Modular Coils: A Promising Toroidal-Reactor-Coil System

by

T. K. Chu, H. P. Furth, J. L. Johnson,*

C. Ludescher, and K. E. Weimer

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

Abstract

The concept of modular coils originated from a need to find reactor-relevant stellarator windings, but its usefulness can be extended to provide an externally applied, additional rotational transform in tokamaks. Considerations of (1) basic principles of modular coils, (2) types of coils, (3) types of configurations (general, helically symmetric, helically asymmetric, with magnetic well, with magnetic hill), (4) types of rotational transform profile, and (5) structure and origin of ripples are given. These results show that modular coils can offer a wide range of vacuum magnetic field configurations, some of which cannot be obtained with the classical stellarator or torsatron coil configuration.

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I. Introduction

The rotational transform in classical stellarators is achieved by a set of 2ℓ toroidally continuous helical windings. The support structure of the windings is usually massive because of the $\vec{J} \times \vec{B}$ forces on the coil. This structure occupies a large magnetic volume which might otherwise be used for plasma confinement or other purposes. The winding configuration also does not lend itself to easy disassembly for maintenance purposes. In torsatrons one set of ℓ helical windings provides both the toroidal and the poloidal field, and, ordinarily, one set of simple toroidal windings provides the vertical field for maintaining plasma equilibrium. The force problem and the complication of two sets of interwoven windings are lessened. The basic nature of the toroidally continuous helical configuration of both the coils and the supporting structure remains.

The term modular stellarator refers to a generalized stellarator configuration of nested magnetic surfaces with multiple helicity achieved by a system of discrete coils which provides both toroidal and poloidal fields. Since there is no net toroidal current, no vertical-field coils are needed. A modular stellarator, therefore, has no toroidally continuous windings. The confinement coil system is modular. Furthermore, there is no inwardly directed force. Thus the support structure can be located outside and on the sides of the coils.

Modular stellarators can be classified into two types: the $\ell = 1$ configuration and the $\ell \geq 2$ configuration. Here ℓ is the poloidal field-period number. The $\ell = 1$ configuration requires only circular, planar toroidal field coils; the rotational transform is obtained by deforming the geometric axis of the coil system from a two-dimensional ring into a threedimensional curve. For $m = 2$, where m is the toroidal field-period number, the configuration is a figure 8 stellarator. For $m > 2$, the geometric axis is a toroidal helix.

For $\ell \geq 2$ configurations, the planar toroidal field coil is deformed (Fig. 1), and successive coils are rotated in the poloidal direction by a given angle as determined from the m number. In terms of the usual stellarator configuration, the deformation of the individual coil is essentially, but not exactly, equivalent to connecting the segments of the helical windings to their neighboring toroidal field coils. (This is similar to the baseball coil which connects the confinement coils and the Ioffe bars in mirror machines.) The relative poloidal rotation between successive coils carries the helical lines around the torus.

This paper describes the configuration of the $\ell \geq 2$ system. The $\ell = 1$ system involves only planar coils and is relatively simple. It should be noted that an $\ell = 2$ configuration can also be obtained by using planar, elliptically shaped coils.[1] Successive coils are rotated and each coil is tilted from the plane perpendicular to the geometric axis. Similarly an $\ell = 3$ configuration can be obtained by using planar triangular coils. The rotational transform obtained with such coil systems is usually small. The modular-coil concept based on deforming the planar coil enhances greatly the rotational transform.

The basic concept of modular coils was first proposed by Rehker and Wobig.[2] An analytic-numerical analysis of a straight system was given by Onasa and Miyamoto.[3] Recent work[4-7] shows that substantial strengthening of the rotational transform can be obtained by improving the helical symmetry in the winding law, and that rotational transforms of confinement interest can be obtained without greatly deforming the toroidal field coils from being planar. The divertor feature of the coil system has also been examined.[8]

Section 2 describes the basic principle of the Rehker-Wobig coil where the winding contains only the fundamental component of the poloidal field-period number ℓ. Section 3 describes the effect of harmonics in the winding law by (a) an analytic-numerical analysis and (b) a geometric analysis. Section 4 discusses types of coils, and Sec. 5 presents the different kinds of magnetic configurations that are achievable in modular stellarators. Types of profiles of rotational transforms are presented in Sec. 6. The structure of ripples and their origin are discussed in Sec. 7.

2. Basic Principle of Rehker-Wobig Coils

In classical stellarators and torsatrons, the windings providing the rotational transform are toroidal helices. These helical lines are shown in Fig. 2 in the unfolded θ-z plane, where z is the distance along the toroidal direction. In a simple modular stellarator, the N windings are sinusoidal curves in the θ-z plane, and they are discrete in z. A good qualitative picture of the nature of these stellarators can be obtained by considering this system and decomposing the currents into superpositions of helical current lines that provide the magnetic field configuration. The current in each coil can be resolved locally into θ and z components. The θ-component of the current provides a toroidal field, and the z-component provides a poloidal field. Since the toroidal field is nearly uniform and is larger than the poloidal field, a qualitative understanding of the effect of changing the shape of the coils can be obtained by considering only the toroidal part of the current in the coils. Figure 3 shows this component of the current I (along z) in each modular coil. The distributed z-components produce an infinite number of helices criss-crossing the θ-z plane. These components can be approximated by equivalent currents at the nodal points of the coil. They can be decomposed along the two directions joining the nearest nodal points (with current flowing in the same direction) in neighboring coils. The decomposed currents make angles α and β with the z-axis, and the (imaginary) lines joining these currents describe the two dominating helical lines. It should be pointed out that, except for certain geometric configurations, the helical lines are not lines joining the sloping parts of the sinusoidal curves in the θ-z plane.

