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MORE ON HIGGS BOSONS IN SU(5)

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Abstract

In the framework of the minimal SU(5) model of Georgi and Glashow the explicit couplings between the various mass eigenstate Higgs bosons and the gauge fields as well as the Higgs boson self couplings are presented. As an application bounds for the parameters of the Higgs potential and for the Higgs boson masses are derived by applying partial wave unitarity to the tree graphs of Higgs-Higgs scattering.

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1. Introduction

During the last years there has been a remarkable interest in Grand Unified Theories (GUTs) [1], theories which attempt to describe the strong, weak and electromagnetic interactions classifying all elementary fields with one unified simple gauge group G. While at laboratory energies the strong, weak and electromagnetic coupling constants differ from each other widely, it is assumed that at a mass scale of O (10^{16} GeV) they all unify into the same coupling constant of the unification group G.

In order to be consistent with present day low energy phenomenology which is widely believed to be a spontaneously broken $SU(3)_c \times SU(2)_L \times U(1)$ gauge theory it is first supposed that at some very large mass scale of O (10^{16} GeV) G is spontaneously broken down to $SU(3)_c \times SU(2)_L \times U(1)$. Finally at the much lower mass scale of O (10^2 GeV) the theory is assumed to break down further to $SU(3)_c \times U(1)$ leaving only color and the electromagnetic charge conserved as suggested by experiment.

One of several prices to be paid in GUTs for the unification is that in general many new particles as well as many unknown parameters have to be introduced: first of all to each generator of the unification group G there corresponds a gauge field, secondly the spontaneous symmetry breaking of G requires many Higgs bosons in general, some of which obtain ultra high masses. It is then the Higgs boson potential where the unknown parameters are involved.

The purpose of this paper is twofold: considering a grand unified model the couplings between the various mass eigenstate Higgs bosons and the various gauge fields as well as the various Higgs boson self couplings are in principle given in the Lagrangian, however in general it is quite a difficult task to extract them explicitly. In this paper we want first to discuss the minimal SU(5) model of Georgi and Glashow [2] in this respect, trying to give insight in the complexity of the couplings between the low mass particles (well known from the Salam Weinberg model [3]) and the new superheavy particles. We will give a short review on the SU(5) model, then present in appendices 1, 2, 3, 4 the calculations for the trilinear and quartic couplings between Higgs bosons and gauge bosons as well as the corresponding Higgs boson self couplings in lowest order of perturbation theory. As an example we use in the second part of the paper the principle of tree unitarity to find bounds for the constants of the Higgs potential [4], [5]: Tree unitarity, closely related to the validity of perturbation theory, is applied to Higgs + Higgs → Higgs + Higgs transitions in the tree approximation at very high energies $s \gg m_{x,y}^2$, (Higgs mass)^2. We get constraints for the effective self coupling constants, depending on 5 primordial Higgs couplings, which can easily be turned into upper bounds for the Higgs boson masses.

2. Couplings of Higgs Bosons and Gauge Bosons in SU(5)

At the beginning we shortly want to review the minimal SU(5) model of Georgi and Glashow [2], concentrating mainly on the Higgs sector of the theory. The minimal Higgs system they use involves an adjoint $\underline{24}$ representation Φ to break SU(5) down superstrongly to $SU(3)_C \times SU(2)_L \times U(1)$ and a fundamental $\underline{5}$ H (similar to the Salam Weinberg model) which causes a further weak breaking to $SU(3)_C \times U(1)$. Imposing for simplicity a discrete symmetry $\Phi \to -\Phi$ the most general Higgs potential of degree 4 takes the form

$$V = V(\Phi) + V(H) + V(\Phi,H) \tag{1}$$

where

$$V(\Phi) = -\frac{1}{2}\mu^2 \operatorname{Tr}(\Phi^2) + \frac{1}{4} a(\operatorname{Tr}(\Phi^2))^2 + \frac{1}{2} b \operatorname{Tr} \Phi^4 \tag{2}$$

$$V(H) = -\frac{1}{2}\nu^2 H^\dagger H + \frac{1}{4}\lambda(H^\dagger H)^2 \tag{3}$$

$$V(\Phi,H) = \alpha H^\dagger H \operatorname{Tr}(\Phi^2) + \beta H^\dagger \Phi^2 H , \tag{4}$$

and Φ is given by

$$\Phi = \sum_{i=1}^{24} \phi^a \frac{T^a}{\sqrt{2}} , \qquad \operatorname{Tr}(T^a T^b) = 2\delta_{ab} . \tag{5}$$

