THE CHIRAL BAG MODEL AND THE LITTLE BAG

by

Vincent VENTO
ABSTRACT: We review the properties of the existing solutions to the Chiral bag equations of motion and discuss how the "little bag" picture could come about in this scheme. Our analysis leads to a model which is qualitatively similar to the naive quark model with pion cloud corrections. We use this latter approach to look for pion cloud signatures in experimental data.

I. INTRODUCTION

The "little bag" is an attempt to incorporate in a predictive scheme, the knowledge on the structure of the hadrons, as seen by short wavelength probes, and the interaction among them in the long wavelength regime.

The first step towards the "little bag" is what we have called the chiral bag model\(^1\). The latter incorporates, not only asymptotic freedom and confinement, as in the MIT bag model\(^2\), but also chiral symmetry, a property of quantum chromodynamics (QCD) for massless quarks. For massless up and down quarks, the procedure consists in introducing a pion field outside the confinement region to assure axial current conservation. The pion field in particular, and chiral symmetry in general are viewed as the link to join the features of the two energy regimes. This pion field represents in some way the non perturbative aspects of QCD beyond confinement. How the pionic degrees of freedom arise from this underlying theory is still an open question. Several attempts in this direction have appeared in the literature\(^3,4\). Our approach is though phenomenological. We construct a model with minimal number of ingredients to preserve chiral symmetry. If our pion field is a complicated vacuum excitation, or some other phenomena of an extremely rich theory, QCD, is a question that remains to be answered.

Other meson-like degrees of freedom might also be included within our scheme, but in this early attempt simplicity has always been the guiding rule.

The emphasis in the little bag philosophy is that the mesonic degrees of freedom introduced via chiral symmetry play a crucial role. Why little and not chiral? Although chirality is the main reason for introducing our approach, chirality can be minimized by perturbing around the MIT solution. The little bag, will arise when
chiral effects are maximal, which in our model will require the whole non-linear structure of the theory. As we shall see there is a connection between size and large pion effects.

We use in our calculations qualitative features of the so-called perturbative expansion, mainly because there is nothing better we can do at this stage, but we are guided in our intuition by the physics coming from the lowest energy classical spherical solution to the non-linear equations we have, the hedgehog\(^{(1)}\).

II. STATIC SOLUTIONS TO THE BAG EQUATIONS

The mathematical formulation of the chiral bag model gives rise to a system of non-linear coupled equation of motion for the fields\(^{(1)*}\). In order to solve them, two procedures have been followed: (1) the perturbative expansion\(^{(5,6)}\); (2) the mean field approximation\(^{(1)}\).

The perturbative expansion emphasizes confinement and assumes chiral effects to be small in comparison. The procedure is to expand the fields in terms of the coupling constant. For the observables this turns out to be an expansion in an effective dimensionless parameter \(\epsilon\),

\[
\epsilon \sim \frac{1}{f_\pi^2 R^2} \tag{II.1}
\]

where \(f_\pi\) is the pion decay constant and \(R\) the bag radius. The method is thus useful for "large" radii.

To first order the pion field vanishes and the quark field satisfies the MIT equations of motion. In order to preserve chirality the second order in the pion field has to be included. This gives rise to a source for linear pions and reproduces the initial little bag picture\(^{(7)}\). The pion source can be used within the reduction formalism to compute pion emission and absorption processes. The coupling of quarks to pions gives rise to a tensor like force that produces to next order a D-state admixture. A problem arising in this scheme is that the pressure balance equation cannot be satisfied locally, unless some kind of explicit surface deformation is assumed to this order. Most calculations proceed to avoid this difficulty by averaging over the angular dependence, which is equivalent to defining the pressure balance equation by

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\*We shall omit in what follows any discussion about the gluonic degrees of freedom, which should be included in the spirit of perturbative QCD inside the bag.
The effect of the pion field in the spectra is relatively small, although qualitatively significant. It gives rise to contributions to the energy and to the pressure which tend to shrink the bag\(^{(6,8)}\). If one remains within the spirit of the large bag, and neglects the deformation of the surface, one achieves a simple predictive scheme. Magnetic moments and other quantities can thus be calculated\(^{(9)}\). The most crucial feature of the calculation, where chirality appears in its full glory is \(g_A\). As shown initially by Jaffe\(^{(5)}\), the model \(g_A\) not only satisfies the Goldberger-Treiman relation, but due to the contribution from the pions becomes bigger than the experimental value of 1.25, no matter what radius one chooses. All other calculated effects disappear in the \(R \to \infty\) limit, but not this one. What appeared as a crudeness of the model might open our understanding of the structure of the hadrons.

The mean field approach has given rise to a classical spherical solution to the equations of motion\(^{(1)}\). Unusual quantum numbers have to be defined in order to satisfy the boundary conditions. The pion field, now a c-number field, points in the radial direction at each point, thus its name: hedgehog. Taken seriously it provides us qualitatively with the picture we are seeking, the little bag.

Let us point out some of its features. For every allowed bag constant we have two solutions at different radii. The small bag solution corresponds to a maximum of the energy functional,

\[
E = E(R)
\]

for fixed \(f_\pi\) and \(B\), where \(B\) is the bag constant, thus unstable. The big radius solution leads to a shallow minimum, i.e., stable. The rising parabola shape for small radii of the non chiral solutions is lost. The big bag solutions are dominated by non chiral effects, MIT like solutions; for the small bag ones, the pionic effects are dominant. Beyond the small radius extremum the system collapses\(^{*}\). A feature of the small bag solution which is extremely exciting, is the non-relativistic behaviour of the quark wave functions. The axial vector coupling \(g_A\), can be calculated exactly, and as a function of bag radius is monotonically decreasing, and always bigger than the experimental value, even for very large radii. From the studied quantities, \(g_A\) is the only one that presents this feature.