The smaller angle α corresponds to the pitch angle of the helical windings in a classical stellarator, and

$$p_\alpha \equiv \tan \alpha = \frac{a}{R}\frac{m}{\ell} \quad . \tag{1}$$

The larger angle β corresponds to a second stellarator helical winding with

$$p_\beta \equiv \tan \beta = \frac{a}{R}\frac{m}{\ell}\left(\frac{N}{m} - 1\right) = \frac{a}{R\ell}(N - m) \quad . \tag{2}$$

Equations (1) and (2) describe the two dominant helices among the complex array of modes in modular stellarators. The characteristics of the modular stellarators can be described approximately by these two stellarator helical components.

The two sets of helical lines give opposing transforms. (The α-angle lines will be designated as the helical lines and the β-angle lines as the antihelical lines.) The helical lines depend only on the geometric configuration of the torus and not on the number of modular coils. The antihelical lines, in addition, depend on the number of modular coils. Both pitch angles are independent of the amplitude of the deformation of the coil. The pitch angle of the antihelical lines corresponds to a set of windings whose number of toroidal field periods is (N/m) - 1 times that of the helical lines. Because of this higher toroidal field-period number, the contribution to the rotational transform from the antihelical lines is relatively small in the interior but becomes significant near the edge. Also, with fewer modular coils, the opposing transform contributed by the antihelical lines is stronger. The lowest integer N/m value which gives non-zero rotational transform is 3. For N/m = 2, $\beta = \alpha$ and there is no transform for the purely sinusoidal coil.

The antihelical lines "unwind" the field lines and give an opposing transform especially near the edge. This has two consequences. First, the rotational transform of a modular stellarator is generally lower than that achieved with the helical lines (α-angle lines) alone. Second, the separatrix is held at a larger radial position than that achieved with the helical lines alone. A modular stellarator thus has lower edge transform and larger usable magnetic volume than a classical stellarator.

In summary, a simple modular stellarator can be regarded as being nearly equivalent to (1) a set of planar toroidal field coils and (2) two sets of stellarator helical windings giving opposing transforms. This decomposition is by no means unique. For instance, for special choices of the geometric parameters, a modular stellarator can also be interpreted as being essentially four sets of torsatron windings, two of the sets giving unequal positive transform, and the other two unequal negative transform. A change of the winding law from being simply sinusoidal modifies these considerations by changing the harmonic content. Only when certain harmonics are present can a modular stellarator produce fields similar to a simple stellarator[9] or fields roughly equivalent to two opposing torsatron windings with different pitch angles.[7,10]

3. Harmonic Content

In this section, we discuss the effect of changing the harmonic content of the winding so as to (1) increase the rotational transform and to (2) obtain field configurations with different helical symmetry and rotational transform profiles. This additional degree of freedom enables the modular stellarator to achieve many different magnetic field configurations, some of which cannot be achieved by classical stellarator or torsatron windings. We will first give some analytic-numerical results for straight modular stellarators. This will be followed by a geometric analysis which provides an intuitive insight into the basic configurations.

3.1 Analytic-Numerical Analysis

Analytic expressions based on Fourier expansion of the currents in the modular coils can be derived for the currents, magnetic scalar potentials, magnetic surfaces, and the rotational transform for a straight system. The coils have a simple periodic deformation,

$$Z = Z_j + \sum_n d_n \sin n\,(\ell\theta - \theta_j) \quad . \tag{3}$$

Here $Z_j = Lj/N$ is the position of the j'th coil, L is the length of the system, N is the number of coils, $\theta_j = 2\pi mj/N$ is the phase angle of the j'th coil, and m is the toroidal mode number. The present analysis is an extension of that of Ohasa and Miyamoto[3] who treated only a simple warping (n = 1) of the coils. A code has been developed which gives the surfaces and transforms when the first three terms of the series are present in any combination. By judicious choice of the d_n's, it is possible to represent quite general winding laws and produce configurations with significant increases in transform over that of the one-term case with $n = 1$. Parameter studies illustrating the effects of the aspect ratio, the number of coils, the toroidal mode number, higher Fourier harmonics, and resonant interactions on the surfaces and transforms can be performed readily.

Excellent agreement with the numerical work[4] for toroidal geometry (based on integration of the vacuum magnetic field lines in a torus) has been obtained for large aspect ratio, $R/a = 9$, $\ell = 2$, $m \geq 4$, $N = 60$, $d_1/a = 0.1$, $d_2/a = -0.025$, and $d_3/a = 0$. In Fig. 4, ι on axis is plotted as a function of the toroidal mode number m. The circles are obtained from field-line integration, and the solid curve is from the analytic-numerical analysis. The transform increases strongly with decreasing m. Profiles, in general, show the transform to be increasing with radius until $r/a \sim 0.6$ where flattening occurs, followed by a decrease due to the increasingly important negative contributions of certain of the higher modes in the Fourier series representation. Transforms of $\iota \sim 1$ can be obtained for reasonable parameters: $\ell = 2$, $m = 6$, $N = 48$, $R/a = 6.667$, $d_1/a = 0.3354$, $d_2/a = -0.05$, and $d_3/a = 0.0354$ (Fig. 5). The agreement with the field line computations for the torus is less good for these parameters since the stellarator expansion used to obtain the expressions for the surfaces and transform is not well satisfied.

The understanding of the importance of the higher Fourier modes is a significant result of this analytic work. What could be even more valuable is the opportunity provided by this work to utilize the magnetic scalar potentials for the fields in equilibrium and stability calculations using the stellarator model[11] to investigate the effect of plasma in these toroidal systems.