The vacuum expectation values and the pattern of symmetry breaking are obtained by studying the absolute minimum of the Higgs potential V and consequently one is interested in the unbroken subgroups of SU(5) left after spontaneous symmetry breaking. These will depend on the direction of Φ and H and not on their norms, hence one has to minimize

$$\frac{1}{2}\, b \;\mathrm{Tr}\; \Phi^4 + \beta\, H^\dagger \Phi^2 H \tag{6}$$

keeping Tr Φ^2 and $H^\dagger H$ fixed. It was shown that the desired symmetry breakdown together with the stability of the potential is realized for [6], [7]

$$15a + 7b > 0\ , \qquad b > 0\ , \qquad \lambda > 0\ , \qquad \beta < 0\ . \tag{7}$$

Additonally the breaking of $SU(5) \to SU(3)_C \times U(1)$ needs $\frac{\beta}{b}\frac{v_o^2}{v^2}$ to be small [7], which in our case for a hierarchically small value of v_o^2/v^2, b at least of O (α_{GUM}) [8], [9] and β bounded by unitarity (see section 3) certainly is fulfilled. We want to emphasize that $|\beta|$ needs not necessarily be smaller than b.

Starting off with a Lagrangian containing elementary gauge and scalar fields we have in the following to extract the physical mass eigenstate gauge and Higgs fields. This is done in a straightforward way [6], [10] by diagonalizing the mass matrices of the vector bosons and of the scalar fields; finally in order to dispose of superfluous degrees of freedom most conveniently the unitary gauge is chosen, which amounts to setting to zero the Goldstone components of the scalar fields.

Thus the 24 gauge bosons V^a of SU(5) split into an octet of color gluons G^a (a = 1,...8), which remain massless as the photon A, further into the weak gauge bosons $W^\pm$, Z and finally into 12 colored superheavy gauge bosons $X_k^\pm$, $Y_k^\pm$ (k = 1,2,3). Their masses are given by

$$m_W^2 = \frac{5}{8}\, m_Z^2 = \frac{1}{4}\, g^2\, v_o^2 \tag{8}$$

and up to corrections of $O(v_o^2)$ by

$$m_X^2 = m_Y^2 = \frac{25}{8}\, g^2\, v^2 \quad . \tag{9}$$

Turning to the physical (mass eigenstate) Higgs bosons, one obtains the usual Salam Weinberg Higgs boson H, which presumably is the only light Higgs boson in minimal SU(5), and the following superheavy Higgs bosons: An uncharged color octet H_8 (consisting of H^a, a = 1,...8), a charged color triplet $H_k^{\pm}$ (k = 1,2,3), an uncolored but charged $H^{\pm}$ and finally the uncolored and uncharged H' and H". Their mass spectrum reads

$$m_H^2 = \frac{1}{2}\lambda v_o^2 - \frac{9}{20}\frac{\beta^2}{b}v_o^2 - \frac{3}{10}v_o^2\frac{(10\alpha + 3\beta)^2}{15a + 7b} \tag{10}$$

and up to (uninteresting) corrections of $O(v_o^2)$

$$m_{H_8}^2 = \frac{1}{4}m_{H^{\pm}}^2 = \frac{1}{4}m_{H'}^2 = \frac{5}{2}bv^2 \tag{11}$$

