses and strains through the body depending on the difference equations and the boundary conditions until the static equilibrium is attained by a suitable choice of the damping of the dynamical system. This solution method is convergent and definite. This method is described in detail for several applications e.g. in references /2/, /3/, /4/ and /5/.

Parallelly to the development of the finite elements, the dynamic relaxation method was optimized with regard to arbitrary boundary conditions, computer time, storage etc. In Germany and Great Britain this method nowadays is used for any problems of thickwalled or thinwalled structures or liner problems. It suggests itself that this method is advantagous for dynamic problems, too.

2. Non-linear material behaviour

The non-linear behaviour of concrete reactor vessels is in general characterized by creep and shrinkage of the concrete, cracks in the concrete structure and non-linear stress-strain behaviour of concrete and steel. The usual way of calculating and designing concrete structures is to neglect the temperature effects, cracks, and massive sizes. This neglecting includes the hydration heat during hardening of the concrete (Fig. 1). This temperature loading combined with the shrinkage of concrete would lead to concrete cracks before being loaded by service loads, if no non-linear material behaviour would be existing and no limitation by a suitable design is provided for. Cracks in abutment piers are well-known examples for neglecting this behaviour of material (Fig. 2).

All rules and regulations concerning non-linear effects concentrate on thin elements. This way allows a special linearization to avoid non-linear calculations. These linearizations cannot be used for thick-walled structures, because material laws are depending on the thickness of the structure and on temperatures etc. Fig. 3 shows creep functions in dependence on temperatures. These non-linear material laws have to be considered within the calculation, because stresses as well as deformations are considerably influenced by these effects. Another kind of non- linearity is the cracking of concrete. The essential effects of this are the anisotropic behaviour in the cracked zones and the integral stress-strain behaviour of the material shown in Fig. 4 for the uniaxial case. This behaviour for example influences the deformations of the structure and reduces the stresses due to temperature and shrinkage. For limite state analyses of PCRVs consideration of cracking and crack propagation is most important.

Another physical non-linearity appears after exceeding the limits of elasticity of the materials, known as plasticity, which is of importance, too.

These few examples show the significance of considering non-linear behaviour of the materials when calculating and designing thickwalled structures.

3. Influences of non-linearities on state of stress and strain in PCRVs

3.1 Creep

Firstly, some basic remarks are given on treating creep problems in calculatons: Creep of concrete depends on many parameters. Comprehensive formulas in order to take into account all influences are still object of research. Thus, normally technical-empirical methods are used. That means, some representative creep measurements are carried out for the actual concrete mix in order to define coefficients in a creep formula. A plot of such a formula is shown in Fig. 3. For considering the effects due to creep, that are most important for PCRVs, the following principles and procedures are existing:

Usually time dependent deformations of concrete are described by a visco-elastic formulation. With such a formulation, creep effects in a three-dimensional continuum can only be exactly considered if a time history calculation is carried out. Hereby, constant creep producing stresses are presumed for each of the time steps, which must be calculated in each of the steps. Due to the linearity of a visco-elastic formula, portions of stresses can be calculated separately for certain times and superposed to those of other times. If this superposition method is used in a calculation, strain compatibility must be gained for each time step by a three-dimensional analysis, and creep strains or stresses are to be stored for all time steps. Furtheron, special considerations are necessary, if stresses decrease with time, since in these cases the superposition method yields too large creep recovery in comparison with test results.

In order to diminish the expenditure in storing data of previous steps it is of advantage to use methods, where changes of strain during time can be defined by the present state of stress only or by a few data for environmental influences and previous load history. Using the "rate of creep method" (Dischinger formula) the change of strain during time is defined only by the present state and thus most easily to be applicated in numerical calculations. With this formula, the increase of strain under increasing stresses and temperatures as well as the creep recovery under decreasing stresses cannot be implemented. These effects can only be considered to some degree by inducing a portion of delayed elastic strain.

Calculations can be simplified by use of an average creep producing stress during a space of time instead of considering variability of stresses by calculating small time steps, as described at first. Thus, for this period only one direct calculation has to be done like in calculations for usual concrete structures. Such a calculation only yields realistic results, if either changes of stress are moderate and these changes only occur during a short initial phase followed by a long period with a few changes of stresses, or if the variations of stresses are well known and can be taken into account.