3.2 Geometric Analysis

The analytic-numerical method for a straight system, outlined in Sec. 3.1, provides a useful modeling and check of the numerical results obtained for toroidal configurations. A geometric analysis can also be made to gain an intuitive understanding of the effect of higher harmonics on the magnetic field configuration. This analysis employs a harmonic multiplication factor s so that, within the segment of the higher harmonic applied at the nodal point, the winding law is described by

$$Z = Z_j + d \sin s\,(\ell\theta - \theta_j) \quad . \tag{4}$$

The rotational transform and its profile, and the location of the separatrix are uniquely related to s. Details of the analysis are given in Ref. [12]. An exact analysis of the field configuration similar to that described in Sec. 3.1 for different s values applied to different segments of the same coil is difficult. However, a reasonable approximation to the coils can be found by properly choosing the parameters d_n in the representation of Eq. (3).

4. Other Types of Coil

The deformation in modular stellarator coils does not have to be restricted to distortions in the θ-z plane; deformation can also be made in the minor-radius direction. For an $\ell = 2$ system, elliptic coils can be used, and for an $\ell = 3$ system, triangular coils can be used. Figure 6 shows the magnetic surfaces of an $\ell = 2$ configuration with elliptical coils. The advantages of using noncircular coils to conform to the shape of magnetic surfaces are (1) a more uniform separation distance between the outermost magnetic surface and the coil, thus allowing more efficient use of available magnetic volume and (2) a stronger rotational transform.

5. Types of Configuration

Since the toroidal current decomposition along the helical line and the antihelical line can be varied by changing the harmonic content of the winding law at and near the nodal points, configurations with varying degrees of helical symmetry and different profile shapes of rotational transform can be obtained. Further, by varying the geometry of the helical and the antihelical lines, magnetic surfaces can be made to contain a multiplicity of ℓ-modes, thus producing a vacuum magnetic field which has a magnetic well (or hill). This section discusses types of magnetic field configurations that can be achieved with modular coils.

5.1 General Configuration

A simple modular stellarator has both helical and antihelical lines. For an $\ell = 2$ configuration, the rotational transform is therefore high near the center and decreases radially outward. In an $\ell = 3$ modular stellarator the contribution to the rotational transform from the helical and antihelical lines are similar, but the rotational transform profile, like that in a classical stellarator, is zero at the center and increases outward.

5.2 Flexibility of Design

A striking feature of these modular coils is that minor changes in shape allow significant modification in the configuration. The angles α and β are affected by geometry and by the positioning of the coils. This can be seen in Fig. 4, where the rotational transform decreases with increasing rotation of the consecutive coils, or increasing m. Similar improvement of the rotational transform with increasing number of coils is shown in Fig. 7. This figure provides a good demonstration that saturation sets in as N gets large; the configuration approaches that of a classical stellarator. It is possible to obtain almost any desired set of values of current along α and β by varying the harmonic content in Eq. (3). This enables one to obtain quite different types of configurations. The limiting cases are the classical stellarator, which has a single helicity and thus possesses highest helical symmetry, and the case with N/m = 3, which has the highest contribution from the antihelical lines.

5.3 Magnetic Well Formation

In modular stellarators, as in classical stellarators and torsatrons, the field configuration is determined by the geometry of the helical lines. By varying the geometry of the α-angle lines, a non-zero local vertical field can be produced. This vertical field causes differential shifts of the magnetic surfaces from (or towards) the major axis thus producing a magnetic well (or hill). A coil deformation which accomplishes this purpose is a deformation in the θ-direction of the θ-z plane so that the α-angle lines are determined from

$$\phi = \frac{\ell}{m} (\theta + \gamma \sin \theta) \tag{5}$$

where ϕ is the toroidal angle. The α-angle lines are not straight in the unfolded θ-z plane, as can be seen in Fig. 8 where the coil windings are shown for a case with $\gamma = 0.6$. Within a toroidal field period, each coil is different. It is possible, however, to use only two types of coils to approximate the specific winding geometry of Eq. (5). In this case the magnetic surfaces are shifted to form a magnetic well as can be seen in Fig. 9. We note that winding laws similar to Eq. (5) have been used in torsatrons. Rau[13] obtained magnetic well and hill formation in torsatrons by using positive and negative γ's, respectively. For negative γ's, a torsatron configuration with no additional vertical field windings for maintaining plasma equilibrium can also be obtained.[14-16]

6. Types of Rotational Transform Profiles

By varying the current components along the α and β angle lines, profiles of rotational transform can be varied from positive $d\iota/dr$ to negative $d\iota/dr$. The profile with negative $d\iota/dr$ cannot be achieved by simple stellarator or torsatron windings. Configurations with this type of profile have a larger usable magnetic volume. If an ohmic-heating current is used, the resultant transform profile will retain the feature of monotonic decrease; no bumpy profile will result.

7. Ripples

Modular stellarators have toroidal, helical and modular ripples. The origins of the modular ripples are the discreteness of the coils and the finite poloidal rotation in successive, discrete coils. Because a modular stellarator inherently contains multiple helicity, its helical ripples have multiple-helicity structure. For the simple Rehker-Wobig coil, the helical ripples consist of those due to the helical lines (the α-angle lines) and the antihelical lines (the β-angle lines). The ripple amplitude for each component of helicity increases radially outward. The amplitude of modular ripples can be reduced by improved winding methods.[5]

8. Summary

Considerable progress has been made in the design of modular stellarator coils since the concept was first proposed. A good qualitative understanding of the fields generated by these coils can be obtained by decomposing the toroidal component of the current into components along directions joining the nodal points (with current flowing in the same direction) of neighboring coils. Improved coil shaping can be designed from geometric studies of these current decompositions as well as from quasi-analytic models obtained by Fourier analyzing the currents into helical components. It is thus possible to design modular systems with large rotational transform. Systems with either radially increasing or decreasing transform as well as with minimum average fields can be obtained.

