INTRODUCTION

These notes summarize the main results on deep inelastic lepton nucleon scattering, a field which has greatly influenced the progress in physics over a period of the last 12 years. In parallel to the original electron scattering experiments at SLAC the quark parton model was developed. Neutrino experiments confirmed basic parton model predictions and greatly extended our knowledge of the structure of hadronic matter. Later muon and neutrino experiments established vital predictions of the new theory of the strong interactions, the quantum chromodynamics.

The notes are divided in three parts. After a brief introduction of the kinematics in part 1 I will summarize all results from electron, muon, and neutrino experiments which have to do with our belief that the quark parton model has some relation to truth. Part 2 discusses the current theoretical interest in "scaling violations" of the structure functions and the experimental situation. Part 3 contains some aspects of the hadronic final state produced in the deep inelastic collision.

1. TESTS OF THE QUARK PARTON MODEL

1.1 Fundamentals

The processes we are concerned with in these lectures are represented by the Feynman diagrams Fig. 1: a) for e,μ scattering, b) for ν scattering with weak charge changing current, c) for ν scattering with weak neutral current.

k and k' are the four momenta of the initial and final lepton, E and E' their laboratory energies, and θ the laboratory scattering angle. q and p are the four momenta carried by the exchanged virtual boson and the target nucleon. Presently most knowledge about the structure of the nucleon comes from the study of processes a) and b)
which are therefore the main subject of these lectures.

A detailed discussion of the kinematics of lepton scattering can be found in Refs. 1, 2, 3, 4. For reasons of consistency I will summarize the main features for the case that only the scattered lepton is detected. From the four momenta \( p \) and \( q \) one can form two Lorentz scalar variables

\[
Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2}
\]

\[
\nu = \frac{p \cdot q}{M_{\text{proton}}} = E - E'.
\]

The mass of the hadronic final state is given by

\[
W^2 = p_F^2 = m^2 - Q^2 + 2M \nu = M^2 + 2MK.
\]

We shall also use the ratios

\[
x = \frac{Q^2}{2M \nu},
\]

\[
y = \frac{\nu}{E}.
\]

For evaluating the cross sections we are interested in the square of the matrix elements corresponding to the diagrams of Fig. 1 summed over all undetected hadronic final states. These are of the form:

a) For \( e_\nu \) scattering

\[
|M|^2 = L_{\mu \nu} \frac{e_\nu^4}{Q^4} W^{\mu \nu}
\]

b) For charged current \( \nu \) scattering

\[
|M|^2 = L_{\mu \nu} \frac{e_\nu^4}{(Q^2 + M_W^2)^2} W^{\mu \nu}
\]

where \( M_W \) is the mass of the exchanged boson.

The first factor \( L_{\mu \nu} \) arises from the coupling to the lepton current and is completely known in all cases.

The second factor contains the contributions from the boson propagator and the couplings to the lepton and hadron currents. For neutrino scattering at available energies \( M_W^2 \gg Q^2 \) and

\[
\frac{e_\nu^4}{(Q^2 + M_W^2)^2} \propto \frac{e_\nu^4}{M_W^4} = \frac{G^2}{2}
\]

where \( G \) is the Fermi constant.

The tensor \( W^{\mu \nu} \) arises from the interaction of the hadron current and is a priori unknown. However, for unpolarized target nucleons, \( W^{\mu \nu} \) can be expressed in the most general case in terms of three independent scalar structure functions \( W_i(Q^2, \nu) \) with \( i = 1, 2, 3 \). The number 3 corresponds to the three helicity states of the exchanged vector boson. In principle the structure functions are different for each scattering process,
but they are connected via the quark parton model as we will discuss in the following.

In the case of the electromagnetic interaction there are only two independent functions \( W_1, W_2 \), since parity conservation relates the amplitudes for the exchange of virtual photons with helicity +1 and -1.

### 1.1.1 Cross Section for Charged Lepton Scattering

Here the tensors \( L_{\mu\nu} \) and \( W^{\mu\nu} \) are of the form

\[
L_{\mu\nu} = \frac{1}{2} \left( k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} \frac{Q^2}{2} \right)
\]

and

\[
W^{\mu\nu} = - (g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{Q^2}) W_1(Q^2, \nu) + (p^\mu + \frac{Q^2}{Q^2} q^\mu) (p^\nu + \frac{Q^2}{Q^2} q^\nu) W_2(Q^2, \nu) / M^2.
\]

To derive this decomposition of \( W^{\mu\nu} \) one needs only Lorentz invariance, current conservation (i.e. \( q_{\mu} W^{\mu\nu} = W^{\mu\nu} q_{\nu} = 0 \)) and parity conservation.

The double differential cross section is

\[
d\sigma_{\nu} = \frac{4\pi^2}{Q^4} \frac{1}{E} L_{\mu\nu} W^{\mu\nu} = \frac{4\pi^2}{Q^4} E^2 \times \frac{2W_1(Q^2, \nu) \sin^2 \frac{Q^2}{2} + W_2(Q^2, \nu) \cos^2 \frac{Q^2}{2}}{E}.
\]

There is also the possibility to consider inelastic electron nucleon scattering as a collision between the exchanged virtual photon and the nucleon. One can then define virtual photon nucleon cross sections for the absorption of transversely (helicity of the photon + 1) and of longitudinally polarized photons, \( \sigma_T(Q^2, \nu) = \frac{1}{2} (\sigma_T + \sigma_L) \) and \( \sigma_L(Q^2, \nu) \). In terms of these cross sections

\[
\frac{d^2\sigma}{dQ^2 d\nu} = \frac{\alpha}{4\pi} \frac{K}{Q^2 E^2} \left( \frac{2}{1+\epsilon} \sigma_T(Q^2, \nu) + \epsilon \sigma_L(Q^2, \nu) \right),
\]

where

\[
\epsilon = (1 + 2(1 + \frac{Q^2}{2}) \tan^2 \frac{\theta}{2})^{-1}
\]

is the virtual photon polarization parameter, \( 0 \leq \epsilon \leq 1 \).

The \( W_{1,2} \) and \( \sigma_{T,L} \) are related by

\[
W_1 = \frac{K}{4\pi^2} \sigma_T, \quad W_2 = \frac{K}{4\pi^2} \frac{Q^2}{Q^2 + \nu^2} (\sigma_T + \sigma_L).
\]

In the limit \( Q^2 \to 0 \), \( \sigma_T \) approaches the total photoabsorption cross section of real photons on nucleons while \( \sigma_L \to 0 \).

It is worthwhile to mention that for the special case of scattering off a pointlike target

\[
\sigma_T = 0 \quad \text{for spin 0,} \quad \sigma_L = 0 \quad \text{for spin } \frac{1}{2}.
\]
1.1.2 Higher Order QED Effects

The aim of the experimentalist measuring inelastic $eN$ or $\mu N$ cross sections is to determine the structure functions. Therefore, he has to separate the cross section due to single photon exchange (Feynman diagram, Fig. 1a) from the measured cross section containing the following additional electromagnetic contributions:

$$2 \text{Re} \left[ \begin{array}{c}
\begin{array}{c}
\text{Fig. 2 Second order contributions to } e,\mu \text{ scattering.}
\end{array}
\end{array}\right]$$

Here only contributions in lowest order of $\alpha$ are shown, in principle one has to add all higher order corrections.

Diagrams 2 (vertex correction) and 6,7 (internal bremsstrahlung) contain the corrections to the lepton current. Their contribution together with the contribution due to diagram 3 (vacuum polarization) is usually included in the "radiative correction" procedure traditionally performed by the experimentalists. Fig. 3 shows the size of the corrections to ep scattering at 20 GeV and up scattering at 200 GeV.

$$\begin{array}{c}
\begin{array}{c}
\text{Fig. 3 Size of radiative corrections.}
\end{array}
\end{array}$$
Plotted are contour lines of constant values of the ratio:

\[ r = \frac{\left( \frac{d^2 \sigma}{dQ^2 dv} \right)_{\text{Born}}}{\left( \frac{d^2 \sigma}{dQ^2 dv} \right)_{\text{measured}}} \]

in the \( Q^2, v \) plane for an incident electron energy of 20 GeV and a muon energy of 200 GeV. Large corrections occur at \( v \) close to \( E \), i.e. at \( y > 0.9 \), a region normally left out when analyzing deep inelastic e and \( \mu \) scattering experiments.

The corrections to the hadron current, diagrams 4 and 8 can be shown to be small and are often neglected.

The contributions due to two photon exchange diagram 5 can be determined by measuring \( e^+/e^- \) (or \( \mu^+/\mu^- \)) cross section ratios

\[ \frac{\sigma^+}{\sigma^-} = 1 + 4 \frac{\text{Re} A_2}{A_1} , \]

where \( A_1 = \) single photon exchange amplitude and \( A_2 = \) two photon exchange amplitude. The single photon exchange amplitude is real, thus the interference term picks out the real part of the two photon exchange amplitude. Fig. 4 shows the ratios of experimental yields measured in SLAC electron experiments\(^6\),\(^7\) and a Fermilab muon experiment\(^8\).

![Fig. 4 Ratios of e^+/e^- yields as a function of Q^2.](image)

The ratios are consistent with unity, and we may conclude that the two photon exchange part of the measured double differential cross section is small.

After having convinced ourselves that we are able to determine the single photon exchange cross section from measurements of deep inelastic eN and \( \mu N \) scattering by performing corrections in the order of a few times 10\%, let us proceed to a discussion of the main features of the structure functions.
1.2 Scaling and the Quark Parton Model

1.2.1 The Gross Features of Deep Inelastic $ep$ Scattering

The first experiments at energies high enough to observe the characteristic features of deep inelastic scattering were performed at the Stanford Linear Accelerator Center (SLAC) at a primary electron energy of typically 20 GeV. Fig. 5 gives an impression of the experimental arrangement.

![Fig. 5 Spectrometers at SLAC.](image)

Electrons scattered off a short hydrogen target are analyzed with an optical spectrometer of high angular and momentum resolution. At fixed spectrometer angle, double differential cross sections for the full range of final electron energies are measured by changing the field of the spectrometer magnets. Several spectrometers optimized for the various ranges of scattering angles enable a complete mapping of the accessible part of the $Q^2,\nu$ plane.

Obviously, at small scattering angles $\theta$ (with $\epsilon$ near 1) one measures essentially $W_2$ while at large $\theta$ ($\epsilon$ small) one measures $W_1$. A collection of data measured at SLAC is presented in Fig. 610).
From the measurements of the double differential cross section with $\epsilon > 1/2$, the values of $W^2$ are determined (assuming $R = R_T = 0$) and plotted versus $x$ for all available values of $Q^2$. In detail the $Q^2$ range of the data depends on $x$, for instance $7 < Q^2 < 14$ GeV$^2$ at $x = 0.7$ and $1 < Q^2 < 5$ GeV$^2$ at $x = 0.2$. The values of $2MW_1$ are obtained from different measurements taken at $\epsilon < 1/2$.

The striking impression from Fig. 6 is that both structure functions are within 30% effects functions of the one variable $x$ only. It was originally predicted by Bjorken\textsuperscript{11)} that in the limit of $\nu$ and $Q^2$ large but $x$ finite the dimensionless structure functions

$$
F_1 = M W_1(Q^2,\nu) \\
F_2 = \nu W_2(Q^2,\nu)
$$

(1.4)

should not depend on any dimensional scale as mass or length and, therefore, be functions of the ratio $x$ only. This property of $F_{1,2}$ is therefore called Bjorken scaling.

One of the important aspects of present experimental research at CERN and Fermilab is the study of small deviations from Bjorken scaling which are of fundamental interest in connection with field theories of the strong interaction. I will discuss the relevant experiments later.
1.2.2 The Quark Parton Model

The prominent features of the data can be easily explained in a model where the proton is composed of pointlike constituents called partons\(^{12\text{),}3\text{)}\). At high energies the interaction time of the virtual photon is assumed to be short enough so that the scattering takes place incoherently off a beam of quasi free partons.

\[ \begin{align*}
\xi p & \rightarrow q \\
p & \rightarrow \text{partons}
\end{align*} \]

Fig. 7 Deep inelastic scattering from partons.

If the initial four momentum of the parton is simply \( \xi p \), i.e. the parton has no transverse momentum with respect to \( p \), one has

\[ (\xi p + q)^2 = m^2 \]

\((m = \text{parton mass})\), and for \( Q^2 \gg M^2, m^2 \)

\[ \xi = \frac{-q^2}{2pq} = \frac{Q^2}{2M} = x \]

The scattering process depends only on \( x \) which is identified as the fraction of the nucleons four momentum carried by the struck parton before the interaction.

If one neglects \( m^2 \) but includes terms with \( M^2 \),

\[ \xi = \frac{-\nu + \sqrt{\nu^2 + Q^2}}{M} = x \frac{2}{1 + \sqrt{1 + 4M^2 x^2/Q^2}}. \]

This is the Nachtmann scaling variable\(^{13\text{)}}\) which includes target mass effects at finite values of \( Q^2 \) and which we will use for later discussion.

