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A FINITE ELEMENT MODEL FOR HEAT CONDUCTION IN JOINTED ROCK MASSES*

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1. SUMMARY

A computational procedure for simulating heat conduction in a fractured rock mass is proposed and illustrated in the present paper. The method makes use of a simple "local" model for conduction in the vicinity of a single open fracture. The distributions of fractures and fracture properties within the finite element model are based on a statistical representation of geologic field data. Fracture behavior is included in the finite element computation by locating local, discrete fractures at the element integration points.

2. INTRODUCTION

Recent interest in the use of underground repositories for the disposal of nuclear waste has dictated the need to analyze heat transfer processes in geologic environments. Typical candidate geologic materials include several types of hard rock, e.g., basalt, granite, shale, and tuff. Hard rock formations are typically characterized as discontinuous media due to the numerous faults and intersecting fracture sets occurring in the formations. Such discontinuities complicate the heat transfer analyses in these materials since the presence of open joints may significantly increase the overall

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thermal resistance of the formation as compared to the resistance of an intact rock mass. A further complication is introduced by the fact that the fracture spacing, magnitude of joint opening, and spatial orientation of the fracture set may vary significantly (but non-randomly), within a particular formation.

Previous work in the finite element analysis of jointed rock masses has not addressed the heat transfer question but rather, has concentrated on prediction of the mechanical response of the formation. Within the context of structural analyses, a jointed rock mass has generally been modeled as one of two limiting cases; the ubiquitous model, which is a continuum approach where the joint spacing is infinitesimal, and the discrete joint model, where the individual behavior of every joint in the formation is considered. The ubiquitous model is most appropriate when the length scale of the problem is large compared to the joint spacing while the discrete fracture model is best suited for very local, small scale analyses. For both of these cases, a comparable treatment of the heat conduction problem can be developed. However, both models become either physically inappropriate or computationally inefficient for intermediate scale problems where the joint spacing is comparable to the length scale of the problem. In the present paper, an alternative to the ubiquitous and discrete fracture models is proposed for the modeling of heat conduction in discontinuous formations. The work reported here closely parallels the work of Thomas [1] in the mechanical modeling of jointed rock formations.

3. MODEL DEVELOPMENT

The development of the heat conduction model for a discontinuous medium is best accomplished in two stages. In the first stage, a local model for the effective thermal conductivity and heat capacity in the vicinity of a single open joint is described. The second portion of the development then describes the manner in which this local model is incorporated into a finite element procedure in order to simulate a heterogeneous material.

3.1 Local Model

Consider the sketch in Figure 1 that shows an idealized set of planar joints imbedded in an infinite mass of otherwise intact rock. For purposes of this discussion, the joint set is assumed to be made

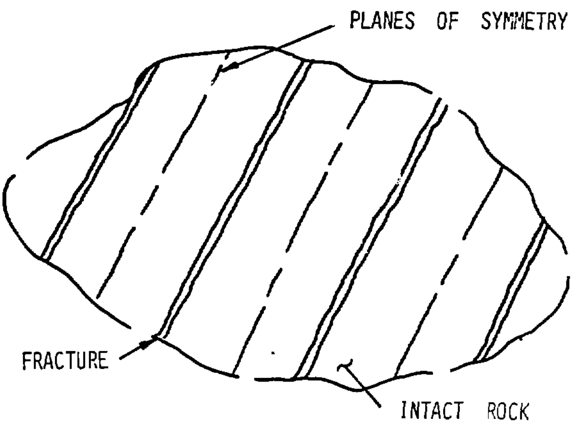


Figure 1. Schematic of Fractured Rock Mass.

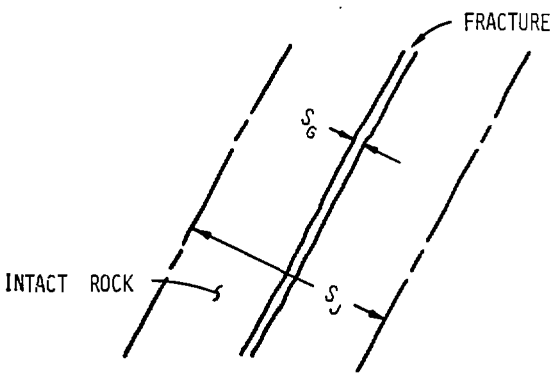


Figure 2. Nomenclature for Single Fracture Model.

up of parallel joints with a uniform spacing, S_j, and a constant joint aperture, S_g. Identifying the planes of symmetry in this infinite array of joints allows a single joint to be considered as shown in Figure 2.