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Figure Captions

Fig. 1 (a) An $\ell = 2$ modular coil, (b) an $\ell = 3$ modular coil, and (c) coil arrangement in an $\ell = 2$ modular stellarator.

Fig. 2 Helical current lines in the unfolded θ-z plane in (a) a classical stellarator, (b) a torsatron. The current lines in a modular stellarator are shown in (c) as solid lines and the dominant helical lines are shown as dashed lines. The classical stellarator configuration of (a) has additional TF coil windings, and the torsatron configuration of (b) has vertical field windings. These windings are not shown in the figure.

Fig. 3 Formation of the helical and the antihelical lines. The coil current is decomposed in the θ-direction (not shown) and z-direction. The distributed z-components of the current are represented by their values at the nodal point, and these are resolved along lines joining nodal points (with current flowing in the same direction) in neighboring coils.

Fig. 4 Analytic-numerical results of on-axis rotational transform for a straight system (curve), and numerical results for a toroidal system (discrete points) when the toroidal field-period number m is varied.

Fig. 5 Analytic-numerical results of rotational transform profiles for a straight $\ell = 2$ system for different combinations of the d_n's.

Fig. 6 Magnetic surfaces of an elliptic-coil modular stellarator with 2 to 1 ellipticity.

Fig. 7 Analytic-numerical results of rotational transform for a straight $\ell = 2$ system for different N.

Fig. 8 Modular coil windings (solid lines) whose helical pitch angle corresponds to Eq. (5). The dash-lines are the helical lines, Eq. (5). The dotted lines are windings having no higher harmonics [s = 1 in Eq. (4)].

Fig. 9 Magnetic surfaces for the winding law of Fig. 8, showing the outward shift of the axis with respect to the shift of the outer surfaces and thus a magnetic well. $N = 36$, $R/a = 10/2.25$, $d/a = 0.3$, $\ell = 2$, $m = 6$, $\gamma = 0.6$, $\iota(0) \simeq 0.35$ and $\iota(a) = 0.15$.

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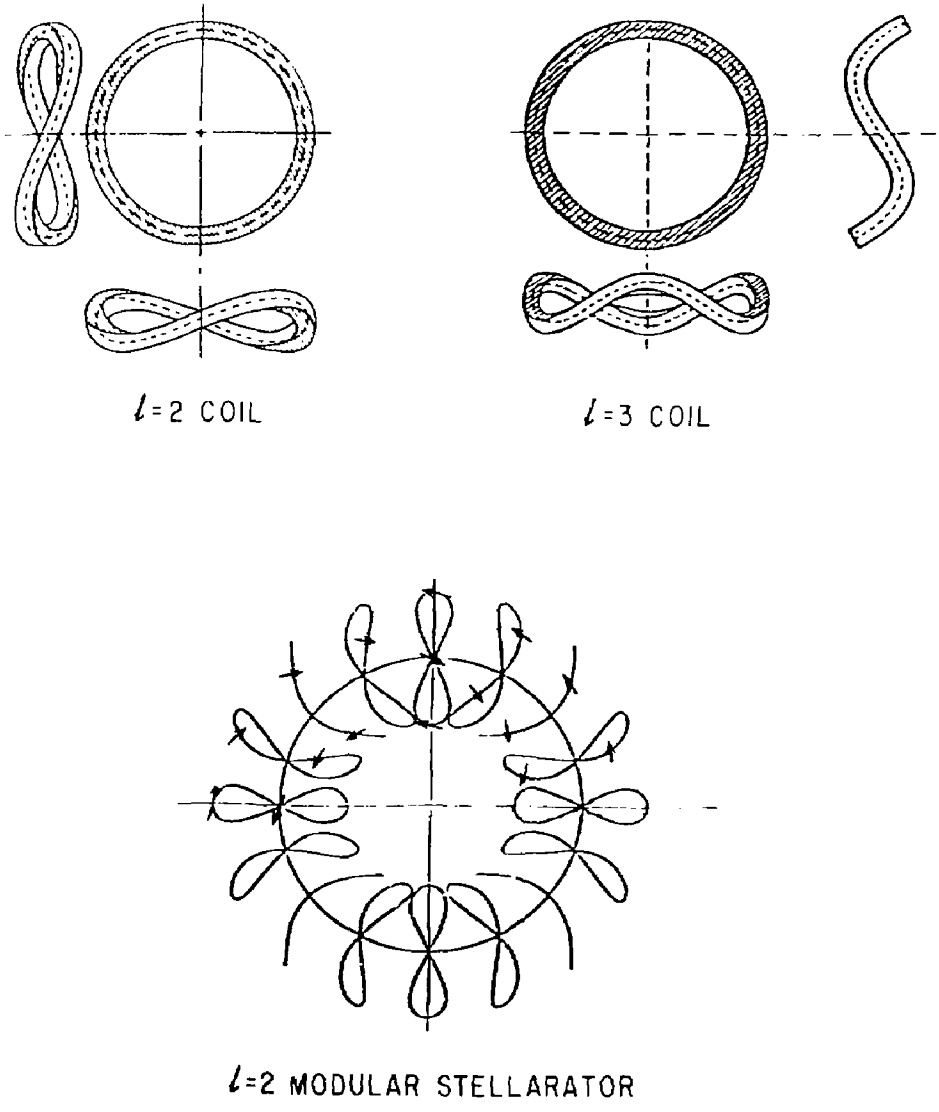