A calculation of the contribution to \( F_2 \) from a single parton yields (writing \( x \) instead of \( \xi \))

\[ F_2 = Z_i^2 x \delta(x - \frac{Q^2}{2M}) \] \hspace{1cm} (1.5)

where \( Z_i e \) is the charge of a type \( i \) parton. Eq. (1.5) is correct for any parton spin. The corresponding contribution to \( F_1 \) depends on the spin

\[ F_1 = 0 \quad \text{for spin 0} \]

\[ xF_1 = \frac{1}{2} F_2 \quad \text{for spin 1/2}. \] \hspace{1cm} (1.6)

As we will see, the experiment supports the hypothesis that charged partons have mainly spin \( \frac{1}{2} \).
For a general distribution of partons one obtains

\[ F_2(x) = \sum_i z_i^2 \times f_i(x) \]  

(1.7)

where \( f_i(x) \) is the probability of finding a parton of type \( i \) with momentum fraction \( x \). Energy momentum conservation requires

\[ \int_0^1 dx \sum_i x f_i(x) = 1 - \varepsilon, \]  

(1.8)

where \( \varepsilon \) is the fraction of the total nucleon momentum carried by neutral partons or gluons. Consequently we obtain

\[ \int_0^1 dx F_2(x) \leq \langle z^2 \rangle = \text{mean square charge of the partons.} \]  

(1.9)

Let us now identify the charged partons as quarks with the usual quantum numbers. Assuming further that the nucleon contains just \( u \) and \( d \) quarks and their antiquarks, we find

\[ \langle z^2 \rangle_{\text{Nucleon}} = \frac{5}{18}. \]

Experimentally,\(^{14}\)

\[ \int_{0.02}^{0.8} dx F_2^N(x) = \frac{1}{2} \int_{0.02}^{0.8} dx (F_2^{e+}(x) + F_2^{e-}(x)) = 0.13, \]

which shows that about 50% of the nucleon's momentum is carried by gluons.

1.3 Further Results from eN Experiments

1.3.1 Determination of \( \sigma_L/\sigma_T \)

As we have already noted, a measurement of \( R = \sigma_L/\sigma_T \) provides information about the spin of the charged constituents, for spin \( \frac{1}{2} \) partons \( R = 0 \) while for spin \( 0 \) partons \( R = \varepsilon \). The reason can be understood easily if we consider the virtual photon absorption process in the parton Breit frame. To the extent that the parton mass is neglected, the parton must conserve helicity. A spin \( \frac{1}{2} \) parton can flip its spin only by coupling to a helicity \( 1 \) (transverse) photon:

A helicity \( 0 \) (longitudinal) photon cannot flip the parton spin. A spin \( 0 \) parton, on the other hand, can couple to a helicity \( 0 \) photon.

The argument \( \sigma_L = 0 \) for spin \( \frac{1}{2} \) partons must be modified if we allow an initial parton transverse momentum \( k_T \) due to the Fermi motion of the partons inside the nucleon.\(^3\) In this case the virtual photon is absorbed by a parton with initial four momentum \( (k_0; k_{1X}, k_{1Y}, x_p) \). Inserting this into the equations of Chapter 1.1, and including also
parton mass effects yields

\[ R = 4 \frac{k_T^2 + m^2}{Q^2} \]

I would like to mention at this point that there are further contributions to \( R \) due to the interaction of charged partons and gluons\(^{15}\). Therefore the measurement of small but finite values of \( R \) at finite \( Q^2 \) does not contradict the conjecture that all charged partons carry spin \( 1/2 \).

In principle it may seem simple to measure \( \sigma_L \) and \( \sigma_T \) separately. One just has to change the polarization parameter \( \varepsilon \) at fixed \( Q^2, v \), i.e. measure at different primary beam energies, plot \( \sigma_T + \varepsilon \sigma_S \) versus \( \varepsilon \) and perform a straight line fit. In practice, however, a measurement of \( R \) is hard for the following reasons:

1. To perform an accurate separation, one has to measure at values of \( \varepsilon \) close to 1 and close to 0. This means measurements at small and also at large values of \( y = v/E \), where radiative corrections are large (Fig. 3). Also various background contributions (e.g. electrons from \( \pi^0 \rightarrow e^+e^- \) decay) increase strongly with increasing \( y \).

2. The measurement of absolute cross sections at different \( \varepsilon \) must be performed with different spectrometers (ep scattering at SLAC) or with different geometrical parts of a large solid angle detector (up scattering at Fermilab). To avoid relative normalization errors between the data, an extremely good knowledge of all apparative details is required.

A summary of data collected at SLAC over a number of years is presented in Fig. 8\(^{16}\), where \( \sigma_T + \varepsilon \sigma_S \) for the proton is plotted versus \( \varepsilon \) for various \( Q^2, W^2 \) bins.

Fig. 8 \( \sigma_T + \varepsilon \sigma_L \) versus \( \varepsilon \) for various fixed \( Q^2, W^2 \).
There is definitely some longitudinal cross section, the results of fits are indicated for each bin.

The average values obtained in this and other eN scattering experiments are collected in Table 1.

Table 1 < R > measurements

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reaction</th>
<th>&lt; R &gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLAC</td>
<td>ep</td>
<td>0.21 ± 0.10</td>
</tr>
<tr>
<td>MIT-SLAC</td>
<td>ep</td>
<td>0.14 ± 0.06</td>
</tr>
<tr>
<td>MIT-SLAC</td>
<td>ed</td>
<td>0.14 ± 0.06</td>
</tr>
</tbody>
</table>

The errors include systematic uncertainties.

Results from a muon experiment performed at higher beam energies and from various neutrino experiments confirm that R is small.

In conclusion: allowing for an initial parton transverse momentum square in the order of some tenth GeV^2, all present R measurements are consistent with the assumption that the charged nucleon constituents have spin 1/2.

1.3.2 Neutron versus Proton

Let us now denote the x distribution functions of the different quarks inside the proton by u(x), d(x), s(x), c(x), ..., and the antiquark distributions by \( \bar{\text{u}}(x), \bar{\text{d}}(x), \bar{\text{s}}(x), \bar{\text{c}}(x), \ldots \). The functions u(x) and d(x) can be divided into two parts, a valence quark part \( u^v(x), d^v(x) \) responsible for the proton quantum numbers and a sea quark part \( u_s(x), d_s(x) \) for which we assume

\[
\begin{align*}
\int_0^1 (u - \bar{u}) \, dx &= \int_0^1 u^v \, dx = 2 \\
\int_0^1 (d - \bar{d}) \, dx &= \int_0^1 d^v \, dx = 1 \\
\int_0^1 (s - \bar{s}) \, dx &= 0 \\
\int_0^1 (c - \bar{c}) \, dx &= 0, \text{ etc.}
\end{align*}
\]

(1.10)

In order to reproduce the proton quantum numbers, the following normalization conditions must be satisfied (writing u instead of u(x), etc.).

\[
\begin{align*}
\int_0^1 (u - \bar{u}) \, dx &= \int_0^1 u^v \, dx = 2 \\
\int_0^1 (d - \bar{d}) \, dx &= \int_0^1 d^v \, dx = 1 \\
\int_0^1 (s - \bar{s}) \, dx &= 0 \\
\int_0^1 (c - \bar{c}) \, dx &= 0, \text{ etc.}
\end{align*}
\]

(1.11)

Because of isospin invariance the corresponding distribution functions for the neutron are

\[
\begin{align*}
u^n &= \bar{d}, \quad d^n = u, \quad s^n = s, \ldots
\end{align*}
\]

and we can write
\begin{equation}
\begin{align*}
F_2^{en}(x) &= x \left[ \frac{2}{3} (u + \bar{u}) + \frac{1}{3} (d + \bar{d}) + \frac{1}{3} (s + \bar{s}) + \frac{4}{9} (c + \bar{c}) \right] \\
F_2^{en}(x) &= x \left[ \frac{1}{3} (u + \bar{u}) + \frac{1}{9} (d + \bar{d}) + \frac{1}{9} (s + \bar{s}) + \frac{4}{9} (c + \bar{c}) \right].
\end{align*}
\end{equation}

Therefore,

\[
\frac{F_2^{en}}{F_2^{ep}} \geq \frac{1}{3}
\]

and

\[
F_2^{ep} - F_2^{en} = \frac{x}{3} (u - d) \text{ if } \bar{u} = \bar{d}.
\]

Experimentally \( \frac{F_2^{en}}{F_2^{ep}} \) is particularly well determined, several sources of errors cancel when the ratio of cross sections is evaluated. Fig. 9 shows the results of various experiments\(^{17}\) plotted as a function of \( x \).

![Fig. 9](image.png)

One observes that the quark parton model bound is nearly reached at high \( x \), i.e. at high \( x \) the momentum of the proton seems to be mainly carried by \( u \)-quarks.

From the normalization conditions Eq. (1.11), we reproduce the Gottfried sum rule

\[
\int \frac{dx}{x} (F_2^{ep} - F_2^{en}) = \frac{1}{3},
\]

which is independent on the amount of the nucleon momentum carried by gluons. In an experiment one measures over a limited \( x \)-range and, because of the \( 1/x \) factor, one can obtain only a fraction of the integral

\[
\int_{0.02}^{0.82} \frac{dx}{x} (F_2^{ep} - F_2^{en}) = 0.20 \pm 0.04.
\]
An attempt to extrapolate to $x = 0$ leads to

$$\int_0^1 \frac{dx}{x} (F_2^{ep} - F_2^{en}) = 0.28 \pm ?$$

which can be considered as consistent with the Gottfried sum rule.

1.3.3 Spin dependent effects in ep scattering

The measurement of spin dependent effects in deep inelastic scattering provides further tests of the quark parton model. Because of parity conservation in the electromagnetic interaction such effects can only be observed if both the incident electron and the target nucleon are polarized. Parity violating contributions due to the interference with the weak neutral current are orders of magnitude smaller at the presently available energies.

For a qualitative understanding of the size of the expected effects let us first consider the scattering of a longitudinally polarized electron off a longitudinally polarized parton at $180^\circ$ in the c.m. system. To the extent that masses are neglected at high energies, the helicities of electron and parton are conserved in the process. An electron with helicity $+\frac{1}{2}$ emits a photon with helicity $+1$ which can only couple to a parton with spin $\frac{1}{2}$ antiparallel to the electron spin. For a parton with parallel spin angular momentum conservation forbids a coupling

\[ e \rightarrow \text{Ze} \rightarrow \text{parton} \]

In terms of photon absorption cross sections for a total spin of the photon-parton system of 1/2 respectively 3/2, the situation is described by

$$\sigma_{1/2} = \frac{Z^2}{18}$$
$$\sigma_{3/2} = 0 .$$

(1.14)

In the simplest quark parton model the spin orientation of the quarks is given by the SU(6) spin-unitarity spin wave function\(^{19}\)

$$\frac{1}{\sqrt{18}} \begin{pmatrix} 2 & |u^+ d^+ u^+ d^+ u^+> + 2 |u^+ u^+ d^+ u^+ d^+ u^+> + 2 |d^+ u^+ u^+ d^+ u^+> \\ - |u^+ u^+ d^+ u^+ d^+ u^+> + 2 |u^+ d^+ u^+ d^+ u^+> + 2 |d^+ u^+ d^+ u^+ d^+ u^+> \\ - |d^+ u^+ u^+ d^+ u^+ d^+ u^+> + 2 |u^+ u^+ d^+ u^+ d^+ u^+> + 2 |d^+ u^+ u^+ d^+ u^+ d^+ u^+> \end{pmatrix}$$

for a proton with spin up. The probabilities to find a quark in the two different spin orientations is:
The virtual photon asymmetry for the proton defined as

\[ A_T^p = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \]

is then

\[ A_T^p = \frac{\left(\frac{4}{9} \cdot \frac{5}{9} + \frac{1}{9} \cdot \frac{1}{9}\right) - \left(\frac{4}{9} \cdot \frac{1}{9} + \frac{1}{9} \cdot \frac{2}{9}\right)}{\left(\frac{4}{9} \cdot \frac{5}{9} + \frac{1}{9} \cdot \frac{1}{9}\right) + \left(\frac{4}{9} \cdot \frac{1}{9} + \frac{1}{9} \cdot \frac{2}{9}\right)} = \frac{5}{9}. \]

For the neutron \( A_T^n = 0. \)

These simple results are modified in more refined parton models\(^{20}\). In general one predicts \( A_T^p \) to be large and positive and to show scaling behaviour, i.e. \( A_T = A_T(x)^{21}. \)

The asymmetry directly determined from the scattering of longitudinally polarized electrons on longitudinally polarized protons is

\[ A = \frac{d \sigma(\uparrow \downarrow) - d \sigma(\downarrow \uparrow)}{d \sigma(\uparrow \downarrow) + d \sigma(\downarrow \uparrow)}. \]

In good approximation \( A \) is related to \( A_T \) by

\[ A = D A_T, \quad (1.15) \]

where \( D \) is the kinematic virtual photon depolarization factor

\[ D = \frac{E - E' \epsilon}{E(1 + \epsilon \sigma_L / \sigma_T)}. \]

Unfortunately even large values of \( A \) lead to rather small values of the directly measured asymmetry

\[ A_{\text{meas}} = P_e P_p F A \quad (1.16) \]

where

- \( P_e \) = electron beam polarization
- \( P_p \) = average polarization of target protons
- \( F \) = fraction of electrons scattered from free protons.

In the SLAC experiments\(^{22,23}\) \( P_e = 0.85 \pm 0.08, P_p = 0.50 \pm 0.04, F = 0.13, \) and \( A_{\text{meas}} = 0.06 A. \) Precision is essential for a successful experiment. The most important achievement was the development of an intensive source of polarized electrons allowing measurements with sufficient statistical accuracy.

The measured values of \( A \) are plotted in Fig. 10 as a function of \( x. \) Within errors the data are consistent with scaling. At large \( x, \) where valence quarks dominate, the most simple quark parton model prediction \( A_T = 5/9 \) agrees with the data. The curve
represents a quark parton model prediction\textsuperscript{26} which takes into account valence and sea quark distributions, the valence quarks carrying the spin information and being described by the SU(6) wave function. Such models predict smaller values of $A_L$ at small $x$.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig10}
\caption{Virtual photon asymmetry $A_L^D$ versus $x$.}
\end{figure}

1.4 Neutrino Scattering

The total cross section for neutrino nucleon interactions is of the order $10^{-37}$ cm$^2$ at a neutrino energy of 10 GeV corresponding to a mean free path in a material of density 1 of about $2 \cdot 10^8$ km. One has to remember this fact in order to fully appreciate the results of present neutrino experiments which compete in statistics with electromagnetic experiments. The progress is due to the development of high quality $\nu$ and $\bar{\nu}$ beams, and of very massive detectors. There are excellent reviews on the subject\textsuperscript{24,25,26}. Here, after a few remarks concerning beams and detectors, I will concentrate on aspects connected with the quark parton model.