$$m_{H''}^2 = 15av^2 + 7bv^2 \tag{12}$$

$$m_{H_k}^2 = -\frac{5}{4}\beta v^2 \ . \tag{13}$$

To obtain the rich structure of the various couplings between the Higgs bosons and the gauge bosons we expand the "kinetic energy" part of the Lagrangian

$$L_{kin} = L_{kin}^{\Phi} + L_{kin}^{H} \tag{14}$$

$$L_{kin}^{\Phi} = \frac{1}{2}\,\mathrm{tr}\,(\partial\Phi - i\frac{g}{2}[T^aV^a,\Phi])(\partial\Phi - i\frac{g}{2}[T^bV^b,\Phi]) \tag{15}$$

$$L_{kin}^{H} = |(\partial - i\frac{g}{2}T^aV^a)H|^2 \tag{16}$$

in the above mass eigenstates. Being interested in the physical Higgs boson self couplings as well (see next section) we subsequently transform the Higgs potential V in a similar manner. The voluminous results of the calculations are presented in the appendices 1, 2, 3, 4.

3. Unitarity Bounds

In this section we want to apply the principle of tree unitarity to find upper bounds on Higgs boson masses [4], [5]. If we consider a partial wave amplitude for a 2-particle scattering process, require the validity of perturbation theory and apply partial wave unitarity, we easily find that the absolute value of the tree graph partial wave amplitude has to be (much) less than 1. A violation of this tree unitarity bound would cause higher order contributions to the scattering amplitude to be of similar magnitude as the lowest order one in order to reestablish the unitarity of the full theory. Thus the breakdown of perturbation theory and the appearance of a strongly coupled theory would be unavoidable. It is well known [11] that spontaneously broken gauge theories fulfil tree unitarity insofar as the correct high energy behaviour of the scattering amplitudes is concerned. However, the scattering amplitudes which have right asymptotic behaviour need not necessarily be small so that the tree unitarity bounds serve to find constraints on unknown parameters, which in our case will be a, b, λ, α, β of the Higgs potential V.

We will concentrate for simplicity on scattering processes of the type Higgs + Higgs $\rightarrow$ Higgs + Higgs at very high energies, s >> (Higgs mass)^2, $m_{x,y}^{\ 2}$, where only contact graphs have to be taken into account so that up to trivial factors the scattering amplitudes are given by the quartic Higgs selfcoupling constants (see appendix 4). The most relevant unitarity constraints for the zeroth partial waves then imply the following relations

$$|a + b| < 8\,\frac{\pi}{3} \tag{17}$$

$$|\lambda| < 32\,\frac{\pi}{3} \tag{18}$$

$$|\alpha| < 8\pi \tag{19}$$

$$|\beta| < 16\pi \tag{20}$$

and

$$|15a + 7b| < 40\pi \ . \tag{21}$$

The constraints (17), (18) and (21) were given by the elastic scatterings of H', H and H", respectively; (19) was obtained from couplings of the type $H_j^+ H_j^- \to H^+ H^-$, H'H' or $H^a H^a \to HH$; (20) followed from the scattering of two color octet and two color triplet Higgs bosons. All the other Higgs couplings cause weaker constraints than those given above or are for our purpose of no interest so that they will be ignored. Using (7) and (17) - (20) we obtain the following unitarity bounds for a, b, λ, α, β:

$$-\frac{7\pi}{3} < a < 8\,\frac{\pi}{3} \tag{22}$$

$$0 < b < 5\pi \tag{23}$$

$$0 < \lambda < 32\,\frac{\pi}{3} \tag{24}$$

$$|\alpha| < 8\pi \tag{25}$$

$$-16\pi < \beta < 0 \quad . \tag{26}$$

There are arguments [8], [9] that all the Higgs couplings are of order c_{GUM} at the unification point and we see that the unitarity bounds (22) - (26) are well above this value. It is now very easy to turn these bounds with the help of (10) - (13) into upper bounds on the Higgs boson masses and we get

$$m_H^2 < \frac{16}{3}\,\pi v_o^2 \tag{27}$$

$$m_{H_8}^2 = \frac{1}{4}\, m_{H^\pm}^2 = \frac{1}{4}\, m_{H'}^2 < \frac{75}{6}\,\pi v^2 \tag{28}$$

$$m_{H''}^2 < 40\,\pi v^2 \tag{29}$$

$$m_{H_k}^2 < 20\,\pi v^2 \quad . \tag{30}$$

These mass bounds mean the following: if the Higgs masses were comparable or greater than the corresponding critical values in eqs. (27) - (30) the interactions among these Higgs bosons would become strong at energies above their masses and higher order effects would necessarily become important.