Fig. 1

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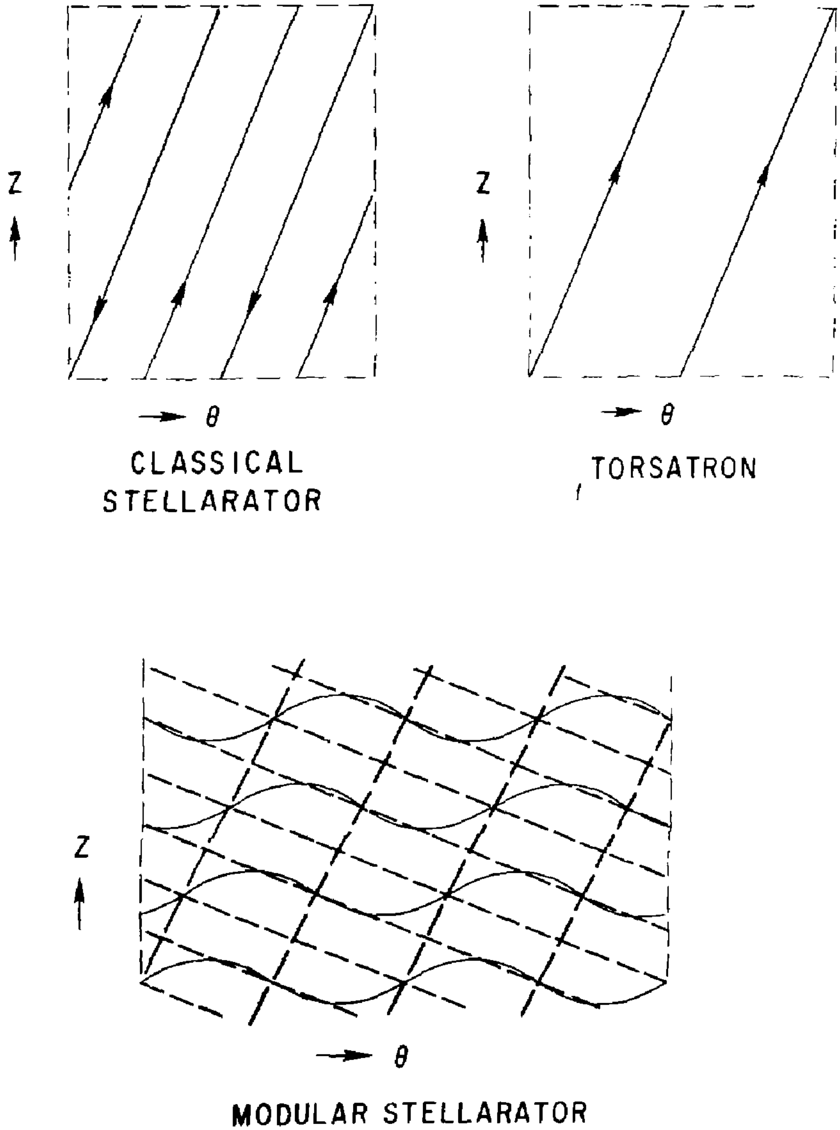

Fig. 2

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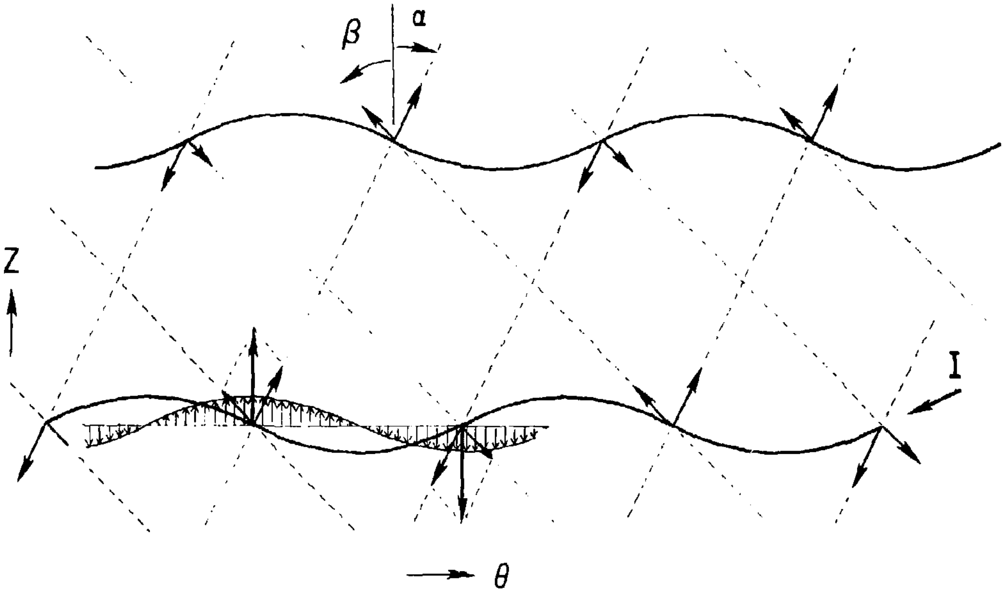