1.4.1 Experiments

a) Neutrino beams

The main features of all neutrino beams are a proton target followed by a pion (and kaon) selecting magnetic system, a decay channel and a massive muon absorber of several 100 m length. The parent mesons are treated in two different ways:

In the wide band beam all mesons of right charge are focused towards the detector by a special magnetic horn. This leads to an intensive beam with a neutrino energy spectrum falling steeply with energy.

In the narrow band beam a narrow momentum band of mesons is selected before decay. In this case the energy of the decay neutrino depends only on its angle with respect to the beam axis and can thus be measured for each event (apart from a $\pi,k$ ambiguity).
Some properties of the two types of neutrino beams (CERN) are collected in Table 2.

**Table 2** Properties of neutrino beams

<table>
<thead>
<tr>
<th></th>
<th>wide band beam</th>
<th>narrow band beam</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parent momentum</strong></td>
<td>all momenta</td>
<td>fixed</td>
</tr>
<tr>
<td><strong>Intensity: v's/GeV for</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^{13} incident protons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E_v = 50 GeV</td>
<td>4 \cdot 10^8</td>
<td>10^7</td>
</tr>
<tr>
<td>E_v = 150 GeV</td>
<td>10^7</td>
<td>3 \cdot 10^6</td>
</tr>
<tr>
<td>Knowledge of E_v</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>v/\bar{v} selection</td>
<td>reasonable</td>
<td>very good</td>
</tr>
</tbody>
</table>

b) Detectors

Due to the small cross section neutrino experiments need very massive targets. The basic types of detectors are therefore either large bubble chambers with masses in the order 1 to 10 tons or electronic detectors with masses up to ~1000 tons.

Bubble chambers have the advantage of very good identification of all charged particles of the hadronic final state. On the other hand at high energies an external muon identifier has to be added to improve the analysis of the outgoing muon. An example is the Big European Bubble Chamber BEBC used at CERN. BEBC has a fiducial volume of about 10 m^3 and a 30 KG magnetic field produced by a superconducting coil. The chamber has been operated with three fillings: H_2, D_2, and a H_2-Ne mixture.

Electronic detectors have the advantage of very high masses and, therefore, allow high statistics experiments with good muon identification by range measurement. They are also capable of measuring accurately the total energy of the hadronic shower produced in the target. On the other hand they cannot supply information about the distributions of the individual final state hadrons.

For the CERN SPS two electronic detectors have been constructed. A collaboration from CERN-Dortmund-Heidelberg-Saclay (CDHS) has built a target calorimeter detector consisting of 1400 tons of magnetized iron toroids. Inserted between the iron plates are scintillator hodoscopes for shower analysis and drift chambers for muon track reconstruction. The data from this experiment have the highest statistical accuracy obtained so far in a neutrino experiment.

A CERN-Hamburg-Amsterdam-Rome-Moscow Collaboration (CHARM) constructed a fine grain target calorimeter composed of marble plates, drift tubes, and scintillator hodoscopes which is particularly designed for the study of neutral current reactions. The instrument is able to measure the direction of the axis of the hadronic shower together with the energy. If also the energy of the incident neutrino is known, one can reconstruct all kinematical variables and study e.g. the x-dependence of neutral current cross sections.
1.4.2 Charged Current Reactions

In this case a charged W* is exchanged between lepton and hadron. Again there are three helicity states ± 1 and 0 of the current (virtual W) and one can define the corresponding absorption cross sections \( \sigma_+ \), \( \sigma_- \) and \( \sigma_L \). However, contrary to the electromagnetic case we have parity violation and \( \sigma_+ \neq \sigma_- \). The neutrino cross section is therefore expressed in terms of three a priori independent absorption cross sections or structure functions:

\[
\frac{d^2 \sigma^\nu}{dx dy} = \frac{G^2}{\pi} M E \left[ xy^2 F_1(x, Q^2) + (1 - y - \frac{M E}{2E}) F_2(x, Q^2) \pm y(1 - \frac{Q^2}{2E}) x F_3(x, Q^2) \right]. \tag{1.17}
\]

The (+) sign is for neutrinos, the (-) sign for antineutrinos. All information about the structure of the hadron vertex is contained in the structure functions \( F_i \).

The corresponding formula for the electromagnetic case is gained by replacing

\[
\frac{G^2 x}{2} \quad \text{by} \quad \frac{4 \pi \alpha^2}{Q^4}
\]

and setting \( F_3 = 0 \). The term with \( \frac{M E}{2E} \) can be neglected at high energies.

In the parton model one expects scaling for the 3 structure functions, i.e.

\[
F_1(x, Q^2) = F_1(x). \quad \text{As immediate consequence}
\]

\[
\sigma_{\nu, \bar{\nu}} = E \quad \text{(for any target)}.
\]

This relation is satisfied by the data in the energy region \( 2 < E_\nu < 20 \text{ GeV} \), as can be seen from Fig. 11 where results from measurements with the heavy liquid bubble chamber Gargamelle at CERN\(^{24}\) are shown.

In the most general case one has to measure the \( F_{1,2,3} \) for each of the processes \( \nu p, \bar{\nu} n, \bar{\nu} p, \bar{\nu} n \). Fortunately, the quark parton model simplified the situation. The quark content of all charged current neutrino structure functions expressed in terms of quark momentum distributions is:

\[
F_2^{\nu p} = 2 x (\bar{u} + d + s + c)
\]

\[
F_2^{\bar{\nu} p} = 2 x (u + \bar{d} + \bar{s} + c)
\]

\[
xF_3^{\nu p} = 2 x (-\bar{u} + d + s - \bar{c})
\]

\[
xF_3^{\bar{\nu} p} = 2 x (u - \bar{d} - \bar{s} + c).
\]

The corresponding expressions for the neutron structure functions are again obtained by replacing \( u \) respectively \( \bar{u} \) by \( d \) respectively \( \bar{d} \) and vice versa. For all processes

\[
2 x F_1 = F_2.
\]

It is convenient to introduce the sums of quark and antiquark distributions:

\[
q = u + d + s + c
\]

\[
\bar{q} = \bar{u} + \bar{d} + \bar{s} + \bar{c}.
\]
Neutrino and Antineutrino Total Cross-Sections

Then for scattering off an isoscalar target \((N = \frac{1}{2} (p + n))\), assuming \(s = \bar{s}\) and \(c = \bar{c}\)

\[
P_{\nu N} = P_{\bar{\nu} N} = x(q + \bar{q})
\]

\[
x P_{\nu N} P_{\bar{\nu} N} = x[q - \bar{q}^2 (c + \bar{c} - s - \bar{s})].
\]

Eqs. (1.19) and (1.20) include the "small" quark distributions \(s, \bar{s}, c\) and \(\bar{c}\). In order to work out some essential points, let us assume now that these can be neglected. Then

\[
\frac{d^2\sigma_{\nu N}}{dx dy} = \frac{G^2_{\nu e}}{\pi} x \left[q + (1 - y)^2 \bar{q}\right]
\]

\[
\frac{d^2\sigma_{\bar{\nu} N}}{dx dy} = \frac{G^2_{\bar{\nu} e}}{\pi} x \left[\bar{q} + (1 - y)^2 q\right].
\]

The form of the \(y\) distribution is easily understood; it reflects the spin structure of the point like scattering. The structure of the charged current interaction is such that neutrinos (left-handed) couple to left-handed quarks and right-handed antiquarks. In the c.m. system a \(\nu q\) reaction has \(I_z = 0\) and leads to an isotropic distribution. A \(\bar{\nu} q\) reaction has \(I_z = -1\) and is forbidden at \(\theta_{\text{cm}} = 180^\circ\). Since \(1 - y = \cos^2 \theta_{\text{cm}}\), one expects a flat \(y\) distribution for \(\nu q\) and a strong \(1-y\) dependence for \(\bar{\nu} q\).
In the following I will summarize the gross features of neutrino scattering using Eqs. (1.18) to (1.21) as a guide line.

a) Total cross section ratios

Integrating Eq. (1.21) over x and y yields \( \frac{\sigma_{\nu N}}{\sigma_{\overline{\nu} N}} > \frac{1}{3} \). The ratio is exactly \( \frac{1}{3} \) if there are no antiquarks in the nucleon. The Gargamelle measurements (Fig. 11) giving a ratio of \( 0.38 \pm 0.02 \) indicate that the antiquark content is indeed small at \( E_\nu \lesssim 20 \text{ GeV} \).

b) x-distributions

A measurement of the x-distributions of both \( \nu \) and \( \overline{\nu} \) scattering (integrating over y) offers a possibility to determine the quark and antiquark distribution functions separately. Fig. 12 shows \( xq(x) \) and \( x\overline{q}(x) \) evaluated from the CDHS data\(^{27}\).

![Fig. 12 x-distribution for quarks and antiquarks in the nucleon.](image)

Note the small error bars obtained in this experiment. Antiquarks contribute only at small x.

c) y-distributions

For the y-distribution (integration over x), we expect for \( \nu N \) scattering a flat distribution with a small \((1-y)^2\) admixture and for \( \overline{\nu} N \) scattering a \((1-y)^2\) distribution with a small constant admixture. This is confirmed by data from the CHHS collaboration\(^ {27}\). Fig. 13.
From a fit to the $y$-distributions, one can determine the fraction of the nucleons momentum carried by antiquarks

$$\frac{\bar{Q}}{Q + \bar{Q}} = 0.15 \pm 0.03,$$

where $\bar{Q} = \int_0^1 dx x\bar{q}(x)$ and $Q = \int_0^1 dx xq(x)$.

d) Callan-Gross relation

All analyses of neutrino scattering base on the Callan-Gross relation $F_2 = 2 xF_1$. It is therefore important to test its validity by studying the $y$-dependence of the sum of the $\nu$ and $\bar{\nu}$ cross sections (which is independent of $xF_3$):

$$\frac{d^2\sigma_{\nu N}}{dx dy} + \frac{d^2\sigma_{\bar{\nu} N}}{dx dy} = \frac{G^2 ME}{\pi} F_2 \left[ 1 + (1-y)^2 - y^2 R' \right]$$

where

$$R' = \frac{F_2 - 2 xF_1}{F_2}.$$

$R'$ is related to the ratio $R = \sigma_L/\sigma_T$ measured in electron or muon scattering by

$$R' = \frac{R - \bar{Q}^2/\nu^2}{1 + R} < R.$$
Collected in Table 3 are the results of groups having explicitly evaluated the radiative corrections to their data (which change somewhat the shape of the $y$ distribution).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$R'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPWFOR$^{26}$</td>
<td>0.18 ± 0.07</td>
</tr>
<tr>
<td>CDHS$^{29}$</td>
<td>0.02 ± 0.12</td>
</tr>
</tbody>
</table>

$R'$ is indeed small but not necessarily zero.

e) Neutrino versus electron.

The relation

$$f_{eN}^2(x) = \frac{5}{18} f_{\nu N}^2(x)$$

is easily derived in the quark parton model since

$$f_{eN}^2 = \frac{5}{18} x (q + \bar{q}) + \frac{1}{6} x (c - s + \bar{c} - \bar{s})$$

and

$$f_{\nu N}^2 = x (q + \bar{q}) .$$

The factor $\frac{5}{18}$ represents the average charge squared of the quarks inside the nucleon. In the region of large $x$ ($x \approx 0.2$) where strange and charm quarks are important, one would expect exactly $f_{eN}^2 = 5/18 \cdot f_{\nu N}^2$.

Experimentally this relation is beautifully satisfied as demonstrated in Fig. 14 where the $\nu N$ Gargamelle measurements$^{2k}$ are compared with curves showing 3.6 times the SLAC $eN$ data.

![Fig. 14 $F_{eN}^2(x)$ compared with $3.6 F_{\nu N}^2(x)$](image_url)
On the first glance this result may appear as striking evidence for fractionally charged partons. For the scepticist, however, it should be pointed out that there are several other explanations possible\textsuperscript{30,31}, in particular one does not necessarily need quarks with fractional charge to derive such a ratio.

\textbf{f) Gross-Llewellyn Smith sum rule}

Using the normalization conditions for the quark distribution functions Eqs. (1.11), one can reproduce the sum rule\textsuperscript{32}

\[
\frac{1}{2} \int_0^1 dx \left( F_3^{up}(x) + F_3^{dp}(x) \right) = 3 \quad \text{number of valence quarks.}
\]

Neglecting again strange and charm quark contributions,

\[
\frac{1}{2} \int_0^1 dx \left( F_3^{up} + F_3^{dp} \right) = \int_0^1 dx F_3^{up} = \int_0^1 dx \frac{3x}{2GMx} \left( \frac{dF_3^{uN}}{dx} - \frac{dF_3^{dN}}{dx} \right).
\]

Experimental evaluations of the integral including attempts to extrapolate to \( x = 0 \) are summarized in Table 4 and are indeed consistent with a value of 3.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Experiment & \( \int dx F_3^{uN} \) \\
\hline
GGM\textsuperscript{24}) & 3.0 \pm 0.6 \\
CIM\textsuperscript{27}) & 3.2 \pm 0.5 \\
HPWFOR\textsuperscript{28}) & 2.8 \pm 0.45 \\
ABCLOS (BEBC)\textsuperscript{33}) & > 2.7 \pm 0.4 \\
\hline
\end{tabular}
\caption{Gross-Llewellyn Smith sum rule}
\end{table}

\textbf{g) Neutron-Proton comparison}

The quark parton model predicts for the ratios of neutron to proton total cross sections

\[
\frac{\sigma^{\text{vn}}}{\sigma^{\text{vp}}} = \frac{\int dx \left( u + \frac{1}{3} d + s \right)}{\int dx \left( d + \frac{1}{3} u + s \right)} = 1.8
\]

and

\[
\frac{\bar{\sigma}^{\text{vn}}}{\bar{\sigma}^{\text{vp}}} = \frac{\int dx \left( \frac{1}{3} d + \bar{u} + \bar{s} \right)}{\int dx \left( \frac{1}{3} u + \bar{d} + \bar{s} \right)} = 0.5.
\]

Here the strange quark contributions are included but the (smaller) charm quark terms are dropped. The numbers on the right-hand side arise from a detailed parton model analysis.