PCRVs are submitted to strong variations of load and temperature only during prestressing, commissioning and shut-down events. Between these periods stress variations due to load and temperature variations are rather small. Consequently it is obvious to use a combination of superposition method and direct calculation with a fictitious material law. For periods of load and temperature variation the superposition method is used, for analysing creep redistributions and reduction of temperature stresses due to creep in the intermediate periods the fictitious material law calculation is applied. Hereby, stresses from prestressing and internal pressure at the moment to be analysed can be used as average creep producing stresses. This procedure mentioned at last leads to a kind of analysis called "effective Young's modulus method" in the literature. If relaxation problems, such as reduction of temperature stresses due to creep, are solved by use of this method, it should be considered that the resulting stress reductions could be too small (whereas the rate of creep method yields a too large stress reduction).

For calculations of deformation behaviour of a PCRV during its whole life it is appropriate to investigate certain extreme design cases instead of many possible load histories, and thus to take advantage of the more simple direct method of calculation. One of such extreme cases for states of stress and deformation after a long period of vessel operation arises, if due to malfunctions during the first years the vessel is unpressurized with heated walls over longer periods. This state produces extreme compressive stresses in the walls and inner slab-wall junctions, leading to deformations and stress redistributions due to creep of significant order of magnitude. By reason of deformations, the highest losses of prestress result from this case. Furthermore, it yields the largest compressive strains, see left part of Fig. 5, on which component design must be based.

The opposite extreme case of design is that of a 30 years operation without interruption, see the right drawing in Fig. 5. This case yields less severe values in the wall and the inner slab-wall junction. But deformations of the end slabs get extreme in this case of permanent operation, as it can be seen in the figure. Stresses and deformations due to sudden load changes in general can be analysed by elastic calculations and be superposed to those of these long time states.

For the first heating during comissioning, however, superposing elastic temperature stresses to stresses due to prestressing would yield extremely pessimistic values. Thus, a step by step calculation with small time steps in order to analyse stresses during the first heating period will be the better way, especially if compressive stresses may be critical. For such a period with initially elevating temperatures and increasing creep producing stresses, rather exact creep data are existing. These creep data and the accuracy gained from such a refined calculation allow good predictions about the vessel behaviour in this important period of changing temperature and stress values. Fig. 6 shows stress profiles during heating a PCRV calculated in that way.

All these creep calculations can be advantageously and economically performed by the dynamic relaxation method.

3.2 Cracking

Basically, cracking of concrete can be analysed by use of two methods. Firstly, definite cracks are assumed; secondly, the effects of cracking are considered summarizing by inducing a fictitious material law. In both cases - like in any numerical calculation of non-linear problems - non-linear behaviour of the structure is approximated by a number of calculation steps, each with linear material behaviour. In each of these steps, changing stiffness conditions due to propagating cracking are taken into account either by changing the geometry (first case) or by changing the elastic material coefficients (second case). Calculations considering cracking can be carried out today for cracks with arbitrary directions with respect to computer meshes.

In calculations assuming definite cracks, reinforcing bars must be defined as separate bar elements coupled by an adequate bond-slip law with the concrete elements. Due to the rather high effort, because it is necessary to chose comparatively fine elements in relation to the crack geometry, calculations like that are only suitable for investigating local detailed problems. Fig. 7 shows crack propagation, concrete stresses, steel stresses and deformations of concrete and steel in comparison for a wall section with instationary surface cooling down. By this way, statements on exspected crack distances, crack widths and reinforcement stresses can be gained.

The fictitious material law method, on the other hand, is more suitable for analysing the overall behaviour of larger regions, in which cracking occurs, or even complete structures. In this case, a fictitious stiffness normal to the crack plane is formulated by use of a coefficient k_m (Fig. 8), which describes the participation of the concrete in carrying tensile stresses between the cracks. The factor k_m can be defined independently of bond characteristics of the steel bars used and is nearly constant up to steel stresses of about 300 N/mm^2. The stiffness coefficients for the other directions can be derived from the boundary conditins at the cracks, too. They can be induced as components of a fictitious elasticity tensor. This tensor describing orthotropic material behaviour with respect to crack-related coordinates is then transformed to the calculation coordinates.