A driving point in our study was the question how severely tree unitarity would restrict the value of the mass of the color triplet Higgs $H_k^{\pm}$. The point is that in proton decay in addition to the decay modes mediated by X, Y-exchange one might expect contributions from H_k exchange. The ratio of the two decay widths is approximately given by [6]

$$\frac{\Gamma(p \overset{H_k}{\rightarrow} \ell + \text{any})}{\Gamma(p \underset{X,Y}{\rightarrow} \ell + \text{any})} = O\left(\frac{m_{X,Y}^4}{m_{H_k}^4} \frac{m_\ell^4}{m_W^4}\right) \tag{31}$$

so that a very small upper bound on $|\beta|$ would give relevance to H_k exchange. With (30) we could check that this is not the case, as we have with $\alpha_{GUM} \sim 1/40$

$$\frac{m_{X,Y}}{m_{H_k}} > \frac{1}{8} \quad . \tag{32}$$

We see that m_{H_k} may lie in such a range that the ratio of the decay widths (31) could be small. In this respect we can conclude that neglect of H_k exchange in proton decay is consistent with our unitary bounds on m_{H_k}.

It is interesting to note that a stronger bound on β (respectively on m_{H_k} can be obtained simply by requiring positivity of the Salam Weinberg Higgs mass m_H! From (10) and (7) we then have

$$-\sqrt{\frac{10}{9} b\lambda} < \beta < 0 \tag{33}$$

and inserting the unitarity bounds for b and λ we get

$$-\frac{40}{9}\sqrt{3}\,\pi < \beta < 0 \quad . \tag{34}$$

Having required positivity of the tree level expression for m_H one might ask whether radiative corrections to (34) are important. But as it was shown in [9] the effect of gauge boson loops on m_H is such that the form of (10) is unchanged, the Higgs parameters however receiving $O(\alpha^2_{GUM})$ corrections. Thus we see that in our case, where the parameters are taking the critical values (23), (24), the radiative corrections can be neglected and we may expect (34) still to be relevant.

The bounds on m_H and β will still be refined, by taking care of the energy dependence of the SW Higgs boson coupling constant λ: Demanding λ to fulfill (24) in the whole energy range up to m_{GUM} one has to consider the unstable character of the renormalization group equations, obeyed by λ. By this an improved bound for λ at the energy scale v_o can be found [12], which has to be used in (27) and (34) and which is approximatively given by

$$\lambda \lesssim 8 \frac{\pi}{3} \tag{35}$$

So we have

$$-\frac{20}{9}\sqrt{3}\,\pi \lesssim \beta < 0 \tag{36}$$

which leads to

$$\frac{m_{X,Y}}{m_{H_k}} \gtrsim \frac{1}{4} \;. \tag{37}$$

We see that although the lower bound on the ratio of the heavy vector boson mass to the color triplet Higgs mass was raised by a factor of 2 our conclusions concerning the neglect of H_k exchange still remain unchanged.

4. Conclusions

Grand unified theories are a fascinating and appealing way to unify electromagnetic, weak and strong interactions. Phenomenology in these theories usually is hindered by an excess of Higgs bosons and gauge bosons. In the first part of this work we presented for the minimal SU(5) model of Georgi and Glashow the explicit calculations for all trilinear and quartic physical Higgs boson and gauge boson couplings.