Some experimental results are collected in Table 5 and show good agreement with the parton model expectations.
<table>
<thead>
<tr>
<th>Experiment</th>
<th>E(GeV)</th>
<th>$\sigma^{\nu\nu}/\sigma^{\nu p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GGM (^{34})</td>
<td>1 - 10</td>
<td>2.08 ± 0.15</td>
</tr>
<tr>
<td>FNAL 15 (^{35})</td>
<td>&gt; 10</td>
<td>1.74 ± 0.25</td>
</tr>
<tr>
<td>GGM (^{36})</td>
<td>&lt; 5</td>
<td>0.46 ± 0.10</td>
</tr>
<tr>
<td>FNAL 15 (^{37})</td>
<td>&gt; 10</td>
<td>0.45 ± 0.08</td>
</tr>
</tbody>
</table>

h) Determination of the strange sea from dimuon events

It is interesting to notice that one can determine the strange quark distributions by studying neutrino induced opposite sign dimuon events of the type $\bar{\nu}N \rightarrow \mu^+\mu^- X$. The underlying mechanism is the production of charmed quarks which according to the Glashow-Iliopoulos-Maiani (GIM) model \(^{38}\) proceeds via the following reactions:

\[
\begin{align*}
\bar{\nu} + d \rightarrow \mu^- + c = d(x) \sin^2 \theta_c \; ; \\
\nu + s \rightarrow \mu^+ + c = s(x) \cos^2 \theta_c \\
\nu + d \rightarrow \mu^- + c = s(x) \sin \theta_c \; ; \\
\nu + s \rightarrow \mu^- + c = s(x) \cos \theta_c ,
\end{align*}
\]

where $\theta_c$ is the Cabibbo angle, $\sin^2 \theta_c \approx 0.05$. The corresponding cross sections are

\[
\frac{d^2\sigma}{dx dy} (\bar{\nu}N \rightarrow \mu^+\bar{c} X) = \frac{G_{\text{ME}}^2}{\pi} [2 \bar{s} \cos^2 \theta_c + (\bar{u} + \bar{d}) \sin^2 \theta_c] \quad (1.23a)
\]

\[
\frac{d^2\sigma}{dx dy} (\nuN \rightarrow \mu^- c X) = \frac{G_{\text{ME}}^2}{\pi} [2 s \cos^2 \theta_c + (u + d) \sin^2 \theta_c] . \quad (1.23b)
\]

The charmed meson of the hadronic final state containing the produced $\bar{c}$ (or $c$) quark decays with about 10% probability into $\mu^-\bar{\nu}k$ (or $\mu^+\nu k$) giving rise to the observed dimuon signal.

Since $\sin^2 \theta_c << \cos^2 \theta_c$, the production of $\bar{c}$ quarks from $\bar{s}$ quarks is heavily favoured and one can neglect in Eq. (1.23a) the contributions from $\bar{u}$ and $\bar{d}$ quarks. Opposite sign dimuon events in $\bar{\nu}$ reactions directly reflect the properties of the $\bar{s}$ sea.

Fig. 15 shows the $x$-distribution of the leading $\mu^+$ as measured by the CDHS collaboration \(^{39}\). The shape is characteristic of a sea quark distribution (see Fig. 12).

From the measurement of both $\bar{\nu}$ and $\nu$ induced $\mu^+\mu^-$ events, one can determine the fraction of the nucleons momentum carried by $\bar{s}$ quarks,

\[
\frac{\bar{s}}{Q^2} = 0.03 \pm 0.01 ,
\]

where $\bar{s} = \int dx x s(x)$. This has to be compared to a value of 0.15 for the total antiquark momentum and indicates that the contribution from $s$ quarks is relatively small $\bar{s}/\bar{u} \approx 0.3$. 
1.4.3 Neutral Current Reactions

Neutrino experiments have meanwhile achieved such a high precision that it is possible to study the $x$-distributions of inclusive processes mediated by the weak neutral current. Let us discuss briefly what is expected on the basis of the Weinberg-Salam-GIM model. For recent summaries on the subject of neutral currents see \(^{260-261}\).

Fig. 15 $x$-distribution of $\bar{\nu}$ induced opposite sign dimuons.

First, the double differential cross section for inclusive neutral current scattering can be expressed in terms of three structure functions $xF_1^{NC}$, $F_2^{NC}$, $xF_3^{NC}$ just in the same way as the charged current cross section is expressed in terms of $xF_1^{CC}$, $F_2^{CC}$, $xF_3^{CC}$. The $F_i^{NC}$ are connected to the chiral coupling constants $u_L$, $d_L$, $u_R$ and $d_R$ and the quark momentum distribution functions. $u_L$ and $d_L$ describe the neutral weak coupling of left-handed quarks of charge $\frac{2}{3}$ respectively $-\frac{1}{3}$, $u_R$ and $d_R$ describe the coupling of right-handed quarks of charge $\frac{1}{3}$ and $\frac{-1}{3}$.

For isoscalar targets:

$$F_2^{NC}(x) = 2 xF_1^{NC}(x) = x(q + \bar{q})(u_L^2 + d_L^2 + u_R^2 + d_R^2) + x(c - s + \bar{c} - \bar{s})(u_L^2 - d_L^2 + u_R^2 - d_R^2)$$

and

$$xF_3^{NC}(x) = x(q - \bar{q})(u_L^2 + d_L^2 - u_R^2 - d_R^2).$$

Eqs. (1.24) and (1.25) are valid for both neutrino and antineutrino scattering.

The chiral couplings depend on the Weinberg angle in the following way:

$$u_L = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W$$

$$d_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W$$

$$u_R = -\frac{2}{3} \sin^2 \theta_W$$

$$d_R = \frac{1}{3} \sin^2 \theta_W$$

(1.26)
Using the above equations it is straightforward to express the cross sections for NC reactions. For reasons of simplicity let us again neglect contributions from scattering off s and c quarks. Then we obtain for the ratio of NC to CC differential cross sections

\[ R = \frac{\frac{d\sigma}{dx}}{\frac{d\sigma}{dx}} = \frac{2 F_{NC}^{2} \pm x F_{NC}^{2}}{2 F_{CC}^{2} \pm x F_{CC}^{2}} = \frac{1}{2} - \sin^{2} \theta_{W} + \frac{10}{9} \sin^{4} \theta_{W} \frac{2(q + \bar{q})}{2(q + \bar{q}) \pm (q - \bar{q})} \]  

(1.27)

Here the (+) sign refers to \( \nu N \), the (-) sign to \( \bar{\nu} N \) scattering.

For \( \sin^{2} \theta_{W} \) the best value obtained from various experiments is \( \sin^{2} \theta_{W} = 0.23 \). Thus we arrive at the following simple prediction for the kinematical region \( x > 0.2 \) where only valence quark scattering is important:

\[ R = 0.31 \text{ for } \nu N \]
\[ \bar{R} = 0.39 \text{ for } \bar{\nu} N \]

Fig. 16 shows \( R(x) \) and \( \bar{R}(x) \) as recently measured by the CHARM collaboration.

Experimentally the cross section ratios are well determined since they are insensitive to some types of systematic uncertainties like acceptance corrections and resolution effects. One observes a flat x-distribution and our expectations are well met by the data.
SUMMARY OF RESULTS DISCUSSED SO FAR

1. The approximate scale invariance of the ep, en, and vN structure functions suggests the parton model.

2. Small ratios $\sigma_L/\sigma_T$ in ep, ed, and vN scattering are consistent with a spin $\frac{1}{2}$ assignment for charged partons.

3. Measurements of numerous relations between the electromagnetic and the neutrino structure functions as well as measurements of the asymmetry in polarized ep scattering are all in agreement with the assumption: The charged partons are quarks.

4. The verification of the Gross-Llewellyn Smith sum rule suggests the presence of 3 valence quarks.

5. From the analysis of $x_F^{vN}$ and $F_2^{vN}$ we learn that the amount of antiquarks in the nucleon is small and noticeably only at small $x$.

6. From the absolute values of $\int_0^1 dx F_2$ (momentum sum rule), we conclude: There are neutral partons carrying about 50% of the nucleon's momentum at $Q^2$ of a few GeV$^2$.

In short, all experimental results measured over a period of 12 years in deep inelastic electron and neutrino scattering are consistent with the quark parton hypothesis.

2. SCALING VIOLATIONS

2.1 Quantum Chromodynamics, The Origin of Logarithmic Scaling Violations

Let us turn now to applications of the new theory of strong interactions, quantum chromodynamics (QCD) to deep inelastic scattering.$^{44}$ QCD has the important property of asymptotic freedom$^{45-47}$, i.e. with increasing four momentum transfer $Q^2$ of the current probe (virtual $\gamma$, $W^\pm$, or $Z^0$) the effective coupling $g_{\text{eff}}$ between colored quarks and colored gluons becomes smaller. More specifically, the effective strong interaction coupling defined as $a_s = \frac{g_{\text{eff}}^2}{4\pi}$ (similar to $a = \frac{g_{\text{eff}}^2}{4\pi}$ in QED) can be shown to behave like

$$a_s(Q^2) = \frac{4\pi}{g_o \ln Q^2/\Lambda^2}$$

with

$$g_o = 11 - \frac{2}{3} f .$$

(2.2)

$f$ counts the number of quark flavors ($u,d,s,c,...$). The mass scale parameter $\Lambda$ (the only free parameter of QCD) has to be determined experimentally by comparing QCD predictions with data. For $Q^2 = \Lambda^2$ the effective coupling vanishes. As a consequence one can use perturbation theory for an exact calculation of the $Q^2$ development of deep inelastic scattering in the region $Q^2 \gg \Lambda^2$ and predict the pattern of scaling violations. Scaling violations of the type predicted by QCD have clearly been observed in the data.
According to Kogut and Susskind\textsuperscript{48j} it is possible to develop the following intuitive picture: Increasing $Q^2$ of the virtual $\gamma$ (or $W^\pm, Z^0$) microscope means probing the structure of the target at smaller and smaller distances. One resolves a richer internal structure since one can see partons created by QCD splitting processes of the type (a) a quark splits into a quark and a gluon, (b) a gluon splits into a quark antiquark pair, (c) a gluon splits into two gluons.

\begin{center}
\begin{tabular}{ccc}
\includegraphics[width=0.3\textwidth]{fig17a}&\includegraphics[width=0.3\textwidth]{fig17b}&\includegraphics[width=0.3\textwidth]{fig17c}
\end{tabular}
\end{center}

\textbf{Fig. 17} Basic Splitting Processes of QCD.

Relative to a reference $Q_0^2$ the current finds at $Q^2 > Q_0^2$ parton distributions created by a more complex hierarchy of splitting processes. Therefore, all parton distributions are functions of $x$ and $Q^2$. Once the $Q^2$ dependent distributions are calculated, they can be inserted in the parton model formulae for $x_F, F_2, x_F^3$ which otherwise remain unchanged (in leading order QCD).

The $Q^2$ development is different for the different parton distributions. Valence quarks can only contribute to splitting process (a). They will therefore lose their momentum which in turn is given to the gluon and sea distributions. Valence quark scattering at large $x$ dies out with increasing $Q^2$. The sea distribution, on the other hand, is influenced by all splitting processes. Since $\gamma$ (or a $W^\pm, Z^0$) cannot couple to a gluon, process (c) contributes in an indirect way, the created gluons can produce a $q, \bar{q}$ pair. In total, more sea quarks with low momentum fraction $x$ are created at higher $Q^2$. Therefore, we expect qualitatively for $F_2$:

\begin{center}
\begin{tabular}{c}
\includegraphics[width=0.5\textwidth]{fig17d}
\end{tabular}
\end{center}
The evolution of $q^{(x,t)}$ is represented by the following equation:

$$\frac{d}{dt} q_{NS}(x,t) = \frac{a_s(t)}{2\pi} \int_0^1 \frac{dy}{y} q_{NS}^{-}(y,t) P_{qq}(\frac{y}{x}) ,$$

where the function $P_{qq}(z)$ is explicitly given by QCD. The change of the quark distribution is due to the bremsstrahlung of a gluon from the initial quark with momentum $y_p$ leading to a quark with momentum $x_p$. In general, one can write for the evolution of the quark and gluon distribution functions $q_i(x,t)$ and $g(x,t)$:
Again, all splitting functions $P$ can be calculated in perturbation theory\cite{17}.