This procedure is particularly useful in combination with the dynamic relaxation method, since variations of the elasticity tensor from one calculation step to the next one according to variations of material behaviour can be performed without extensive increase of computational effort. If, on the other hand, finite element technique is used, generally the stiffness matrix is kept constant during the whole calculating process, and the step by step adjusting to material behaviour is done by formulating suitable initial states of stress or states of strain, respectively.

As an example for a calculation considering cracking by this secondly mentioned way, a series of states of stress in a section of a computer mesh for calculating the THTR vessel shall be presented. Fig. 9 shows the elastic solution. Figures 10 and 11 show the evaluation of the cracked region in the junction between cylinder and bottom slab under 1.6 times operating pressure. Fig. 11 points up the final state, when material behaviour has stabilized. Concrete tensile stresses outside the cracked region remain now below the assumed concrete tensile strength.

3.3 Non-linear stress-strain behaviour

Describing non-linear stress-strain behaviour of reinforcing and prestressing steel and its implementation in numerical calculations is not very difficult, since here the important characteristics are sufficiently represented by stress-strain curves gained from uniaxial tension tests.
Some more problems arise with the attempt to describe non-linearity of concrete strains under triaxial states of stress with significant deviatoric components. Here, normally formulations of theory of plasticity are used, although microscopic processes in concrete are fully different from hardening processes of crystalline materials. This proceeding, however, is acceptable because of the macroscopic view of continuum theory. Hence, concrete is considered here as a hardening material. For describing this phenomenon, an incremental or a finite hardening law can be used.

For formulation of non-linear deformations, it is reasonable to split the constitutive equations into hydrostatic and deviatoric terms using the bulk modulus K and the shear modulus G as material parameters. Test results gained from concrete discs and cubes show, that for high strength concretes as applied for PCRVs K can be used as a constant with sufficient accuracy. The plastic shear modulus Ψ , however, induced instead of the constant G, is depending not only on the second invariant of the stress deviator as usually, if arbitrary three-dimensional stress states are regarded. Additionally a dependence on the direction of the stress deviator and of the hydrostatic pressure has to be regarded. Based on available test data, the interrelation shown in Fig. 12 has been developed, using as additional parameter the ratio of largest and smallest principal stress σ_1/σ_3.

As an example for an analysis in the overload domain, where a sufficiently good description of non-linearity in compression zones has been necessary additionally to the formulations for crack development, some results of calculations corresponding to ultimate load tests with a concrete closure model for a boiling water reactor vessel shall be shown. Model and test arrangement can be seen in Fig. 13.
Fig. 14 demonstrates stresses and cracked zones of the closure exposed to 80 bar above atmospheric internal pressure, which is corresponding approximately to the assumed operating pressure. The existance of cracks already under this load indicates a not optimal closure design.

The following figures represent the corresponding phenomena with about two times and four times operating pressure (Fig. 15 and Fig. 16), that is 160 bar and 320 bar above atmospheric. In the latter state shown, the maximum principal compressive stresses are about 110 N/mm^2, uniaxial concrete strength in compression being 45 N/mm^2. That means, the material law used yields correctly, that in the center of a dome like this non-linearity of deformations is small on the pressurized side, since the deviatoric component of the states of stress existing there is not very large. Thus, the difference between measured and calculated ultimate strength is only 5 %, computed results being conservative, and the deformations resulting from the analysis fit the test results rather well, see Fig. 17. Ultimate load of this closure is more than four times operating pressure.

4. Final remarks

It was the aim of this paper, to show some possibilities of calculating PCRVs regarding non-linear material behaviour. A short paper like this only allows a brief review of these complex questions without discussing details. This review was given with examples of non-linear calculations which have been performed with the dynamic relaxation method.