Fig. 3

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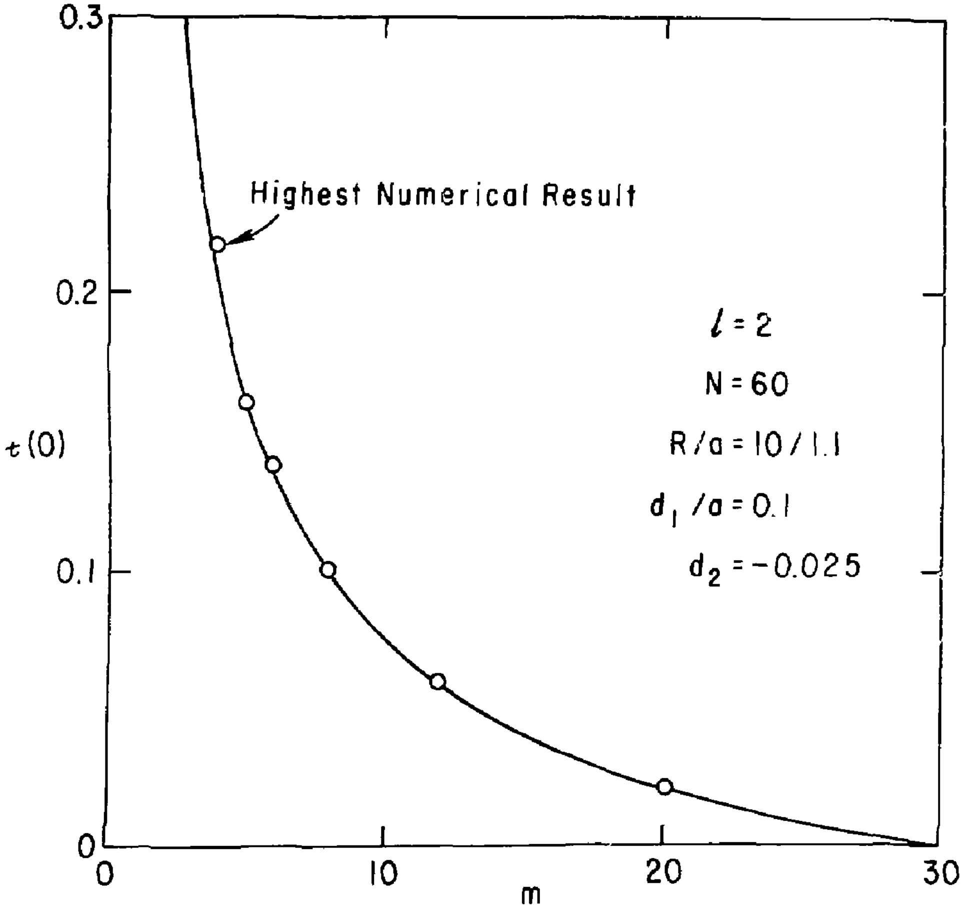
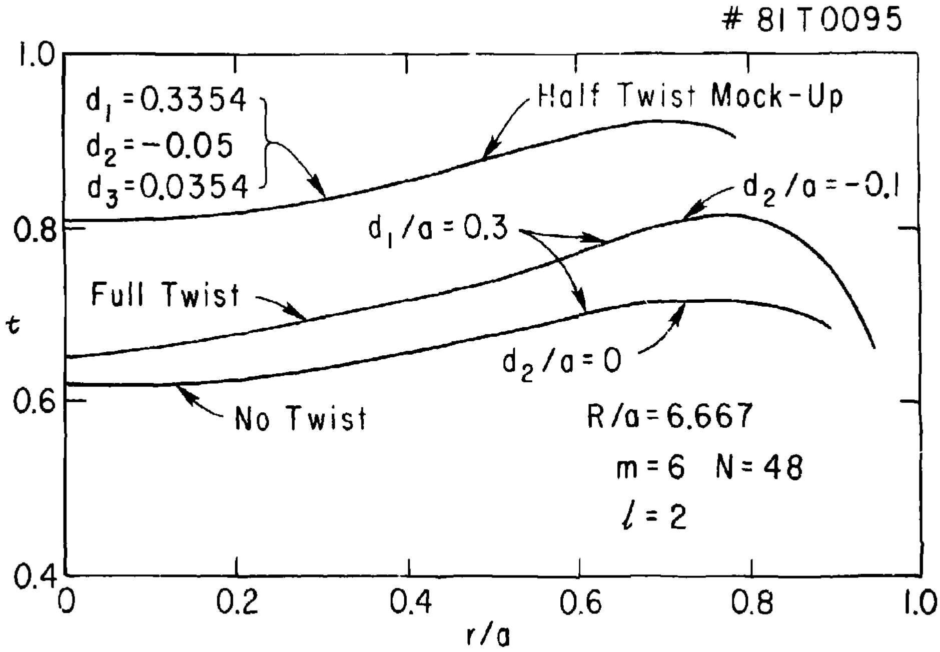


Fig. 4



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No Twist
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r/a