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\* Notice that for fixed \(B > 0\) and \(f_\pi\) there exist in the energy functional two extrema, a maximum and a minimum, but only one of them satisfies the Goldberger-Treiman relation with fixed pion nucleon coupling constant \(f_\pi^2 = .081\).
Other aspects of the solution, specially in relationship with magnetic moments have been discussed elsewhere\(^{(10)}\), and we shall omit them for brevity.

III. THE LITTLE BAG

The previous section has taught us how the pion field affects the bag picture once it is included via chiral symmetry. The main problem we face at this stage is how to eliminate the collapse feature of the energy functional at small radii. Let us conjecture that there exists some kind of mechanism in the true solution of the field equations including quantum effects, that recovers the negative slope at small radii. Three possibilities can be envisaged: (1) only the perturbative minimum remains. (2) the maximum of our energy functional is just a signal for a second minimum beyond it\(^{(11)}\). (3) only this second minimum remains and the perturbative minimum is washed out\(^{(12)}\).

If we believe in the first possibility, it is clear that the theory as defined initially by the MIT group would be close to reality, although chirality enriches it tremendously by including the effects we have mentioned before, i.e., pion self-energy, pion pressure, contribution to \(g_A\), D-state admixture etc... If the second alternative is the chosen one, hadrons will behave differently according to determined initial conditions. Two extreme alternatives might occur in this case. One, the energy functional for fixed \(B\) and \(f_\pi\) is unaltered and the hadron just jumps from one minimum to the other by changing its coupling to pions. The second one is that \(B\) and/or \(f_\pi (g_A)\) change while leaving the pion coupling fixed, i.e., the functional changes. If we consider the energy as a function of all its degrees of freedom \(R, B\) and \(f_\pi\) the different alternatives one could envisage represent motions along different paths in the space of the parameters.

If we believe blindly in the model as it stands, apart from the stabilization mechanism, charge radii tell us that large confinement regions would be a property of free nucleons and small bags would arise in nuclear matter where the "quark soup" has not been observed at nuclear matter densities.

Finally and this is the realization of the little bag idea, the third possibility implies that the complicated bag model we started from, transforms into an almost non relativistic quark model with pion cloud corrections. Effectively this leads to a quasi-particle interpretation in which the classical solution would provide a mechanism to generate the almost free quasi-quarks endowed with a mass of \(\sim \frac{1}{3} m_N\). Sphericity though, produces a large deviation of \(g_A\) from its experimental value, thus the true baryons should have a large amount of D-state admixture.

This picture we envisage when properly formulated should lead to the non-relativistic quark model\(^{(13)}\), with some corrections due to pion cloud contributions and
IV. SOME RESULTS WITHIN NAIVE QUARK MODEL APPROACH

The previous section has been mainly one of conjectures as a result of some model calculations. But while the search for the theory that provides us with some of the answers continues, let us look from a more pragmatic prospective to signatures of some of the ideas we have advocated.

We have performed a series of calculations within the naive quark model assuming a D-state admixture in the usual quark wave function $\psi$. In the case of $g_A$ the result is encouraging:

$$g_A = \frac{5}{3} \left( 1 - \frac{6}{5} P_D \right)$$

(IV.1)

Glashow obtained this result arguing by analogy to the nuclear three body problem. Other quantities within the same approach show also the right tendency $\frac{g_{A\pi N}}{g_{\pi NN}}$ and $\frac{D+F}{D-F}$. The amount of D-state admixture depends strongly on the initial spheri­cal state one starts from. We shall not make at this moment a strong point on its magnitude, just say it is large compared to similar situations in nuclear physics, i.e. 7% for triton. We advocate in our calculation for a tensor force like interaction produced by quark pion coupling in the spirit of the chiral bag model results mentioned earlier.

Pion cloud effects in magnetic moments can also be significant as was shown in a calculation for strange baryons in which deformation effects were not included.

Preliminary results in the $\Delta-\gamma N$ E2 transition in the same spirit look promising. The fact that this calculation depends on the explicit radial wave function one chooses, makes predictions though, less universal.

In our opinion a complete reanalysis of the naive quark model results with the new ingredients will most certainly not only shed some light in our understanding of the properties of the baryons but point out the experiments that need to be performed to clarify these conjectures.

V. CONCLUSION

The results obtained with the naive quark model approach, as well as the systematic deviation of $g_A$ from the experimental value point towards a more complicated structure of the baryons than initially suspected. Chiral bag models provide qualitative understanding for these features. The duality between large and small bags is still unresolved. Most of the static properties are almost radius independent and the
one that is not, is difficult to calculate within the chiral bag approach beyond lowest relevant order in the perturbative expansion.

The collapse feature of the energy functional is a deep problem. We might just overcome it by ignoring its existence and accepting the energy functional close to its stability point but not beyond it; or try to understand it. This might bring new physics into the picture and the success of the non relativistic model points to some of us that the direction towards maximal chirality, i.e., little bag, is worth pursuing.

The calculations within the naive quark model have also shown, that this extremely successful approach might need revision, and that experimental deviations from its predictions might open our understanding for phenomena that were omitted in its original formulation. This signals the possibility of beginning to understand the overlap between the "old" and the "new" degrees of freedom and points towards new theoretical and experimental work that needs to be done. A concrete experimental proposal would be to determine more accurate $E2$ matrix elements.

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