Concluding it should be stated that today more knowledge about calculation methods is available than about material behaviour. Because of the high costs of the PCRV calculations it is necessary to decide about the accuray of the numerical calculation. It is senseless to do an expensive and accurate calculation with less precise informations about the input data. It is the task of future activities in this field to reduce the immense calculation efforts to a level which ist sufficient for the safety aspects and adapted to the knowledge about the material and to the demands of the structural design.

References

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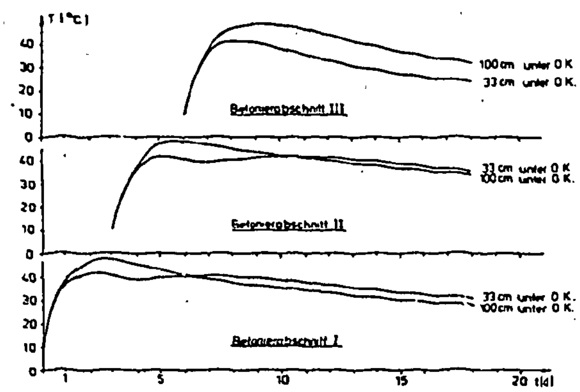


Fig. 1

Temperatures during hydration

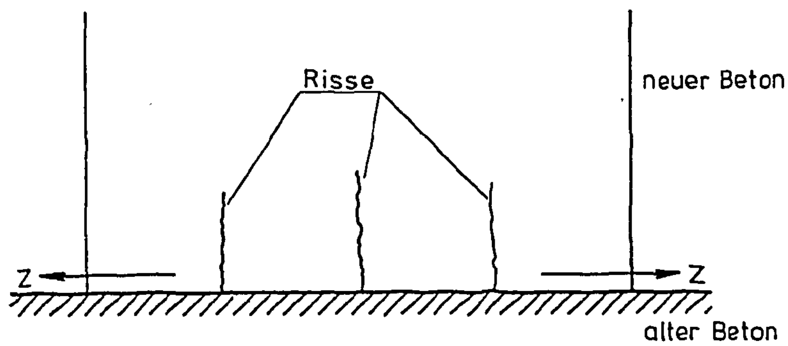
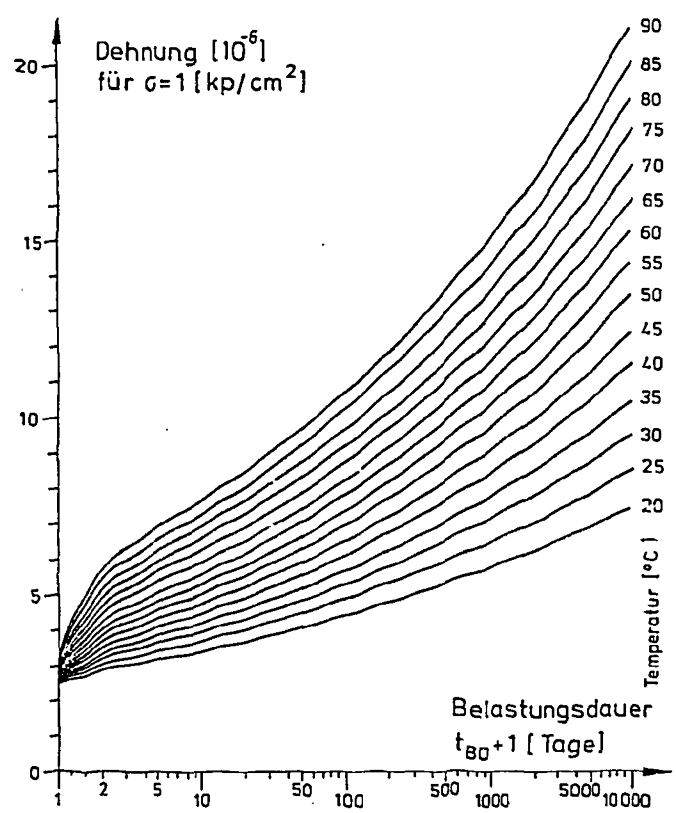