Fig. 18 illustrates the above equations in terms of the basic splitting diagrams\cite{50}.

\[ \frac{d q_i(x,t)}{dt} = \frac{a_s(t)}{2^n} \int x \frac{dy}{y} \left[ q_i(y,t) \frac{\partial}{\partial y} \left( \frac{\bar{q}}{q} \right) + f(y,t) \frac{\partial}{\partial y} \left( \frac{\bar{q}}{q} \right) \right] \quad (2.6) \]

\[ \frac{d g(x,t)}{dt} = \frac{a_s(t)}{2^n} \int x \frac{dy}{y} \left[ \bar{q} \left( \frac{\partial}{\partial y} \left( \frac{\bar{q}}{q} \right) + g(y,t) \frac{\partial}{\partial y} \left( \frac{\bar{q}}{q} \right) \right) \right]. \quad (2.7) \]

The change of a quark distribution with respect to $t$ is in general due to gluon bremsstrahlung of the initial quark and due to quark antiquark pair creation from an initial gluon. The change of the gluon distribution arises from gluon radiation from all species of initial quarks and from gluons radiated from initial gluons. Only the change of the non-singlet quark distributions does not depend on the gluon distribution since by taking the difference $q(x,t) - \bar{q}(x,t)$ the gluon terms drop out.

The above integro-differential equations are equivalent to a set of evolution equations for the moment integrals of the parton distribution functions or the structure functions. A summary of all formulae can be found in Ref.\cite{47}. Here I will reproduce the evolution equations for the moments of the structure functions which are defined as:
\[ M^2(n,Q^2) = \int_0^1 dx \: x^{n-2} \: \tilde{F}_1(x,Q^2) , \]  

(2.8)

where \[ \tilde{F}_1 = xF_1, \: \tilde{F}_2 = F_2 \: \text{and} \: \tilde{F}_3 = xF_3 \: . \]

For the \( Q^2 \) dependence of the moments of any non-singlet structure function like \( xF_{3N} \) or \( F_{2n}^2 - F_{2}^n \) (presented by pure non-singlet quark distribution functions), one finds:

\[ \eta^2(n,Q^2) = \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \cdot (2.9) \]

The numbers \( \eta_n^2 \) (called anomalous dimensions) are connected to the probability function \( P_{qq} \):

\[ \eta_n^2 = \frac{2}{\beta_0} \int_0^1 dz \: z^{n-1} \: P_{qq}(z) , \]  

(2.10)

and can be explicitly calculated:

\[ \eta_n^2 = \frac{4}{35 - 2\pi^2} \left[ 1 - \frac{2}{n(n+1)} \right] + \frac{4}{3} \sum_{j=2}^{n} \frac{j}{j^j} . \]  

(2.11)

With increasing order \( n \) the powers \( \eta_n^2 \) increase, i.e. the moments decrease more strongly with increasing \( Q^2 \).

In the general case the evolution equations for the moments (e.g. of \( F_2 \)) are more complicated:

\[ M^2(n,Q^2) = \sum_{i=1}^{3} A_i/(\ln Q^2/\Lambda^2) \: \eta_n^2 \]  

(2.12)

where the \( \eta_n^2 \) are again numbers predicted by QCD. The constants \( A_i \) must be determined from a fit to the data. The three terms correspond to the flavor non-singlet part of the quark distributions (valence quarks), and the two flavor singlet parts of the sea quark-antiquark and the gluon distributions.

From the integro-differential Eqs. (2.5) - (2.7) or the corresponding moment evolution equations one can numerically compute the parton distributions at any \( Q^2 \) once they are given at \( Q_0^2 \). As an example, Fig. 19 shows a typical result of a QCD calculation for \( F_2 \), the valence quark distributions \( u \) and \( d \), the non-charmed sea, and the charmed sea at \( Q^2 = 1.8 \: \text{GeV}^2 \) and \( 22.5 \: \text{GeV}^2 \).
Second, new contributions to $F_2$ can rise from their threshold $W_{th}$, e.g. charm production. Then a new piece has to be added to $F_2^{\text{old}}$:

$$F_2(x) = F_2^{\text{old}}(x) + F_2^{\text{new}}(\bar{x}) \Theta(W^2 - W_{th}^2) ,$$

where

$$\bar{x} = \frac{Q^2}{2M_0 - W_{th}^2 + M^2}$$

with $W_{th} = 4$ GeV for charm. As a consequence one will observe an increase of $F_2$ at fixed (small) $x$ leveling off to a constant as $\bar{x} \rightarrow x$.

There are three more subtle modifications to the logarithmic scaling violations expected from lowest order QCD (here I follow a presentation of Barnett) originating from:

1. Target mass corrections,
2. Coherent phenomena,
3. Higher order QCD corrections.

As discussed earlier, the Bjorken scaling variable $x$ is the relevant scaling variable in the parton model only if target and quark mass effects are neglected. The inclusion of target mass effects leads to the scaling variable:

$$\xi = \frac{2x}{1 + \sqrt{1 + 4 M^2 x^2/Q^2}} = x(1 - \frac{M^2 x^2}{Q^2}) \text{ for large } Q^2 .$$

If $M^2$ is negligible compared to $Q^2$, then $\xi = x$. Much of the present data are, however, at $Q^2$ where this assumption is not justified and there are correction terms of order $M^2/Q^2$. To illustrate the difference between $\xi$ and $x$ scaling, let us assume for a moment that $F_2$ would scale exactly in $\xi$ and behaves like

$$F_2 = \text{const } (1 - \xi)^k \text{ for } \xi > \xi_0 .$$
A value of $a = 3$ gives a reasonable parametrization of the data. Then, $F_2$ taken at fixed $x$ will decrease with increasing $Q^2$ and eventually reach a finite asymptotic value. Fortunately, Nachtmann has developed a formalism$^{53}$ to take full account of target mass effects in the comparison of structure function moments with QCD predictions. I will return to this point later.

More serious problems arise from the presence of coherent phenomena such as diquark scattering, elastic scattering, resonance production and transverse momentum effects. These effects are called higher order twist effects (the name is due to non-leading twist operators in the Wilson operator product expansion). For our discussion it is sufficient to note that they represent corrections to the parton model picture leading to additional terms of order $1/Q^2, 1/Q^4 \ldots$. It is not possible at present to calculate the higher order twist effects, but one can assume that their presence can be parametrized in the following form$^{54}$:

$$F_2^{NS}(x,Q^2) = C(1-x)^3 \left[ 1 + \frac{\mu_1^2}{Q^2(1-x)} + \frac{\mu_2^2}{Q^4(1-x)^2} \right],$$

where $\mu_1, \mu_2$ are unknown constants. Obviously, the corrections are largest at large $x$, where one also expects large scaling violations due to leading order QCD.

Higher order QCD corrections have been calculated for the next order beyond the leading logarithms$^{55,56}$. They lead to additional terms of order $1/(\ln Q^2/\Lambda^2)^2$. The effective strong interaction coupling constant takes the improved form:

$$a_s(Q^2) = a_s^0(Q^2) \left( 1 - \frac{\beta_1}{\beta_0} \ln \frac{Q^2}{\Lambda^2} \right),$$

with $\beta_1 = 102 - 38 \frac{3}{\pi} f$.

$a_s^0(Q^2)$ is given by Eq. (2.1). The lowest order QCD predictions for the structure functions and their moments are therefore also modified. E.g. the non-singlet moments expressed in next to leading order are given by:

$$M_n^{NS}(n,Q^2) = M_n^{NS}(n,Q_0^2) \left( \frac{a_s(Q^2)}{a_s(Q_0^2)} \right) \cdot \left( 1 + K_n(a_s(Q^2) - a_s(Q_0^2)) \right), \quad (2.15)$$

where the $K_n$ are known numbers. The next to leading order equations for the moments of $F_2$ can be found in$^{55}$.

A fit to the data measured at $Q^2 \approx$ few GeV$^2$ using the leading order QCD formulae will result in a definite numerical value of the parameter $\Lambda$. However, as shown by Bac$^6$, the connection between the fit value $\Lambda$ and the mass scale of QCD is not unambiguous as long as only leading order is considered. It is, therefore, important to include higher order effects in a thorough QCD analysis of the data.

From the preceding discussion one may understand why the problem of the scaling violations is still an active field of research. A mere observation of the scale breaking pattern predicted by leading order QCD allows by no means a conclusion about the strength of QCD effects. Only if scaling violations persist at $Q^2 \gg \Lambda^2$ (let's say at $Q^2$ of order 100 GeV$^2$), one can hope to get unambiguous results concerning the crucial QCD
features and can reliably determine the mass scale.

2.3 The Experimental Situation

Scaling violations have been reported for the first time in 1975 from a muon experiment performed by the Cornell-Michigan State-Berkeley-La Jolla Collaboration at Fermilab\textsuperscript{59}). In principle, deviations from exact Bjorken scaling were observed earlier in the SLAC experiments. However, at that time the effect was parametrized by using modified scaling variables of the type \( x_s = x/(1 + x a^2_p/\nu^2) \) with \( a^2_p \approx M^2 \), which have some similarity with the Nachtmann variable \( \xi \). Exact scaling in \( x_s \) is equivalent to a decrease of \( F_2 \) with \( \nu^2 \) at fixed \( x \) in the kinematical region covered by the SLAC data\textsuperscript{10}).

In the following I will try to summarize the well-established experimental facts, the results of comparisons between data and QCD predictions and also discuss some problems of the QCD analysis. I will not quote all papers relevant to the subject and instead refer to summaries presented at Conferences or earlier Schools of Physics.

2.3.1 Fully Analyzed Electron and Muon Experiments

a) Structure functions

All electron or muon scattering experiments using hydrogen, deuterium, carbon or iron targets have confirmed the expected scaling violations. Unexpected structures observed by a Michigan State-Fermilab Collaboration\textsuperscript{60}) seem to disappear in a more complete analysis of the experiment.

The most precise published data at muon beam energies in the 100 GeV region come from the Chicago-Harvard-Illinois-Oxford (CHIO) Collaboration\textsuperscript{61}), working at Fermilab. Their apparatus is a large spark chamber spectrometer using the magnet of the former Chicago cyclotron for momentum analysis. The target is 1.2 m liquid hydrogen or deuterium. The results obtained for \( F_2^{EP} (F_2^{DP} = F_2^{DP} = F_2^{DP}) \) at various beam energies (96, 147 and 220 GeV) are plotted in Fig. 20 as a function of \( \ln \nu^2 \) for various \( \omega = \frac{1}{x} \) bins. It should be remarked that a knowledge of \( R = \sigma (\omega) \) is necessary for an extraction of \( F_2 \) from the directly measured double differential cross section (see Eqs. (1.1), (1.3)). In this case a constant value of \( R = 0.52 \) has been assumed which corresponds to the average value of \( R \) measured by the CHIO collaboration. The horizontal bars indicate the effect of changing \( R \) by its standard deviation to 0.37 (lower bar) respectively 0.69. Only the data at large \( x \) and \( \nu^2 \) are affected. Plotted are also ep data from SLAC and SLAC-MIT\textsuperscript{62}). One observes a clear decrease of \( F_2 \) with increasing \( \nu^2 \) at small \( \omega \) (\( x > 0.5 \)) and an increase at large \( \omega (x < .2) \), precisely in the manner predicted by QCD. The increase at high \( \omega \) persists even if uncertainties in \( R \) are taken into account near \( \omega \approx 4, F_2 \) is independent of \( \nu^2 \).

Several QCD fits in leading order with or without target mass corrections find good agreement with these or earlier data and lead to a mass scale \( \Lambda \) in the range 0.3 to 0.5 GeV (see for instance Ref.\textsuperscript{51}). However, most of the data is in the region \( \nu^2 \lesssim 20 \text{ GeV}^2 \), where corrections to leading order QCD effects may be important.
b) Analysis of moments

From the experimental side there is a substantial amount of data for evaluating the moments of $F_2^P$ and $F_y^d$ up to $Q^2 \sim 30$ GeV$^2$. Fig. 21 shows the range covered in $x$ at various values of $Q^2$ and indicates the excellent agreement between electron and muon data.

For a comparison with QCD predictions usually the Nachtmann version of the moments is evaluated using the variable $\xi$ instead of $x$ and taking into account target mass effects. The $\xi$ formalism also allows a reasonable averaging over the contributions from elastic scattering and resonance production and may thus include some higher order twist corrections. This point is discussed for instance in Ref. 58. The Nachtmann moments take a more complicated form than the ordinary $x$ moments, namely

\begin{equation}
M_2(n,Q^2) = \int_0^{\xi_{\text{max}}} d \xi \left[ 1 + \frac{M^2}{Q^2} \left( \frac{4 n(n+2) x^2 + 6(n+1) x \xi}{(n+2)(n+3)} \right) \right] \cdot \left( 1 - \frac{M^2}{Q^2} \right)^{n-2} F_2(\xi,Q^2).
\end{equation}

$\xi_{\text{max}}$ corresponds to elastic scattering ($x = 1$) $\xi_{\text{max}} = 2/(1 + \sqrt{1 + 4 M^2/Q^2})$.

At first comparison to the leading order QCD prediction for both proton and deuteron moments performed by Anderson et al.\textsuperscript{63} showed impressive agreement as presented in Fig. 22. It is advantageous to fit $p$ and $d$ moments at the same time since two of the three coefficients $A_i$ of Eq. (2.12) to be determined from the data at each $n$, namely the two singlet coefficients are the same for $M_2^{\text{prot}}$ and $M_2^{\text{deut}}/2$. The best value of the mass scale resulting from the fit is $\Lambda = 0.60 \pm 0.08$ GeV.
Fig. 21 F₂P data from FNAL and SLAC versus x for various fixed Q².