The local conduction model requires the effective conductivity tensor and heat capacitance for the combined joint and rock mass shown in Figure 2. Considering first the conductivity problem, a local coordinate system is established with its principal axes parallel and perpendicular to the fracture. Denoting the conductivity of the joint material as K_j and the intact rock conductivity by K_r, then the effective conductivity parallel to the joint is given by

$$K_{11} = K_r + \frac{S_g}{S_j}(K_j - K_r) \tag{1}$$

The effective conductivity perpendicular to the joint is

$$K_{22} = \frac{K_r}{1 - \frac{S_g}{S_j}\left(1 - \frac{K_r}{K_j}\right)} \tag{2}$$

These relationships are easily derived by considering the electrical network analogy to the indicated conduction problems, *i.e.*, parallel and series connections of "thermal resistances." Note that these components of the conductivity tensor are written in terms of the principal "material" axes of the jointed rock; use of the conductivity tensor in a global reference frame requires the specification of the angular orientation of the joint.

Procedures similar to the above may be used to derive the effective heat capacity for the region around the fracture. Such an analysis allows the effective capacity to be expressed as

$$\rho C = (1 - \phi)(\rho C)_r + \phi(\rho C)_g \tag{3}$$

where $\phi = S_g/S_j$ is the joint porosity. Equation (3) is a well known formula for computing effective properties in porous materials.

Equations (1)-(3) describe the effective material properties for a single open joint located in an otherwise intact rock mass. The problem remains as to how to incorporate such a "local" model into an overall description of a multiple fractured formation.

3.2 Finite Element Implementation

Material constitutive relations and material properties are normally evaluated in a finite element model at the integration (Gauss quadrature) points for each element in the model. In the present approach, discrete fractures can be arbitrarily assigned to individual element integration points. This approach permits the continuum nature of the heat conduction problem to be maintained while still incorporating the effects of a discrete fracture system.

In actual implementation, the computational procedure begins with the decision as to whether or not a fracture is to be assigned to a Gauss point. For the present work, it is assumed that all Gauss points for all elements have a joint located at the integration point, i.e., a ubiquitous model is being invoked. The mechanism for relaxing this assumption is straightforward and would involve a statistical model for the frequency of occurrence and location of fractures in the model.

Having once decided that a fracture exists at a Gauss point, there remains the problem of assigning a number of parameters to each fracture. In particular, joint angle (with respect to a global reference), joint aperture and joint spacing must be specified to allow use of the previously defined local joint conductivity and capacitance models. Field data for various hard rock formations suggests that the above properties are best represented in a statistical manner [2,3,4]. Again, for simplicity and brevity, the present work will consider in detail only the statistical representation of joint angle. All joints in a model are assumed to have the same aperture, S_g, and a common spacing, S_j.

The statistical representation of local joint angle is based on the cumulative work of a number of investigators [5,6,7,8,9,10] and is summarized by the equation

$$P(\phi) = 1 - \exp[k(\cos \phi - 1)], \quad k > 6 \quad . \qquad (4)$$

In Equation (4), P is the probability that an observed fracture orientation lies within the solid angle, ϕ, measured from the mean orientation of the global joint pattern. The parameter, k, is the dispersion coefficient (dispersivity) and describes the scatter in the observations. Large values of dispersivity indicate small scatter in the data. The function given in Equation (4) is plotted in Figure 3 for two dispersion coefficients. From the figure, it is seen for example, that a dispersion coefficient of k = 10 implies that 50% of the joint planes are expected to be orientated within 22° of the mean direction of the joint set.