Fig. 2

Cracking in a concrete slab due to cooling and shrinkage when cast onto old concrete member



$$\lg \varepsilon_{sp} = [1-\frac{T}{25}\cdot 10^{-(t-t_{BA}+1)}]\cdot[0{,}34+0{,}0045\cdot T+(0{,}146-(90-T)^4\cdot 1{,}43\cdot 10^{-9})\cdot \lg(t-t_{BA}+1)]$$

Fig. 3

Creep function

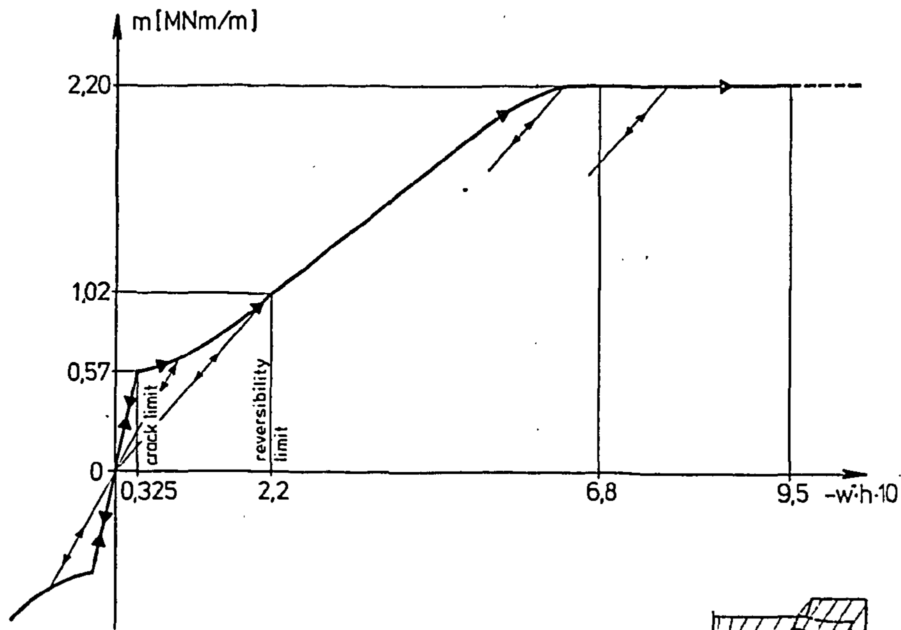


Fig. 4

Influence of cracking on stress-strain behaviour (bending moment m versus bending curvature w)