Fig. 22 Fits to the Nachtmann moments of the proton and deuteron structure functions F₂.
In a more refined analysis performed by the CHIO group with the full set of electron and muon data, it was found, however, that leading order QCD cannot fully account for the observed $Q^2$ dependence of the moments of order $n = 2$ to $n = 10^{61}$. There are strong indications that next to leading order effects are present in the data. This is also observed in a careful comparison of data and QCD performed by Duke and Roberts. The authors find in particular excellent agreement between data and next to leading order predictions for the moments of the electromagnetic non-singlet structure function $F_2^{\pi^+} - F_2^{\pi^-}$. Typical values of the mass scale $\Lambda$ obtained from fits are 0.5 to 0.6 GeV (using the so-called MS scheme of Bardeen et al.).

On the other hand there are possible problems. The moments of order $n \geq 4$ heavily weight the large $x$ region due to the factor $x^{n-2}$. This is the region where the statistics of the data is poorest. As discussed in chapter 2.2 at large $x$ also corrections due to coherent effects are largest. I will return to this point after summarizing the results from neutrino experiments.

2.3.2 Neutrino Data

a) Total cross sections

In the energy region $2 < E_\nu < 20$ GeV, the total $\nu$ and $\bar{\nu}$ cross sections are found to increase linearly with energy in agreement with the expectation of the quark parton model. The ratio $\sigma_\nu/\sigma_{\bar{\nu}}$ of about 1/3 indicates scattering mainly from valence quarks. At increasing $E_\nu$ (i.e. increasing $Q^2$) scattering off sea quarks will become more and more important as a result of the QCD parton splitting processes. Therefore, one would expect deviations from the linear relation between $\sigma_{\nu}$ and $E_\nu$. In the limit of $Q^2 \to \infty$ when valence quark scattering has completely died out $\sigma_{\nu}/\sigma_{\nu} \to 1$.

The data measured by 7 neutrino collaborations show such effects (Fig. 23) in particular $\sigma_{\nu}/E_\nu$ clearly decreases with $E_\nu$. The relatively slow variation of $\sigma_{\nu}/E_\nu$ is due to the fact that $\int dx F_2^N$, measuring the fraction of the nucleon's momentum carried by quarks, approaches a constant for $Q^2 \to \infty$. The ratio $\sigma_{\nu}/\sigma_{\nu}$ increases from $0.38 \pm 0.02$ at $2 < E_\nu < 10$ GeV (measured by Gargamelle) to $0.48 \pm 0.02$ at $30 < E_\nu < 200$ GeV (measured by CHRS). The solid curve represents a QCD calculation with $\Lambda = 0.5$ GeV including the effect of charm production. The curve with $\Lambda = 0$ indicates the increase due to charm production alone.

b) Neutrino structure functions

Better QCD tests are possible by looking directly at the $Q^2$ and $x$ distributions of the structure functions. The data obtained from recent neutrino experiments are indeed accurate enough to study the variation of the structure functions with $Q^2$ and fixed $x$. In particular, the CHRS Collaboration has separated $F_2^{\pi^+}(x,Q^2)$ from the sum of $\nu + \bar{\nu}$ measurements and $x F_2^{\pi^+}(x,Q^2)$ from the difference of $\nu - \bar{\nu}$ measurements in a $Q^2$ region extending to $Q^2 \approx 200$ GeV$^2$. In Fig. 24 their results for $x F_2^{\pi^+}$ are plotted versus $\log Q^2$ for various $x$ bins. Also shown are leading order QCD fits with $\Lambda = 0.55$ GeV which again describe well the data.
Fig. 23 Neutrino and antineutrino total cross sections compared with a QCD prediction.

Fig. 24 $x_F^3$ versus $Q^2$ for various $x$ bins. The lines represent QCD fits.
c) Analysis of moments of $xF_3^{\mu N}$

The BEBC and CDHS Collaborations have also performed an analysis of the moments of the non-singlet structure function $xF_3$ using combinations of BEBC/Gargamelle data and CDHS/SLAC data, respectively. The SLAC $F_2$ data multiplied by $9/5$ are used at high $x$ (where only valence quarks contribute) to fill a gap in the CDHS data. As in the electromagnetic case the moments are found to decrease strongly with increasing $Q^2$, the larger $n$ the steeper the $Q^2$ dependence quite in agreement with expectation.

Using a simple trick one can test the $n$ dependence of the powers $\delta_n^{\mu N}$. Remember, the QCD prediction for the $xF_3$ moments takes the simple form:

$$M_3(n,Q^2) = M_3(n,Q_0^2) \left( \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \right)^{n/2}$$

If the logarithm of a moment $m$ is plotted versus the logarithm of another moment, one expects a straight line:

$$\ln M_3(m,Q^2) = \frac{\delta_m^{\mu N}}{\delta_n^{\mu N}} \ln M_3(n,Q^2) + \text{const}$$

(2.17)

where the slope

$$\frac{\delta_m^{\mu N}}{\delta_n^{\mu N}} = \frac{1 - \frac{2}{m(m+1)} + 4 \frac{m}{2} \frac{1}{2}}{1 - \frac{2}{n(n+1)} + 4 \frac{n}{2} \frac{1}{2}}$$

(2.18)

is independent of the number of flavours $f$ and the mass scale $\Lambda$. Fig. 25 shows such a log-log plot for the two pairs of moments with $m = 6$, $n = 4$ and $m = 5$, $n = 3$. Clearly the data follow the expected linear relation, the slope predicted by QCD is indicated by the lines close to the data points.

Fig. 25 log-log plot of two Nachtmann moments. The slopes are predicted by QCD.
Fits of the slopes of various logarithmic moment combinations result in the \( \frac{d_m}{d_n} \) ratios collected in Fig. 26. The \( \frac{d_m}{d_n} \) values obtained by the CDHS collaboration for the relation between two Nachtmann moments are lower than those obtained for the relation between the ordinary \( x \) moments (also called Cornwall-Norton moments). The values determined from the BEBC/GGM data are somewhat higher than the corresponding values from CDHS. The solid horizontal lines indicate the prediction of Eq. (2.18). It turns out that all theories with spin 1 gluons would predict the same \( \frac{d_m}{d_n} \) ratios. Theories with scalar gluons would predict values indicated by the dashed lines. The data seem to prefer theories with vector gluons, in particular all experimental points are above the scalar gluon theory predictions.

A further leading order QCD test can be performed by taking the roots of the \( xF_3 \) moments:

\[
M(n, Q^2) = \ln \frac{Q^2}{\Lambda^2}
\]

(2.19)

Plotting \( M(n, Q^2) \) versus \( \ln Q^2 \) for various \( n \) should result in straight lines all with a common intercept \( \ln \Lambda^2 \). This is done in Fig. 27 for \( n = 3, 4, 5, 6 \); in part (a) for the ordinary and in part (b) for the Nachtmann moments. In both cases one observes good agreement with Eq. (2.19) but the numerical values of \( \Lambda \) obtained from fits are different giving some measure of the uncertainty of the procedure:

\[
\Lambda_{\text{Ordinary}} = 0.60 \pm 0.15 \text{ GeV} \\
\Lambda_{\text{Nachtmann}} = 0.33 \pm 0.10 \text{ GeV}
\]

Here, \( f = 4 \) flavors have been assumed.

Unfortunately, also these QCD tests, though looking very elegant on the first view, suffer from the ambiguities of corrections. To take this point to an extreme, Abbott et al.\(^5\) have shown that coherent effects alone (parametrized as in Eq. (2.13)) can completely fit the BEBC/GGM and CDHS neutrino data and also the SLAC-MIT electron data. The present measurements are unable to distinguish between \( 1/\ln Q^2 \) and \( 1/Q^2 \) or \( 1/Q^4 \) effects. For example, Fig. 28 shows \( xF_3 \) at various fixed values of \( Q^2 \) obtained from an interpolation of the CDHS measurements together with a QCD prediction (solid curves) and a fit including only coherent effects (dashed curves). Obviously, both fits describe well
the data. Furthermore, the slopes fitted from log moment versus log moment plots can be reproduced by pure higher twist phenomenology.

![Graphs showing moments of $x_F^3$ raised to the power $-1/d^2$ versus $Q^2$.](image)

**Fig. 27** Moments of $x_F^3$ raised to the power $-1/d^2$ versus $Q^2$.

![Graphs showing $x_F$ versus $x$ at various fixed $Q^2$.](image)

**Fig. 28** $x_F$ versus $x$ at various fixed $Q^2$ together with a QCD fit and a fit including only higher twist effects.
2.3.3 Remark concerning QCD corrections

All fully analyzed electromagnetic and neutrino data can be completely understood in terms of leading order QCD or even better in terms of leading and next to leading order QCD ignoring corrections due to coherent effects. Values of the mass scale $\Lambda$ obtained from leading order fits range from 0.33 to 0.74 GeV. Including next to leading order results in $\Lambda$ values from 0.20 to 0.46 GeV. For a summary see $^6$.1

On the other hand, most observed scaling violations can also be fitted by coherent effects alone. This is, by no means, an argument against QCD. But it might be dangerous to neglect such corrections. An argument against fits including only an ansatz for coherent effects comes from the rise of $F_2^{\nu\nu}$ with increasing $Q^2$ at large $\omega$ as observed by the CHIO Collaboration (Fig. 20).

Certainly, it is also possible to fit QCD and coherent effects simultaneously, for instance, by modifying $F_2$ in the following ways$^{3,1}$:

\[ F_2 = F_2^{\nu\nu} \left[ 1 + \frac{\mu_1^2}{(1-x)Q^2} \right], \quad i = 1, \]

or

\[ F_2 = F_2^{\nu\nu} \left[ 1 + \frac{\mu_2^2}{(1-x)^2Q^2} \right], \quad i = 2, \]

where $F_2^{\nu\nu}$ is composed of quark distribution functions obeying the Altarelli-Parisi Eqs. (2.6),(2.7). In Fig. 29 the correlation between $\Lambda$ and $\mu_1$ or $\mu_2$ is presented, obtained from fits to the SLAC $F_2^{\nu\nu}$ data. With increasing $\mu_i$ the fit values $\Lambda$ becomes smaller, e.g. for $\mu_1 = 1$ GeV $\Lambda \sim 0.1$ GeV. Without a solid calculation of coherent effects, the exact value of $\Lambda$ cannot be determined from the presently published data.

Fortunately, there are new muon experiments hoping to measure the electromagnetic structure functions at high $Q^2$ with high precision.

![Fig. 29 Correlation between $\Lambda$ and $\mu_1$, $\mu_2$ obtained from fits to the SLAC $F_2^{\nu\nu}$ data.](image)
2.3.4 Recent muon experiments at Fermilab and CERN

During the last year, three new muon experiments presented preliminary data on $F_2^{68,69,70)$. A common feature of the new measurements is the high luminosity which is achieved by the use of very massive targets and at CERN also by a higher quality muon beam. Compared to earlier muon experiments a gain in luminosity by a factor of 10 to 100 was achieved. A summary of these experiments is given in Table 6.

Table 6 Recent μ experiments at FNAL and CERN

<table>
<thead>
<tr>
<th>Experiment / Collaboration</th>
<th>Particles Analyzed</th>
<th>Apparatus</th>
<th>Target</th>
<th>Beam (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNAL E 203A/391 Berkeley-Fermilab-Princeton</td>
<td>$μ$, Hadronic Energy</td>
<td>Magnetized Calorimeter Spectrometer</td>
<td>Fe/Scin</td>
<td>213</td>
</tr>
<tr>
<td>CERN NA2 European Muon Collaboration</td>
<td>$μ$, $π^\pm$, $K^\pm$, $p$, $π^0$, $γ$</td>
<td>Air Gap Drift and Proportional Chamber Spectrometer, Lead Glass, Cerenkov</td>
<td>Fe/Scin</td>
<td>$H_2$, $D_2$</td>
</tr>
<tr>
<td>CERN NA4 Bologna-CERN-Dubna-Munich-Saclay</td>
<td>$μ$</td>
<td>Magnetized Iron Torus (surrounding target)</td>
<td>C</td>
<td>250</td>
</tr>
</tbody>
</table>

As an example Fig. 30 shows the spectrometer of the European Muon Collaboration (EMC). This instrument is capable of accepting and analyzing muons together with charged hadrons and photons produced over a wide range of angles and momenta. The target can be 6 m liquid
hydrogen or deuterium, 2 kg/cm² iron-scintillator calorimeter or polarized butanol. The central part of the detector is a large air gap magnet. A system of drift and proportional chambers before, inside, and behind the magnet determines the particle tracks. Muons are identified by the ability to traverse a 2.5 m thick iron absorber and to produce a track in the down-stream drift chambers. Scintillation counter hodoscopes behind the magnet and the absorber allow to trigger on selected muons. A threshold Čerenkov counter filled either with nitrogen or with neon identifies pions or kaons.

The following figures present some preliminary results on \( F_2 \) obtained by the two CERN groups.

Figs. 31 show \( F_{\gamma p}^P \) as measured by the EMC at two muon beam energies of 120 and 280 GeV plotted versus \( Q^2 \) at various fixed \( x \).

Scaling violations are clearly observed at small \( x \) (\( x < 0.1 \)) where \( F_2 \) increases with increasing \( Q^2 \). At large \( x \) one observes a slight decrease of \( F_2 \) with \( Q^2 \). The solid curves indicate a QCD fit based on the Altarelli-Parisi Equations which results in a relatively small value of \( \Lambda \approx 0.1 \) GeV.

Fig. 32 contains a comparison of \( F_{2N}^N \) data measured by the EMC with an iron/scintillator target\(^{71}\) and by the BCDMS Collaboration with a carbon target\(^{72}\). Both data
sets agree well within errors. At large $x$ one observes a small but significant decrease of $F_2$ with increasing $Q^2$. QCD fits to the EMC data again result in a small value of $\Lambda \sim 0.1$ GeV.