In the second part of this paper we applied partial wave unitarity to the tree graphs of Higgs-Higgs scatterings and derived bounds on the parameters a, b, λ, α, β as well as on the Higgs boson masses. We checked that neglect of the color triplet Higgs $H_k^{\pm}$ contributions to proton decay is consistent with the upper bound we found for m_{H_k}.

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Appendix 1

In the appendices we use the following notation: $\delta = \frac{10\alpha + 3\beta}{15a + 7b}$, f_{abc} and d_{abc} are the usual SU(3) coefficients; λ^a_{ik} are the Gell-Mann matrices.

Trilinear Couplings between Higgs Bosons and Gauge Fields from L_{kin}

a) SW-Higgs, SW-gauge bosons

$$\frac{2}{5} v_o\, g^2\, H\, Z\, Z + \frac{1}{2} v_o\, g^2\, H\, W^+\, W^- \; .$$

b) SW-Higgs and superheavy gauge bosons

$$\frac{1}{8} v_o\, g^2\, H\, Y^+_k\, Y^-_k (4 - 3\,\frac{\beta}{b} - 10\delta) + \frac{1}{8} v_o\, g^2\, H\, X^+_k\, X^-_k (3\,\frac{\beta}{b} - 10\delta) \; .$$

c) SW-Higgs, superheavy Higgs and superheavy gauge bosons

$$\frac{1}{2} g\, H\, [i(\partial H^-_k) Y^+_k + h.c.] + \frac{1}{2} g\, (\partial H)(iH^+_k\, Y^-_k + h.c.)$$

d) Superheavy Higgs bosons and gluons

$$g\, f_{abc}\, H^a (\partial H^b) G^c - \frac{g}{2} [i(\partial H^+_j) \lambda^a_{jk}\, H^-_k + h.c.] G^a \; .$$

e) Superheavy Higgs and SW-gauge bosons, superheavy gauge bosons

$$\frac{1}{2\sqrt{6}} g\, [3iH^+(\partial H^-) + iH^+_k(\partial H^-_k) + h.c.] A - \frac{1}{2\sqrt{10}} g\, [5iH^+(\partial H^-) - iH^+_k(\partial H^-_k) +$$

$$+ h.c.] Z - \frac{2}{\sqrt{10}} v_o\, g^2 (H^+_k Y^-_k + h.c.) Z + gH' [i(\partial H^-) W^+ + h.c.] +$$

$$+ g(\partial H')(iH^+ W^- + h.c.) + \frac{1}{\sqrt{2}} v_o g^2 (H^+_k\, X^-_k\, W^+ + h.c.) \; .$$

f) Superheavy Higgs and superheavy gauge bosons

$$\frac{5}{2\sqrt{2}} v\, g^2\, H^a (Y^+_j \lambda^a_{jk} Y^-_k + X^+_j \lambda^a_{jk} X^-_k) + \frac{5}{2\sqrt{2}} v\, g^2\, H' (Y^+_k Y^-_k - X^+_k X^-_k) +$$

$$+ \frac{25}{2\sqrt{30}} v\, g^2\, H''(Y_k^+Y_k^- + X_k^+X_k^-) - \frac{5}{2} v\, g^2 (H^+Y_k^+X_k^- + \text{h.c.})$$

Appendix 2

Quartic Couplings between Higgs Bosons and Gauge Fields from L_{kin}

a) SW-Higgs, SW-gauge bosons

$$\frac{1}{4} g^2 (H)^2 W^+W^- + \frac{1}{5} g^2 (H)^2 Z^2 \quad .$$

b) SW-Higgs, superheavy gauge bosons

$$\frac{1}{4} g^2 H^2 Y_k^+ Y_k^- \quad .$$

c) SW-Higgs, SW-gauge bosons, superheavy gauge and superheavy Higgs bosons

$$\frac{1}{4} \frac{1}{\sqrt{6}} g^2 HA(Y_k^- H_k^+ + \text{h.c.}) - \frac{3}{4\sqrt{10}} g^2 HZ(Y_k^- H_k^+ + \text{h.c.}) + \frac{1}{2\sqrt{2}} g^2 H(W^- X_k^+ H_k^- + \text{h.c.}) .$$