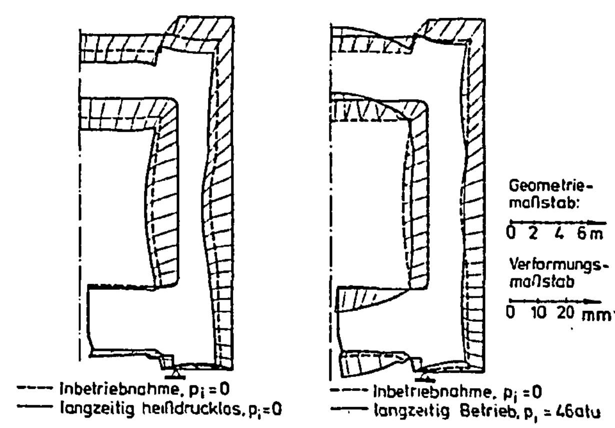


Fig. 5

Extreme cases of deformations during PCRV life

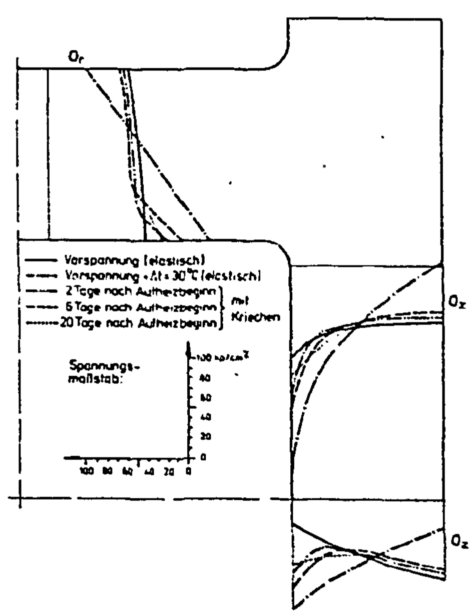


Fig. 6

Stresses during heating a PCRV

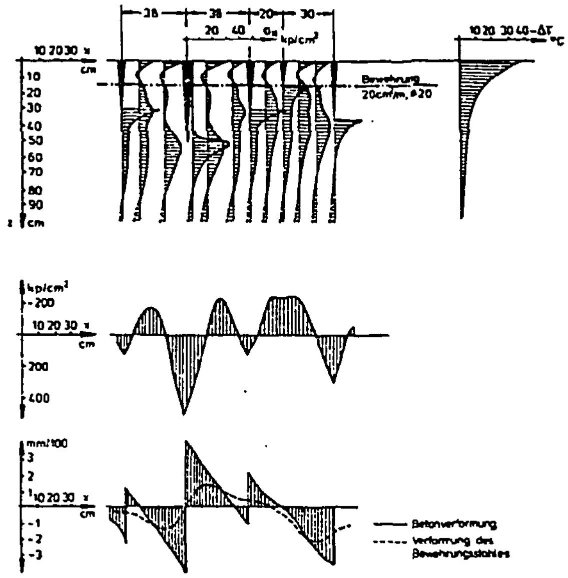


Fig. 7

Cracking under non-stationary surface cooling down

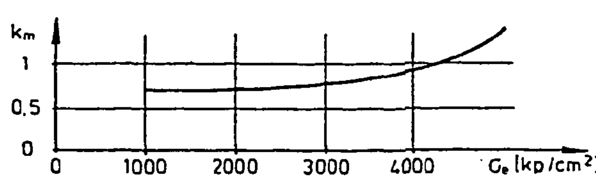


Fig. 8

Coefficient k_m for crack formula