Since these data are preliminary, one has to be careful in drawing conclusions, but so much can be said: There is some tendency that $F_2$ becomes flatter at fixed $x$ with increasing $Q^2$. In principle such an effect is expected by QCD but the data seem to indicate even smaller values of the logarithmic slope than previously expected. If this should be confirmed it would indeed mean that the QCD mass scale $\Lambda$ is smaller than determined from earlier data at lower $Q^2$.

With a value of $\Lambda$ in the order of 0.1 GeV, $Q^2 \gg \Lambda^2$ at $Q^2 \sim 1$ GeV$^2$, i.e. perturbation theory works at values of $Q^2$ where the onset of approximate Bjorken scaling was originally observed in the SLAC experiments.

**SUMMARY**

1. Scaling violations in the manner predicted by QCD are observed in all electron, muon, and neutrino data.

2. The measured $Q^2$ dependence of the structure functions and their moments can be well fitted by QCD without any further corrections.

3. Still corrections due to coherent phenomena may be present and sizable. Therefore, it is not possible to determine precisely the mass scale parameter $\Lambda$ with the presently published data. To quote R.D. Field\(^\text{73}\), "Any $\Lambda$ in the range $0.01 < \Lambda < 0.8$ GeV is acceptable".

4. The variation of $F_2$ with $Q^2$ at high $Q^2$ as measured by recent muon experiments at CERN and Fermilab is considerably weaker than observed in earlier experiments at lower $Q^2$. This may indicate that $\Lambda$ is smaller than the previously favored value of about 0.5 GeV.
3. THE HADRONGIC FINAL STATE

Let us now turn to a discussion of some aspects of the hadronic final state produced in deep inelastic lepton nucleon scattering. The experimental information obtained at beam energies in the 100 GeV region comes in case of neutrino induced reactions from bubble chambers (BEBC, FNAL 15') and in case of muon reactions from large magnetic spectrometers (CHIO, EMC). In the 10 GeV region various electron and muon experiments using mainly spectrometers and/or streamer chambers have been performed at Cornell, DESY, and SLAC, an excellent summary of the results has been given by L.N. Hand at the Lepton-Photon Conference, Hamburg 197774).

At high energies the prominent feature is the development of jets with axes in the direction of the incident current ($\gamma$, $W^\pm$, $Z^0$) and with limited transverse momentum relative to the current axis. As an example, Fig. 33 shows a picture taken with BEBC in the 200 GeV narrow-band neutrino beam75).

We can understand the main characteristics in an extended quark parton model as sketched in Fig. 34.

The new aspect we have to add is the hypothesis of quark fragmentation describing the evolution of the interacting quark and the remaining spectator quarks into hadrons. By picking up an antiquark from a virtual $q\bar{q}$ pair the original fast quark is turned into a hadron, the remaining $q$ partner of the $q\bar{q}$ pair continues the procedure until all

Fig. 33 Charged current neutrino event taken with BEBC filled with a Ne/H$_2$ mixture.
energy is used up in the hadron cascade. The hadrons originating from the interacting quark are called current fragments, the hadrons connected with the spectator quarks are called target fragments.

What has been mostly studied are the distributions of the individually produced hadrons, i.e. the semi-inclusive reaction,

\[ e + N + e' + h + X. \]

Each hadron is then described by the following three variables:

a) One longitudinal variable, either

rapidity \[ y = \frac{1}{2} \ln \frac{E^* + p_{\parallel}^*}{E^* - p_{\parallel}^*} = \ln \frac{E^* + p_{\parallel}^*}{\sqrt{m^2 + p_T^2}}, \]

or

Feynman \[ x_F = \frac{2p_{\parallel}^*}{W}, \]

or

energy fraction \[ z = \frac{E^*}{W}, \]

where the indices * and L refer to the current nucleon c.m. system respectively to the laboratory system (for \( z > 0.1, x_F = z \) if \( p_T \) small);

b) two transverse variables, either the transverse momentum \( p_T \) with respect to the current axes and the azimuthal angle \( \phi \) around the current direction or the transverse momentum components in the lepton plane \( p_{\text{Tin}} \) and perpendicular to the lepton plane \( p_{\text{Tout}} \).

I shall now summarize some general properties of the final hadrons and then turn to a discussion of current fragmentation.
3.1 Rapidity Distributions

We consider a reaction with medium or small Bjorken $x$, i.e. sufficiently distinct from elastic processes. When viewed in the c.m. frame immediately after the interaction, a single quark with rapidity $y \approx \frac{1}{2} \ln s$, where $s = W^2$ in GeV$^2$, moves in the current direction while the spectator quarks move in the opposite direction (for reasons of simplicity we assume here $p_T^2 + p_T^2 = 1$ GeV$^2$).

The hadronic final state arising from this configuration is supposed to show quite generally a rapidity distribution of Fig. 36.

We can distinguish five regions. The target fragmentation region (low $y$) depends only on the properties of the target, one would expect to observe the same situation here as in ordinary hadronic reactions (e.g. pp reactions at the ISR), where e.g. the length of the target fragmentation region is of order $\Delta y = 2$. The adjacent hadron plateau containing

Fig. 35 Definition of variables.

Fig. 36 Shape of rapidity distribution.
soft hadrons from current fragmentation should again be characteristic of hadronic collisions. The current fragmentation region at high \( y \) depends only on the details of the quark fragmentation process. One also expects to find a current plateau whose height should be roughly the height of the hadron plateau but must not necessarily agree with it. The observation of two plateaus with different heights would indeed be characteristic of deep inelastic lepton scattering and indicate that the object moving immediately after the hard interaction to the left has a different structure than the one moving to the right.

Since a plateau can only be observed if the rapidity range \( \Delta y > 4 \), one expects to see such features only for \( W \gtrsim 8 \text{ GeV} \). Fig. 37 shows results from measurements of \( \nu p \) reactions with BEBC, in part (a) the rapidity distribution for positive hadrons, in part (b) the difference between positive and negative hadron distributions for \( 8 \text{ GeV} < W < 16 \text{ GeV} \) is plotted. In Fig. 37b one finds indeed a steep at \( y^* = 0 \) indicating a difference between current and hadron plateau and suggesting that a separation between current and target fragments should be performed in the c.m. system.

3.2 Quark Fragmentation

We now turn to a more detailed discussion of quark fragmentation. Since the struck quark moves away from the spectator quarks with large momentum, one assumes in the quark parton model that the fragmentation is independent of the mechanism by which energy is transferred to the quark and obeys:

1. Scaling in \( z \). The only variable describing the fragmentation into a hadron is the energy fraction \( z \), i.e. the ratio of hadron energy to quark energy.

2. Environmental independence. The \( z \) distribution of hadrons of type \( h \) is given by

\[
\frac{dN^h}{dz} = \frac{1}{\sigma_{tot}} \frac{d \sigma^h}{dz} = \sum_i \epsilon_i \eta^h_i(z),
\]

where \( \epsilon_i \) is the probability that the fragmenting quark is of type \( i \). \( \eta^h_i(z) \) represents the probability for a quark of type \( i \) to fragment into a hadron \( h \) with energy fraction between \( z \) and \( z + dz \).

Eq. (3.1) gives a powerful tool for comparing the hadron distributions observed in various processes which, at a given \( z \), should only depend on the probability \( \epsilon_i \) for fast quark generation. The \( \epsilon_i \) for various relevant reactions are collected in Table 7.

![Fig. 37](image-url) Rapidity distributions of particles per event and unit rapidity in the c.m. system, \( 8 < W < 16 \text{ GeV} \).
In electro and neutrino-production, the $\epsilon_i$ depend on the quark distribution functions.

Most of the produced hadrons are pions. From isospin and charge conjugation invariance one can obtain the following relations between the quark fragmentation functions which greatly simplify comparisons with the data:

$$
\begin{align*}
D_u^+ &= D_d^- = D_d^+ = D_u^- \\
D_d^+ &= D_u^- = D_d^+ = D_d^- \\
D_s^+ &= D_s^- = D_s^+ = D_s^- .
\end{align*}
\tag{3.2}
$$

We see, for instance, that

$$
\frac{dN^+}{dz} (\nu p) = \frac{dN^+}{dz} (\nu n) = \frac{dN^-}{dz} (\overline{\nu} n) = \frac{dN^-}{dz} (\overline{\nu} N) = D_u^+ (z)
$$

and

$$
\frac{dN^-}{dz} (\nu p) = \frac{dN^-}{dz} (\nu n) = \frac{dN^+}{dz} (\overline{\nu} n) = \frac{dN^+}{dz} (\overline{\nu} N) = D_u^- (z) .
$$

Such relations are in good agreement with the data shown in Fig. 38. It should be remarked that the data plotted in Fig. 38 are for charged hadrons and thus even in the quark parton model only approximate equality for $\nu + h^+$ and $\overline{\nu} + h^-$ etc. is expected.

It is also satisfying to find the ratio $D_u^+/D_u^- > 1$ with tendency to increase with $z$, since an $u$ quark is contained in the $\pi^+$ but not in the $\pi^-$. The dash-dotted curve shows a quantitative quark parton model description of Field and Feynman\textsuperscript{80}.

Environmental independence allows a direct comparison of neutrino-production, muo-

production, and $e^+ e^-$ annihilation. With a little calculation one can prove:

$$
D_u^+ + D_u^- = \left[ \frac{dN^+}{dz} + \frac{dN^-}{dz} \right]_n N = \left[ \frac{dN^+}{dz} + \frac{dN^-}{dz} \right]_n N \approx \left[ \frac{dN^+}{dz} + \frac{dN^-}{dz} \right]_{e^+ e^-} .
\tag{3.3}
$$
Fig. 38 Compilation of neutrino data, $\frac{dN}{dz}$ versus $z$ for (a) $\nu N \rightarrow \mu^+ h^- X$ and $\bar{\nu} N \rightarrow \mu^- h^+ X$, (b) $\nu N \rightarrow \mu^+ h^- X$ and $\bar{\nu} N \rightarrow \mu^- h^+ X$. The figure is taken from Ref. 78. The differential multiplicity $\frac{dN}{dz}$ defined in Eq. (3.1) is equal to the quantity $\frac{1}{N_{\text{event}}} \frac{dN}{dz}$ used in Ref. 78.

Here the contributions from $s$ and $c$ quarks have been neglected. Fig. 39 shows the $z$ distributions of charged hadrons measured for the various processes. For the case of $e^+ e^-$ annihilation, the nearly equivalent quantity $\frac{dN}{dX||}$ obtained from an analysis of one of the two jets is plotted, $X|| = 2 p || / \sqrt{s}$ being the scaled momentum transfer along the jet axis. Note that absolute values of the differential multiplicities are compared. The general agreement is certainly striking.
In electron (or muon) induced reactions the production of hadrons of a fixed charge depends on Bjorken $x$ in the way predicted by Eq. (3.1) and Table 7. As an example, let us consider the charge ratio $\pi^+/\pi^-$. In the region of large $x$, where only valence quark scattering is important, one would expect:

$$\frac{N^+}{N^-} = \frac{4 u(x) n(z) + d(x)}{4 u(x) + d(x) n(z)}$$

for $ep$, and

$$\frac{N^+}{N^-} = \frac{4 d(x) n(z) + u(x)}{4 d(x) + u(x) n(z)}$$

for $en$, where

$$n(z) = \frac{D^+}{D^-}(z) .$$

In the region of very small $x$, where only sea quark scattering is important, $N^+/N^-=1$.

From Eq. (3.4) one expects an excess of positive charges for $ep$ scattering. But also for $en$ scattering, where a neutral current hits a neutral target, an excess of positive charges is expected as long as $u(x) < 4 d(x)$.

Fig. 40 shows a collection of charge ratios measured in electro and muoproduction plotted as a function of $\omega = 1/x$.

The FNAL data are for $0.3 < z < 0.85$, the SLAC electron data are for $0.4 < z < 0.85$ which may explain the higher $N^+/N^-$ values. The electron data from DESY, SLAC, and Cornell are $\pi^+/\pi^-$ ratios, the FNAL and SLAC muon data are just for charged hadrons.

So some differences between the various measurements are no surprise. However, the tendency is clearly what is expected from the quark parton model (solid curves), in particular there are at least indications of a small excess of positive charges for the neutron target at $\omega$ near 10. This is hard to explain without fractionally charged quarks.

Fig. 40 Charge ratios as a function of $\omega = 1/x$ for proton, deuteron, and neutron. The curves represent a quark parton model prediction. The figure is taken from Ref. 81.
3.3 QCD Effects of Quark Fragmentation

Gluon radiation due to the processes of Fig. 41 where the hadron \( h \) could originate from the outgoing quark or hard gluon results in modifications of our simple picture. If the transverse momentum of the emitted gluon (transverse with respect to the current momentum) is sufficiently large then the quark and gluon will be the parents of two distinct jets. If the transverse momentum is small, there will appear only one jet but with a \( z \) distribution of the final hadrons depending on \( Q^2 \). Similar as in the case of the quark distribution functions the probability to find a high \( z \) hadron is reduced, the probability to find a small \( z \) hadron is increased at \( Q^2 \) larger than some reference \( Q^2_0 \). In leading order QCD it is possible to describe the \( Q^2 \) dependence by quark and gluon fragmentation functions \( D^q(z,Q^2) \) and \( D^g(z,Q^2) \) obeying integro differential equations of the Altarelli-Parisi type\(^{86}\). Fig. 42a shows the predicted \( Q^2 \) variation for the fragmentation of an \( u \) quark in a \( \pi^0 \), i.e. \( zD_u^{\pi^0}(z,Q^2) \). The distribution at the reference value \( Q^2_0 = 4 \) GeV\(^2 \) is used as an input into the calculation. Fig. 42b shows the same for the fragmentation of a gluon into a \( \pi^0 \). Though the expected \( Q^2 \) dependence of \( D_u^{\pi^0}(z,Q^2) \) shows similarities to \( F_2(x,Q^2) \) the predicted scaling violations are smaller than for \( F_2 \).