d) SW-Higgs, gluons, superheavy gauge and superheavy Higgs bosons

$$\frac{g^2}{4} HG^a (Y_j^- \lambda_{jk}^a H_k^+ + \text{h.c.}) \quad .$$

e) Superheavy Higgs bosons and gluons

$$\frac{g^2}{4} H_j^+ (\frac{2}{3} \delta_{ab} \delta_{jk} + d_{abc} \lambda_{jk}^c) H_k^- G^a G^b + \frac{1}{2} g^2 f_{abe}\, f_{cde}\, H^b H^d G^a G^c \quad .$$

f) Superheavy Higgs bosons, gluons and SW-gauge bosons

$$\frac{1}{2\sqrt{6}} g^2\, H_j^+\, \lambda_{jk}^a\, H_k^-\, G^a\, A + \frac{1}{2\sqrt{10}} g^2\, H_j^+\, \lambda_{jk}^a\, H_k^-\, G^a\, Z \quad .$$

g) Superheavy Higgs bosons and SW-gauge bosons

$$\frac{15}{4} g^2 H^+ H^- (\frac{1}{10} AA + \frac{1}{6} ZZ - \frac{1}{\sqrt{15}} AZ) + \frac{1}{4} g^2 H_k^+ H_k^- (\frac{1}{6} AA + \frac{1}{10} ZZ + \frac{1}{\sqrt{15}} AZ) +$$

$$+ \frac{3}{2\sqrt{6}} g^2 H'(H^+W^- + h.c.)A - \frac{5}{2\sqrt{10}} g^2 H'(H^+W^- + h.c.)Z +$$

$$+ g^2[H^+H^- + (H')^2]W^+W^- - \frac{1}{2} g^2[(H^+)^2 W^-W^- + h.c.] \ .$$

h) Superheavy Higgs bosons and superheavy gauge bosons

$$\frac{1}{4} g^2 H^a H^b (\frac{2}{3} \delta_{ab} \delta_{jk} + d_{abc}\lambda^c_{jk})(Y^+_j Y^-_k + X^+_j X^-_k) + \frac{1}{2} g^2 H' H^a Y^+_j \lambda^a_{jk} Y^-_k +$$

$$+ \frac{5}{2\sqrt{15}} g^2 H'' H^a (Y^+_j \lambda^a_{jk} Y^-_k + X^+_j \lambda^a_{jk} X^-_k) - \frac{1}{\sqrt{2}} g^2 H^a (H^- X^+_j \lambda^a_{jk} Y^-_k + h.c.) +$$

$$+ \frac{1}{2} g^2 H^+_j H^-_k (Y^+_j Y^-_k + X^+_j X^-_k) + \frac{1}{2} g^2 H^+ H^- (Y^+_k Y_k + X^+_k X^-_k) -$$

$$- \frac{5}{\sqrt{30}} g^2 H'' (H^+ Y^+_k X^-_k + h.c.) + \frac{5}{2\sqrt{15}} g^2 H' H'' (Y^+_k Y^-_k - X^+_k X^-_k) +$$

$$+ \frac{1}{4} g^2 (H')^2 (Y^+_k Y^-_k + X^+_k X^-_k) + \frac{5}{12} g^2 (H'')^2 (Y^+_k Y^-_k + X^+_k X^-_k) \ .$$