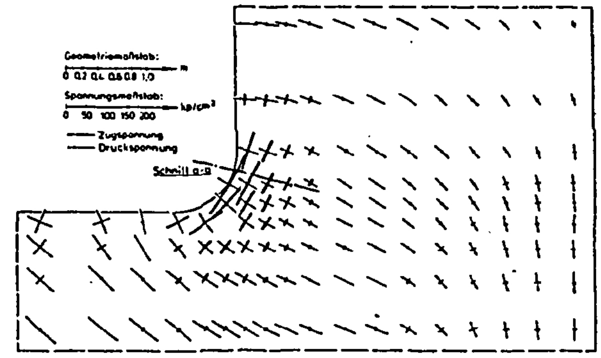


Fig. 9

Haunch region of PCRV, Calculation step 1: no cracking

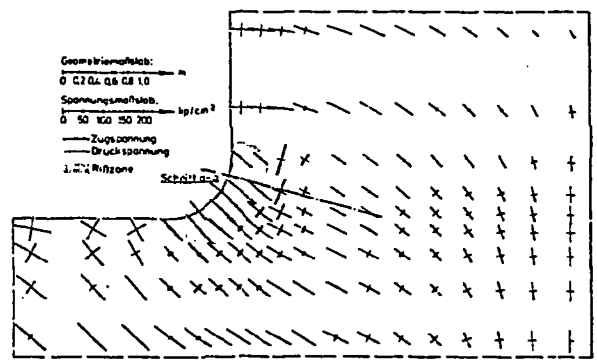


Fig. 10

Haunch region of PCRV, calculation step 2

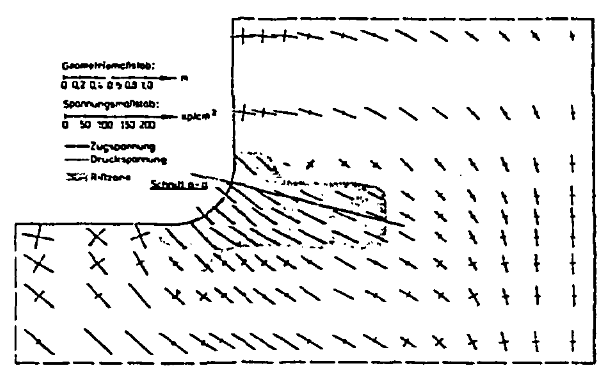


Fig. 11

Haunch region of PCRV, calculation step 3: cracks stabilized

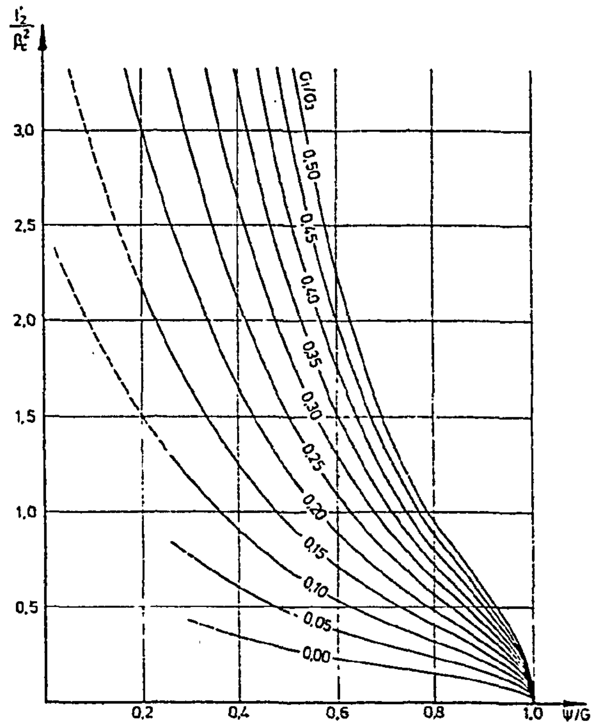
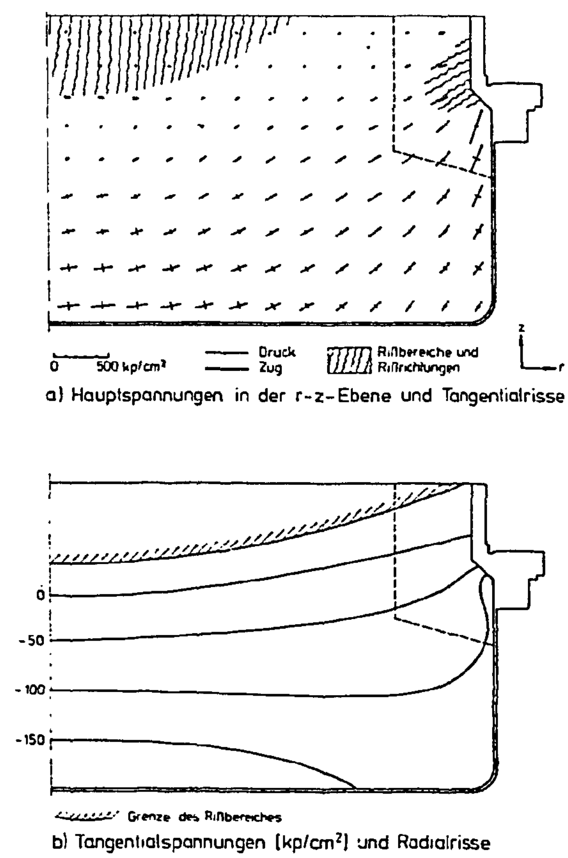