Experimentally the breaking of \( z \) scaling was investigated by the neutrino bubble chamber collaborations. Fig. 43 presents BEBC data\(^{87}\) for positive and negative hadrons. Plotted is

\[
D^h_\pm(z,Q^2) = \frac{dN^\pm_\pm(z,Q^2)}{dz}
\]
as a function of \( z \) for low and high \( Q^2 \). Part (a) is for all hadrons, Part (b) for those hadrons going forward in the current nucleon c.m. frame. In both cases one observes

\[ zD(z, Q^2) \text{ VERSUS } z \]

\[ (a) u = \pi^+ \quad \Lambda = 0.4 \]
\[ \text{Q} = 10 \]
\[ \text{Q} = 50 \]
\[ \text{Q} = 500 \]
\[ (b) \text{gluon} = \pi^+ \quad \Lambda = 0.4 \]
\[ \text{Q} = 10 \]
\[ \text{Q} = 50 \]
\[ \text{Q} = 500 \]

**Fig. 42** Expected \( Q^2 \) variation of parton fragmentation function\(^{50} \), (a) for u quark into \( \pi^0 (D^0_{u} (z, Q^2) = \frac{1}{2} (D^+_u (z, Q^2) + D^-_u (z, Q^2))) \), (b) for gluon into \( \pi^0 \). The \( z \) distributions at high \( Q^2 \) are calculated from the \( z \) distribution at the reference value \( Q_{0}^2 = 4 \text{ GeV}^2 \).

This effect is clearly seen if one calculates the moments of the fragmentation functions \( D^H_{Q}(z, Q^2) \) defined as:
\[ \mathcal{D}^h(m, Q^2) = \int_0^1 dz z^{m-1} \mathcal{D}^+_q(z, Q^2) \]  

(3.5)

For the non-singlet fragmentation functions like

\[ \mathcal{D}^{\text{NS}} = \mathcal{D}^{h^+}_u - \mathcal{D}^{h^-}_u = \mathcal{D}^{h^+}_u - \mathcal{D}^{h^+}_u, \]

(applying charge conjugation invariance) QCD predicts in leading order

\[ \mathcal{D}^{\text{NS}}(m, Q^2) = \mathcal{D}^{\text{NS}}(m, Q^2) \left[ \frac{\ln Q^2/A^2}{\ln Q^2/A^2} \right], \]  

(3.6)

where the \( \mathcal{D}^{\text{NS}}_m \) are given by Eq. (2.11). Like the \( F_2 \) moments the D moments are expected to decrease with increasing \( Q^2 \). The results obtained for the BEBC data, using only events with \( W > 4 \) GeV, are shown in Fig. 44 for various orders \( m \). No \( Q^2 \) dependence remains. A careful discussion of all details can be found in[78].

![BEBC vp](image)

**Fig. 44** Non-singlet moments \( \mathcal{D}^{\text{NS}}(m, Q^2) \) determined from events with \( W > 4 \) GeV plotted versus \( Q^2 \) for various \( m \).

An impression of the amount of information that can be obtained from muon experiments with regard to the \( Q^2 \) variation of the fragmentation functions is given in Fig. 45 where EMC results for

\[ \frac{dN^{h^+}}{dz} + \frac{dN^{h^-}}{dz} \]

are plotted versus \( z \) for various \( Q^2 \) and \( x \) bins[88]. Also these data show no obvious \( Q^2 \) dependence.

To summarize the present situation: QCD effects of parton fragmentation functions have not been firmly observed yet, this is an active field of investigation.
Fig. 45 Differential multiplicities
\[ \frac{dN^+}{dz} + \frac{dN^-}{dz} \]
versus \( z \) for various \( x \) and \( Q^2 \) ranges.

3.4 Hadron Transverse Momentum

The study of the transverse momentum in semi-inclusive hadron production has so far revealed the clearest indications of QCD effects in deep inelastic leptoproduction. Before discussing the data, let us first list the possible sources of transverse momentum:

1. The initial partons inside the target nucleons may have some primordial transverse momentum \( k_T^0 \). In the quark parton model this contributes a term of the form \( \langle k_T^0 \rangle z^2 \) to the average transverse momentum squared of the final hadrons. The factor \( z^2 \) is due to the fact that only the fraction \( z \) of the parton momentum is transferred to the hadron.

2. A further contribution \( P_{T,QCD} \) to the transverse momentum of the parent parton with respect to the current axis is due to gluon radiation or quark-antiquark pair production from a gluon according to the diagrams of Fig. 41.

3. The observed hadron originating from the fragmentation cascade acquires a transverse momentum \( P_{T,Frag} \) with respect to the momentum of the initiating parton. A good guess for the average value, including possible effects of decays of originally created vector mesons into
some of the later detected pseudo-scalar mesons is \( \langle p_T \rangle_{\text{frag}} = 0.32 \text{ MeV}^{80} \).

The average \( p_T^2 \) of the detected hadron is thus the sum of three terms\(^9\):

\[
\langle p_T^2 \rangle = \langle k_T^2 \rangle + \langle p_T^2 \rangle_{\text{QCD}} z^2 + \langle p_T^2 \rangle_{\text{frag}}.
\]  

(3.7)

Of these only the QCD term can be calculated exactly in lowest order perturbation theory. In the fundamental two body processes, e.g. \( \gamma \gamma + q + q + g \), higher \( p_T \) are generated with increasing \( W \), and one obtains roughly\(^80\)

\[
\langle p_T^2 \rangle_{\text{QCD}} = W^2 / \ln \frac{Q^2}{\Lambda^2},
\]

where an averaging over some region of \( x \) is included. The non-perturbative QCD contributions \( \langle p_T^2 \rangle_{\text{frag}} \) and \( \langle k_T^2 \rangle \) can be assumed to be nearly independent of \( W^2 \).

From Eq. (3.7) one expects an increase of \( \langle p_T^2 \rangle \) with increasing \( z \). This is known as the seagull effect (the name is due to the fact that by plotting \( \langle p_T \rangle \) versus \( x_F \) one gets a figure looking like a seagull). I will show here recent EMC data taken at rather large \( W \) (Fig. 46). Beside the clear rise of \( \langle p_T^2 \rangle \) with \( z \) there is also a rise with \( W \). Note the large value of \( \langle p_T^2 \rangle = 1 \text{ GeV}^2 \) at \( z = 0.75 \) and \( \langle W^2 \rangle = 385 \text{ GeV}^2 \).

The rise of \( \langle p_T^2 \rangle \) with \( W^2 \) is also presented in Fig. 47 where data measured by a BEBC collaboration\(^3\) and the EMC\(^4\) are plotted for various \( z \) intervals. The curves show the \( W^2 \) dependence of the QCD contribution \( \langle p_T^2 \rangle_{\text{QCD}} \). They are obtained by adding to the \( W \)-dependent QCD part \( \langle p_T^2 \rangle_{\text{QCD}} \) a \( W \)-independent non-perturbative contribution \( \langle p_T^2 \rangle_{\text{NP}} \) with the latter being separately fitted for each \( z \) interval. Most of the rise of \( \langle p_T^2 \rangle \) with \( W^2 \) can be attributed to the QCD term.

The dependence of \( \langle p_T^2 \rangle \) on \( Q^2 \) is, on the other hand, rather weak as can be seen from Fig. 48. The curves in Fig. 48 are obtained in the same way as for Fig. 47 and describe again well the observed behaviour.

To summarize, the measured dependence of \( \langle p_T^2 \rangle \) on \( W^2, Q^2 \) and \( z \) agrees well with expectations from QCD.

Fig. 46 \( \langle p_T^2 \rangle \) versus \( z \) for an average \( W^2 \)
of 100 and 385 GeV\(^2\).
Fig. 47 $W^2$ dependence of $\langle p_T^2 \rangle$ for various $z$ intervals as measured by EMC and ABCDLOS. The curves are fits of the form $\langle p_T^2 \rangle_{QCD} + \langle p_T^2 \rangle_{NP}$, where $\langle p_T^2 \rangle_{NP} = \text{const}$ is fitted for each $z$ interval, and $\langle p_T^2 \rangle_{QCD}$ is calculated in leading order QCD.

Fig. 48 $Q^2$ dependence of $\langle p_T^2 \rangle$ for the same $z$ intervals. The curves are calculated using the same values of $\langle p_T^2 \rangle_{NP}$ as in Fig. 47.
3.5 Production of Charmed Mesons by Photon-Gluon Fusion

Muoproduction offers a unique possibility to study the production of heavy quarks from gluons. At \( Q^2 \gg \Lambda^2 \) (so that one can use perturbation theory) but \( Q^2 \) not too large (so that c-quark mass effects are still important), photon-gluon fusion according to the first order QCD process of Fig. 49 can be shown to be the most important source of heavy quark production.

The dynamics of this process can be completely calculated in QCD perturbation theory\(^9\). Once the parton cross section \( \sigma_{\gamma g \to cc} \) is known, one can calculate the cross section for \( \gamma p \to cc X \) as in the parton model by convoluting the parton cross section for a gluon carrying momentum fraction \( x \) of the target nucleon with the gluon distribution function \( g(x) \):

\[
\sigma(\gamma p \to cc X) = \int x_2 \ dx \ g(x) \ \sigma(\gamma g \to cc) .
\]  

(3.8)

For open charm production

\[
x_1 = \frac{Q^2 + 4 \ m_D^2}{2M} , \quad x_2 = 1 ,
\]

where \( m_D \) is the D meson mass. In model calculations the gluon distribution function is typically assumed to be

\[
g(x) = \frac{5}{x} \ (1-x)^5
\]

normalized such that the total momentum fraction carried by gluons is

\[
\int_0^1 dx \ x \ g(x) = 0.5 .
\]

Experimentally one can test this model by measuring muon induced multimun final states, e.g. dimuon events of the type \( \mu^+ N \to \mu^+ \mu^\pm X \). The produced \( \mu \) originates from the decay of a charmed meson \( D \to \mu \nu \bar{\nu} \) (branching ratio 10%). Here some energy is carried away by the
neutrino, so an experimental signature of such events should be the measurement of missing energy

$$E_{\text{miss}} = E_{\text{beam}} - E_{\nu_1} - E_{\nu_2} - E_{\text{calorimeter}}.$$  

In the EMC and BFP measurements $\langle E_{\text{miss}} \rangle = 20$ GeV.

After subtraction of various backgrounds, the most important being due to muons from decays of $\pi, k$ mesons produced in the event, one obtains the results shown in Fig. 50. Plotted are the differential cross sections for dimuon production in $Q^2$, $W$, $p_T$ (the transverse momentum of the produced $\mu$ relative to the virtual photon) and $z = E_2/\nu$ where $E_2$ is the energy of the produced $\mu$) using the kinematical cuts $y < 0.91$, $Q^2 > 1$ GeV$^2$, $\theta_1 > 7$ mrad, $E_{\nu_2} > 16$ GeV, $p_T^2 > 0.25$ GeV$^2$. These cuts are necessary to separate a clear dimuon signal due to charm production. The solid curves represent absolute predictions of the photon-gluon model for the same cuts in the kinematical variables. They agree very well with the data.

![Fig. 50](image)

Differential cross sections for $\mu^+N \rightarrow \mu^+\mu^+X$ in the kinematic region defined in the text. The background from $\pi, k$ decay has been subtracted. The curves present absolute values from the photon-gluon fusion model in the same kinematic region.

In the photon-gluon fusion model the energy of the virtual photon is on the average equally shared by the two $c$-quarks. One should therefore also observe trimuon events $\mu^+N \rightarrow \mu^+\mu^+X$ with large missing energy originating from $c\bar{c}$ production. A study of such events leads to the differential cross sections presented in Fig. 51. Again, the data are compared with model predictions using the same kinematical cuts as the data. The quantitative agreement is quite convincing.
To summarize, inelastic multimun events observed in high energy muon scattering support the hypothesis of heavy quark production by the QCD process of photon-gluon fusion.

Using the dimuon data it is possible to evaluate the contribution of $c\bar{c}$ production to the structure function $F_2$. The analysis of the Berkeley-Fermilab-Princeton Collaboration\(^{37}\) shows that the $c\bar{c}$ piece $F_2(\psi, \chi)$ of $F_2$ contributes up to 4%. However, the contribution of $F_2(\psi, \chi)$ to the scaling violation of $F_2$ at small $x < 0.1$ is relatively large, a detailed study shows that for $2 < Q^2 < 13$ GeV\(^2\) and $50 < y < 200$ GeV centered at $x \sim 0.025$ charm production is responsible for about 1/3 of the total scaling violation of $F_2$. Thus the opening of the charm channel can explain only a fraction of the observed increase of $F_2$ with $Q^2$ at small $x$, there is room for a clear QCD effect.

CONCLUDING REMARKS

Deep inelastic lepton nucleon scattering has proven to be the powerful tool for investigating the structure of hadrons. Most of our knowledge about parton distributions inside the nucleon and our only direct evidence of asymptotic freedom of QCD is due to such experiments.

From the analysis of the hadronic final state we started to understand how the energy transferred to the partons is converted to the final hadrons. The first indications of QCD
effects, e.g. the increase of $p_T^2$ with $W$ and charm production due to photon-gluon fusion became available during the last year. Still a number of questions remain to be answered, some of these are:

- Do fragmentation functions depend on $Q^2$ in the manner expected by QCD?
- What is the decomposition of the produced hadrons, what is the role of baryons?
- What are the correlations between the fast hadrons?
- Are jets originating from hard gluon emission observable at the present available energies?
- How do the spectator quarks fragment?

The high statistics muon experiments presently in progress will answer such questions in the near future.

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