Fig. 5

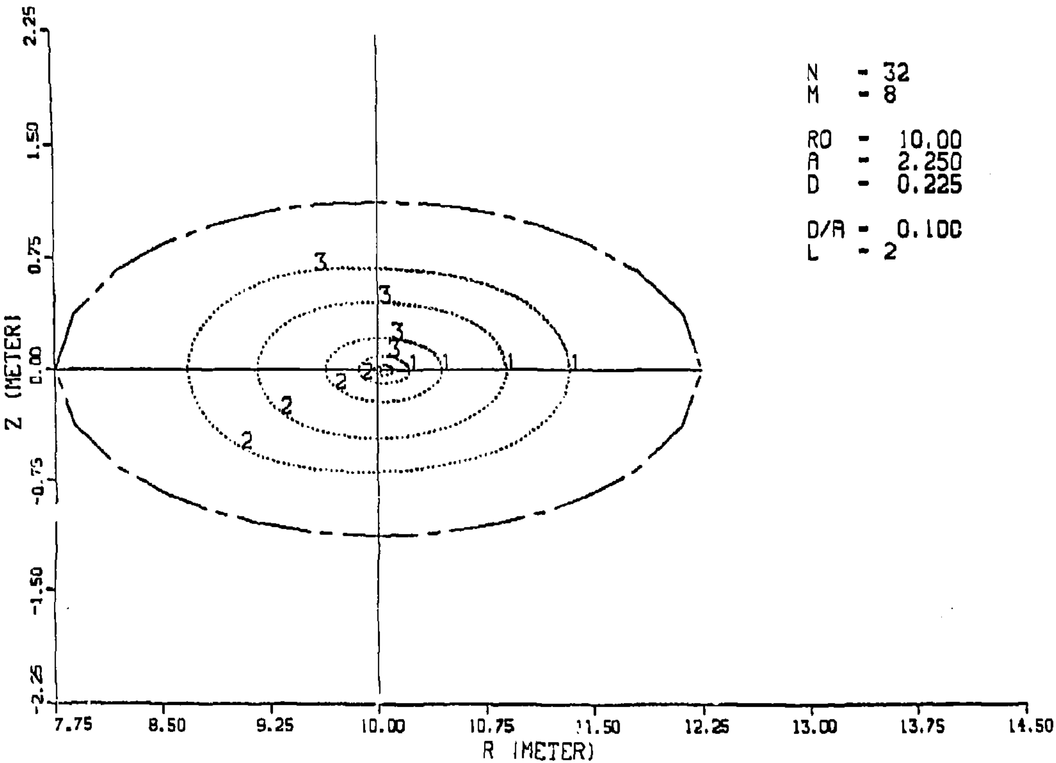
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Fig. 6