Fig. 12

Plastic shear modulus versus state of stress



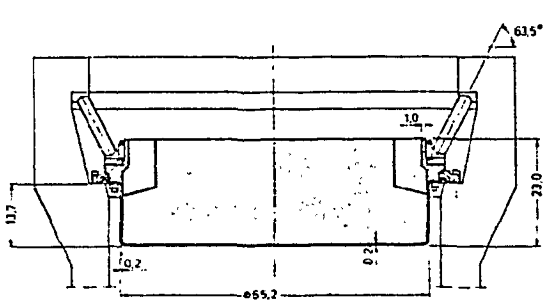


Fig. 13

Top closure model, test arrangement

Fig. 14

Top closure model, stresses and cracking under 80 bar pressure

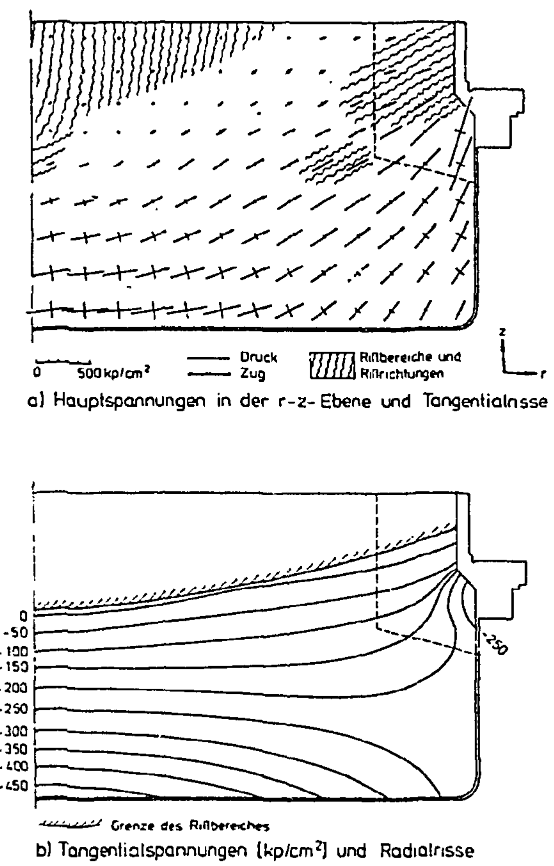


Fig. 15

Top closure model,
stresses and cracking under
160 bar pressure

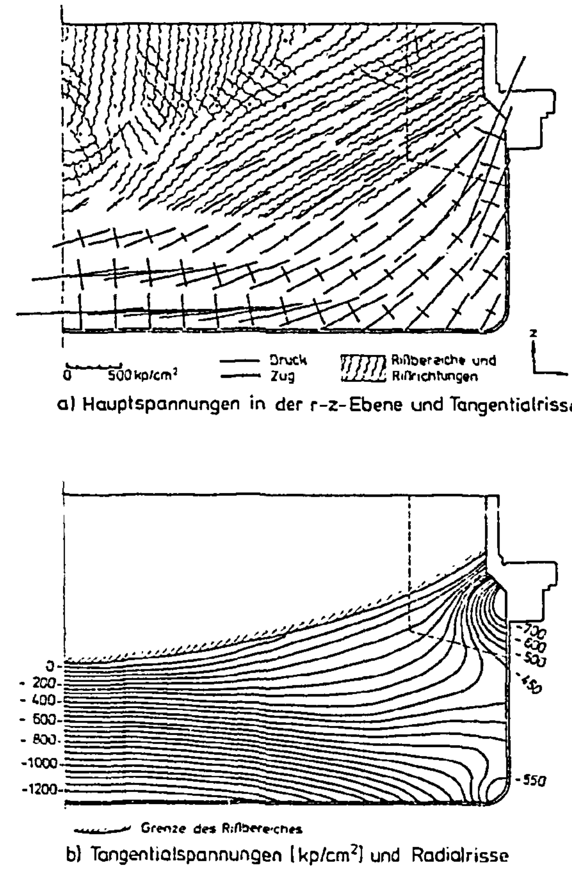


Fig. 16

Top closure model,
stresses and cracking under
320 bar pressure

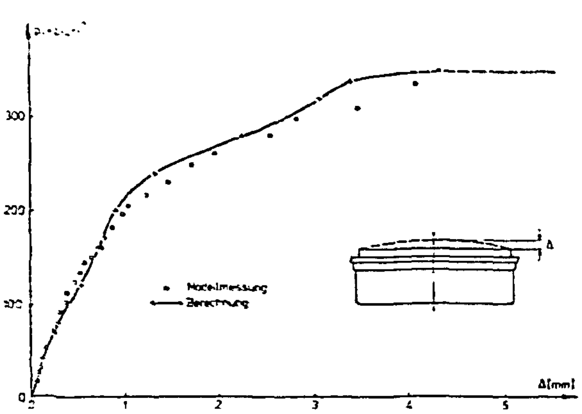


Fig. 17

Top closure model,
vertical displacement
of center