The procedure for initializing fracture angles within the context of a finite element computer code is straightforward. Both the mean orientation, α, of the joint set to be modeled and the dispersivity, k, are assumed to be known. For each element integration point at which a fracture exists, a random number is generated from a uniform distribution defined over the interval (0,1). This value is assigned to the cumulative probability, $P(\phi)$. The deviation from the mean, ϕ, is derived from Equation (4). For two-dimensional models, the solid angle, ϕ, is then projected onto a plane by a direction cosine having the angle $p\pi$, where p is again a random number generated from a uniform distribution on (0,1). Finally, the local joint angle at an integration point is given by

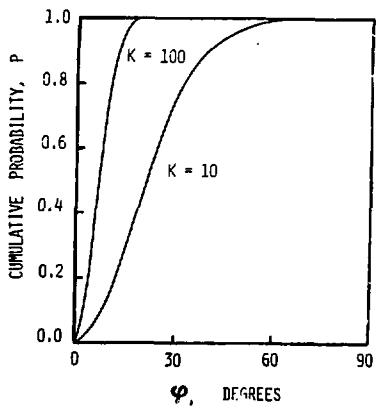


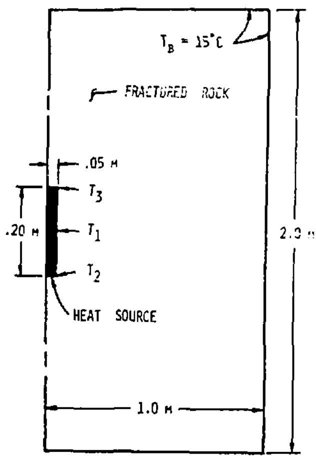
Figure 3. Normal Distribution Function Used to Model Joint Orientation Field Data.

$$\beta = \alpha + \phi \cos (p\pi) \quad . \tag{5}$$

The angle given by Equation (5) along with the specified values of S_g and S_j are then sufficient to evaluate Equations (1)-(3) for the effective conductivities and capacitance at each element integration point.

4. NUMERICAL EXAMPLE

In order to demonstrate the utility of the previously proposed procedure, a simple two-dimensional conduction problem was considered. Shown in Figure 4 is a schematic of a volumetrically heated source imbedded in a planar block of jointed material. The assumed thermal properties of the intact material are typical of welded tuff; the joint material is assumed to be air. The planar block was held at a constant outside temperature of 15°C. The heat source had a constant power output.



Rock Properties (Tuff)

ρ = 2280.0 kg/m^3

C_p = 1600.0 Ws/kg-C

K = 2.4 W/m-C

Joint Properties (Air)

ρ = 1.0 kg/m^3

C_p = 1.0 Ws/kg-C

K = 0.024 W/m-C

Figure 4. Schematic of Heat Conduction Problem in a Fractured Block of Rock.

Figures 5 and 6 show sample finite element meshes used in the computations where the angular orientation of the joints at each integration point are shown by the short lines within each element. The mean joint set angles were 0° and 90° for Figures 5 and 6, respectively; the dispersivity for both cases was $k = 10$.

To illustrate the effects of various parameters in the model, a series of eight cases were analyzed. These runs are summarized in Table I. Case 1 is a base case that was executed without the fracture model and used only intact rock properties. For cases 2-4, the mean angle of the joint set was varied between 0° and 90° while holding the dispersivi (k = 10) and joint aperture ($S_g = 0.001$ m) const . Cases 5-7 were similar to 2-4 with the joint aperture increased to $S_g = 0.005$ m. Finally, case 8 demonstrated the effect of decreasing the variation in the local joint angle by increasing the dispersion coefficient to $k = 1000$.

In order to roughly quantify the effects of the above parameter variations, Table I also lists the temperatures computed at several points along the heat source/rock interface. The location of these points is shown in Figure 4. As expected, the increase in thermal resistance due to the presence of even small fractures is seen to significantly increase the maximum temperature (T_1) found in the problem. Also, for a given problem geometry, the orientation of the overall joint set (α) can have a pronounced effect on the temperature maximums. Comparing cases 7 and 8 indicates that the primary effect of the dispersion coefficient is to decrease the asymmetry of the solution as the variation in the local joint angles are decreased.

Finally, to allow a more qualitative picture of these solutions, a series of isotherm plots are reproduced in Figure 7 for cases 1, 5, 6, and 7. The isotherms for each plot represent the same temperature levels, thus permitting an immediate comparison. The figures clearly show the changes in the temperature field due to the presence of a fracture distribution (compare case 1 with 5, 6, and 7). Also noticeable is the shifting of the maximum temperature gradients in response to changes in the mean joint angle.