Appendix 3

Trilinear Self Couplings between Higgs Bosons from the Higgs Potential V

a) Involving SW-Higgs bosons

$$
\begin{aligned}
& v_o H\, H^+ H^- [-\tfrac{3}{5}(5a+9b)\delta + 2\alpha + \beta] + \\
& + \frac{9}{2\sqrt{15}}\, \beta\, v_o H\, H' H'' [-\frac{1}{10b}(10a-10\alpha-3\beta) + \delta - \frac{17}{15}] + \\
& + \frac{1}{2} v_o H (H')^2 [-\frac{3}{5}(5a+9b)\delta + 2\alpha + \beta + \frac{9}{10}\frac{\beta^2}{b}] + \\
& + \frac{3}{10} v_o H (H'')^2 (10\alpha+3\beta)(\delta - \frac{2}{3}) + v_o H\, H^a H^a [-\frac{3}{10}(5a+4b)\delta + \alpha] \\
& - \frac{1}{5} v_o H\, H_k^+ H_k^- [(15\alpha+2\beta)\delta + \beta - \frac{5}{2}\lambda] + \\
& + \frac{3}{2\sqrt{2}}\, \beta v (H)^2 H' + \frac{3}{2\sqrt{30}}\, v (H)^2 H'' (10\alpha+3\beta) - \frac{1}{40} v_o (H)^3 [9\frac{\beta^2}{b} + 6(10\alpha+3\beta)\delta - 10\lambda].
\end{aligned}
$$

b) Superheavy Higgs bosons

$$
\begin{aligned}
& \frac{3}{\sqrt{30}} v (H')^2 H'' (5a+9b) + \frac{3}{\sqrt{30}} v\, H'' H^a H^a (5a+4b) + \frac{2}{\sqrt{30}} v\, H'' H_k^+ H_k^- (15\alpha+2\beta) + \\
& + \frac{6}{\sqrt{30}} v\, H'' H^+ H^- (5a+9b) + \frac{1}{\sqrt{30}} v (H'')^3 (15a+7b) + \sqrt{2}\, b\, v\, d_{abc}\, H^a H^b H^c + \\
& + \sqrt{2}\, \beta\, v\, H_j^+ \lambda_{jk}^a H_k^- H^a .
\end{aligned}
$$

Appendix 4

Quartic Self Couplings between Higgs Bosons from the Higgs Potential V

a) Involving SW-Higgs Bosons

$$\frac{1}{2}\,\alpha(H)^2H^aH^a + \frac{1}{4}\,\lambda(H)^2H_k^+H_k^- + \frac{1}{2}(H)^2H^+H^-(2\alpha+\beta) + \frac{3}{2\sqrt{15}}\,\beta(H)^2H'H'' +$$

$$+ \frac{1}{4}(H)^2(H')^2(2\alpha+\beta) + \frac{1}{20}(H)^2(H'')^2(10\alpha + 3\beta) + \frac{1}{16}\,\lambda(H)^4 \quad .$$

b) Superheavy Higgs bosons

$$aH^+H^-H^aH^a + 2\alpha H^+H^-H_k^+H_k^- + (H^+)^2(H^-)^2(a+b) + \frac{a}{2}(H')^2H^aH^a + \alpha(H')^2H_k^+H_k^- +$$

$$+ (H')^2H^+H^-(a+b) + \frac{1}{10}(H')^2(H'')^2(5a+9b) + \frac{1}{4}(H')^4(a+b) +$$

$$+ \frac{2}{\sqrt{15}}\,b\,d_{abc}\,H^aH^bH^cH'' + \frac{2\beta}{\sqrt{15}}\,H''H_j^+\lambda_{jk}^aH_k^-H^a + \frac{1}{10}(H'')^2H^aH^a(5a+4b) +$$

$$+ \frac{1}{15}(H'')^2H_k^+H_k^-(15\alpha+2\beta) + \frac{1}{5}(H'')^2H^+H^-(5a+9b) + \frac{1}{60}(H'')^4(15a+7b) +$$

$$+ \frac{1}{4}\,H^aH^aH^bH^b(a+b) + \frac{1}{2}\,\beta\,H^aH^bH_j^+\left(\frac{2}{3}\,\delta_{ab}\delta_{jk} + d_{abc}\lambda_{jk}^c\right)H_k^- +$$

$$+ \quad \alpha\,H^a\,H^a\,H_k^+\,H_k^- + \frac{1}{4}\,\lambda\,H_j^+\,H_j^-\,H_k^+\,H_k^- \; .$$