81T0094

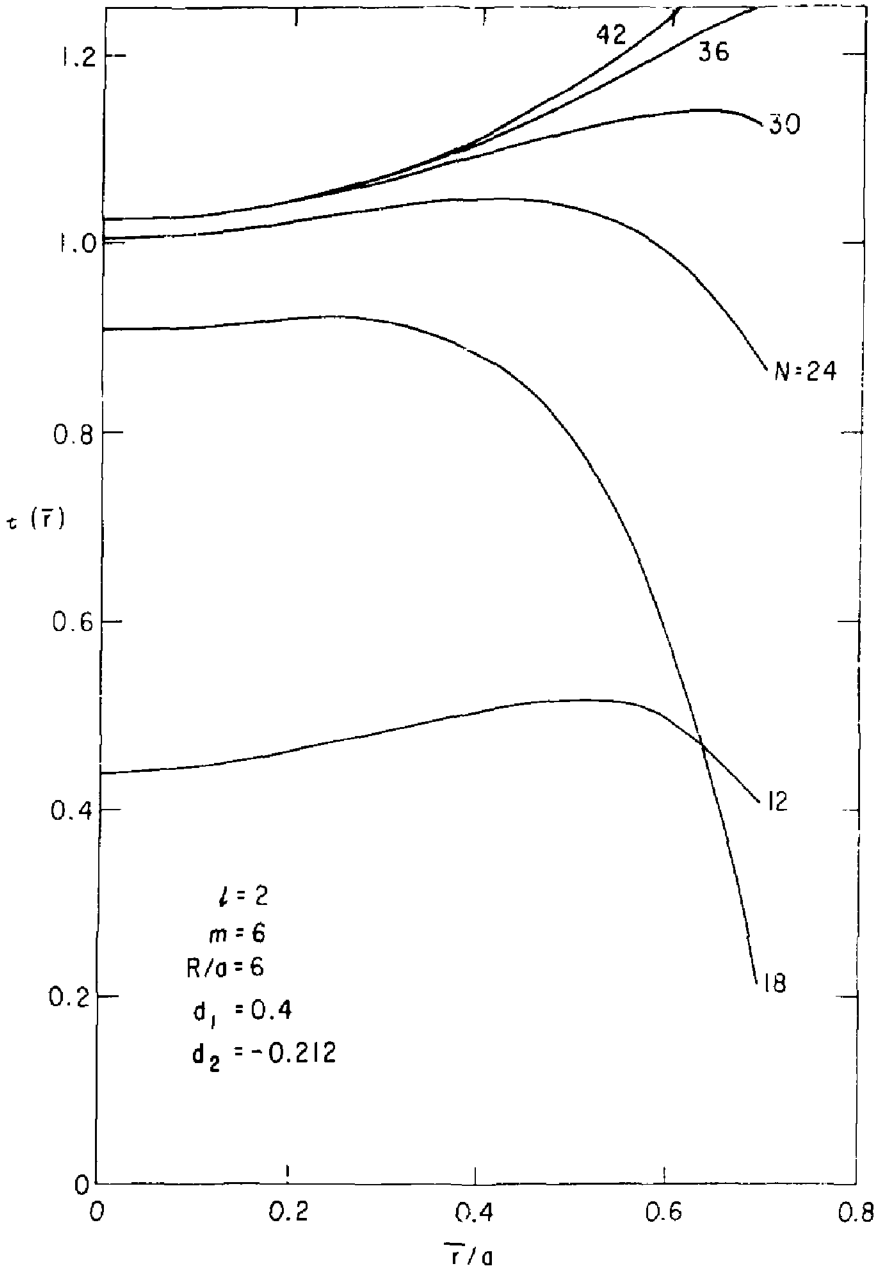

Fig. 7

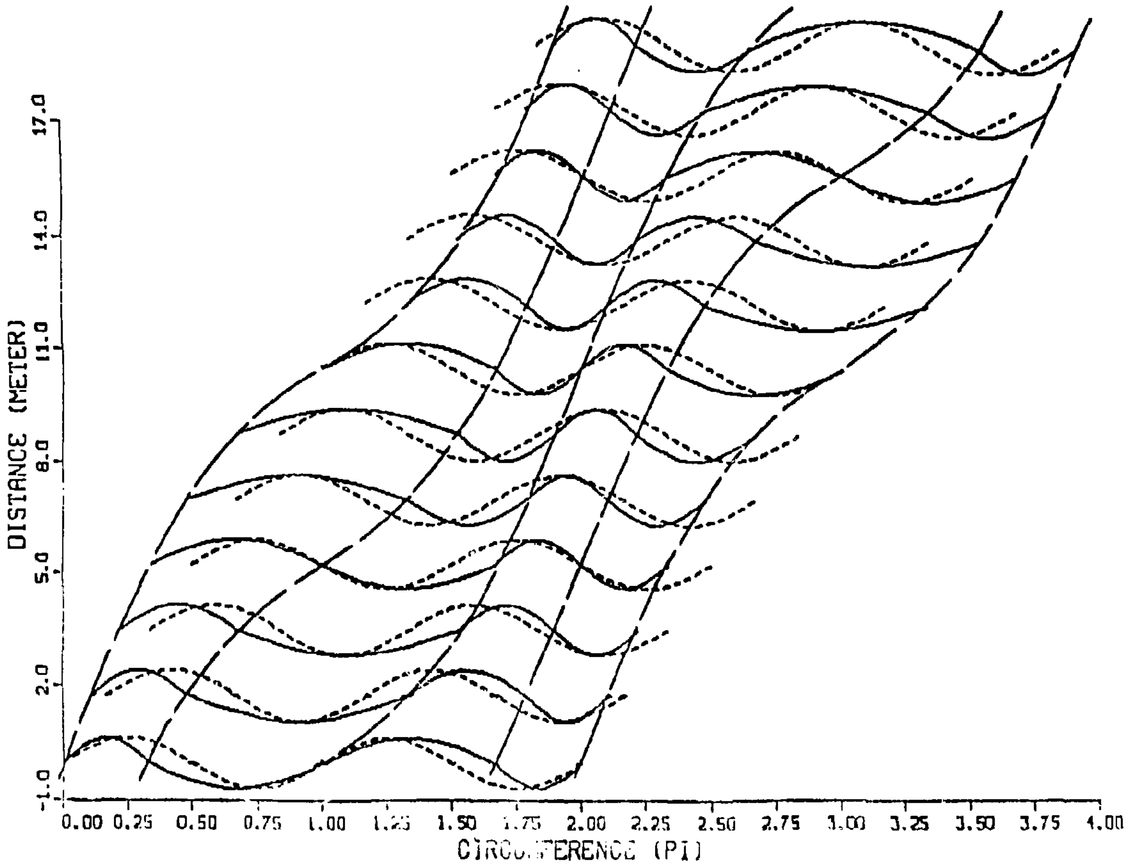
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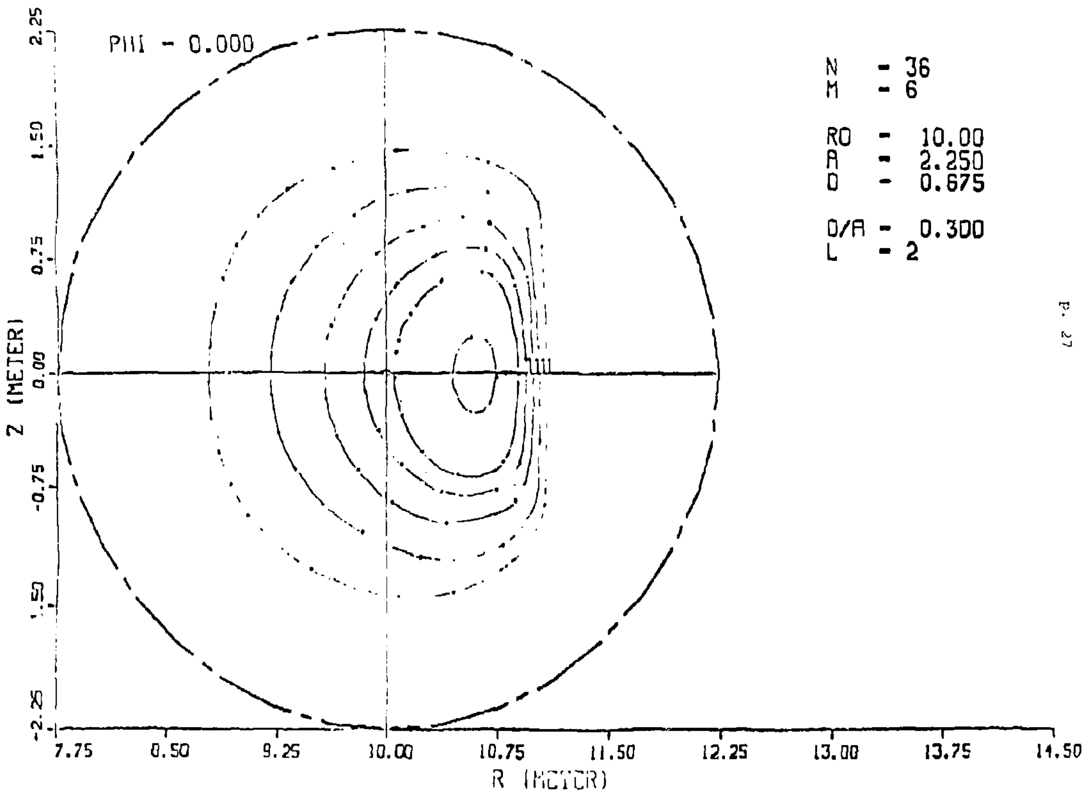
Fig. 3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R (METER)

Fig. 9