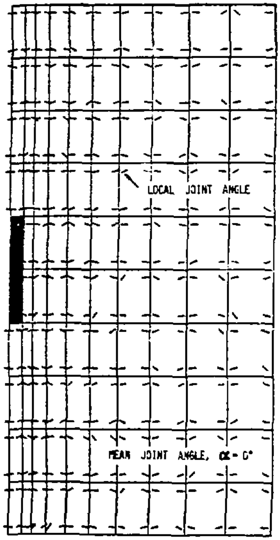
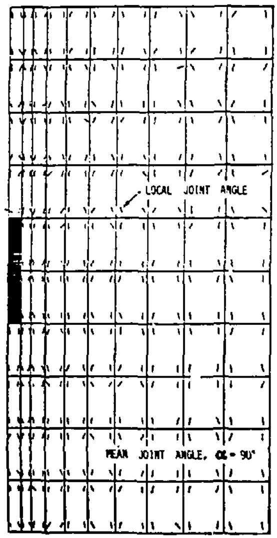


Figure 5.
Finite Element Mesh
for Fractured Block,
α = 0°, k = 10.

Figure 6. Finite Element
Mesh for Fractured Block,
α = 90°, k = 10.



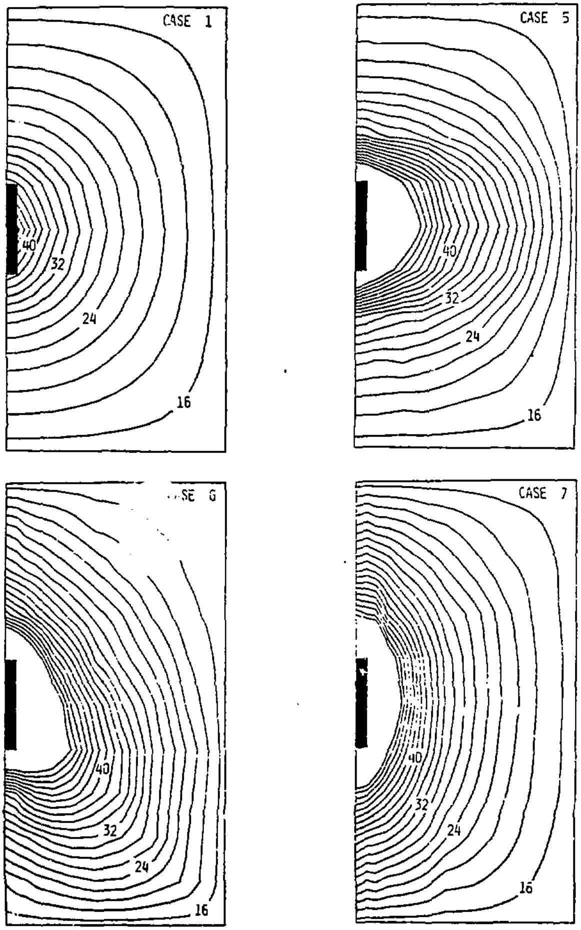


Figure 7. Typical Isotherm Plots for Fractured Block Example Problem.

Table I. Summary of Example Problems.

Case	α	k	S_g	T_1	T_2	T_3
1	-	-	-	46.67	38.41	38.41
2	0°	10.	0.001	55.45	46.37	46.38
3	45°	10.	0.001	59.20	47.56	48.22
4	90°	10.	0.001	60.52	48.88	48.06
5	0°	10.	0.005	73.01	60.43	62.75
6	45°	10.	0.005	81.22	65.42	68.79
7	90°	10.	0.005	88.25	70.40	73.85
8	90°	1000.	0.005	89.96	71.67	71.60

Note: For cases 2-8, the joint spacing S_j was held constant at S_j = 0.10 m.

5. CONCLUDING REMARKS

The present work has attempted to briefly outline a fairly general procedure for simulating heat conduction in a fractured medium. The proposed method is capable of treating many of the important statistically distributed features of a jointed rock mass such as joint orientation, aperture, spacing, and length. Implementation of the procedure into existing finite element software is straightforward and has proved to have no significant effect on computational efficiency. There remain, however, a number of topics for future work in this area. The question of interpretation and verification of solutions from a model with statistically based input has not been addressed. Also, the extension of this method of computation to other important physical processes in geomechanics such as fluid flow and convective heat transfer has yet to be considered.

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