CONF-800723--27

# STUDY OF TURBULENT NATURAL-CIRCULATION FLOW

# AND

# LOW-PRANDTL-NUMBER FORCED-CONVECTION FLOW

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by

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Prepared for

National Heat Transfer Conference

AIChE

July 29-30, 1980

U of C-AUA-USDOE

ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

Operated under Contract W-31-109-Eng-38 for the

U. S. DEPARTMENT OF ENERGY

# STUDY OF TURBULENT NATURAL-CIRCULATION FLOW AND LOW-PRANDTL-NUMBER FORCED-CONVECTION FLOW

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## ABSTRACT

Calculational methods and results are discussed for the coupled energy and momentum equations of turbulent natural circulation flow and low Prandtl number forced convection flow. The objective of this paper is to develop a calculational method for the study of the thermal-hydraulic behavior of coolant flowing in a liquid metal fast breeder reactor channel under natural circulation conditions. The two-equation turbulence model is used to evaluate the turbulent momentum transport property. Because the analogy between momentum transfer and heat transfer does not generally hold for low Prandtl number fluid and natural circulation flow conditions, the turbulent thermal conductivity is calculated independently using equations similar to the two-equation turbulence model. The numerical technique used in the calculation is the finite element method.

## NOMENCLATURE

- BF — ratio of body force to inertia force, $Gr/Re^2$
- C — constants
- $C_p$ — specific heat
- D — effective channel width
- f — driving force
- g — gravity force
- Gr — Grashof number, $\rho^2 g\beta\Delta T D^3/\mu^2$
- h — film coefficient
- H — enthalpy
- I — turbulent energy flux, $\frac{\overline{T'^2}}{2}$
- k — thermal conductivity
- K — turbulent kinetic energy, $\frac{\overline{u'^2} + \overline{v'^2}}{2}$
- Nu — Nusselt number, $\frac{hD}{k}$
- P — pressure
- Re — Reynolds number, $\frac{\rho U D}{\mu}$
- T — local mean temperature
- u — x-directional local mean velocity
- U — channel inlet velocity
- v — y-directional local mean velocity
- $\alpha$ — thermal diffusivity
- $\beta$ — thermal expansion coefficient
- $\sigma$ — constants
- $\varepsilon$ — rate of dissipation of turbulent kinetic energy
- $\eta$ — rate of dissipation of turbulent energy flux
- $\mu$ — absolute viscosity
- $\nu$ — kinematic viscosity
- $\rho$ — density

## SUBSCRIPTS

- primes — fluctuating turbulent values
- e — effective value
- i — inlet condition
- I — turbulent energy flux
- K — turbulent kinetic energy

t turbulent

x x-coordinate

y y-coordinate

$\mu$ viscosity

w hot wall

1,2,.. constant values

## INTRODUCTION

Natural circulation phenomena in reactors have received considerable attention for many years because of the reactor safety considerations associated with the self-sustained decay heat removal after reactor shutdown. There exists a substantial amount of literature reporting advancements in the study of decay heat removal by natural circulation at the reactor postscram condition [1,2,3]. The majority of the earlier research is limited to scoping-nature calculations using a large scale computer code or detailed analyses usually performed for laminar flow cases [4]. In the study of turbulent natural circulation phenomena, uncertainties related to the turbulent transport property evaluation pose a serious difficulty to obtaining analytical solutions. The methods for calculating turbulent flows are diversified [5 to 12]. The simplistic approach which has been used in many turbulence calculations employs a universal eddy diffusivity correlation based on the extrapolation of experimental data near the wall. However, when the flow pattern cannot be described by the well defined flow motion such as channel flow or boundary layer flow, this approach cannot be expected to provide a proper solution. Furthermore, not all boundary layers possess a turbulence structure near the wall which conforms with this universal law. Examples of nonconformance are turbulent structures under the influence of severe streamwise pressure gradients, mass influx into the stream, buoyancy, centrifugal or coriolis forces. As a recent development, extensive studies have been performed for flows with such complicated force balances by using the various turbulence models [6,8].

In the present study, the two-equation turbulence model [9 to 12] is employed to evaluate the turbulent momentum transport property for the combined natural circulation flow and the low Prandtl number forced convection flow. Furthermore, the turbulent thermal transport property is also calculated by using equations similar to the two-equation turbulence model.

## ANALYSIS

The main driving force for natural circulation flow in a reactor system is the temperature difference at various locations in the reactor loop. The body force generated by this temperature difference is large enough to accelerate flow up to the state of turbulent motion. The flowrate produced by natural circulation depends largely on the reactor power level. At postscram conditions, the flowrate is about 2-4% of the rated flowrate, and is approximately proportional to the reactor decay heat power. Because the density difference is the driving force for the natural circulation phenomena, flow properties are strongly temperature dependent. Furthermore, because the developed calculational method will be used for heat transfer calculation of irregularly shaped reactor structures, the finite element method is chosen as the calculational tool.

By the hypotheses of J. Boussinesq and Prandtl [5], the governing equations for a two-dimensional turbulent flow motion between parallel plates, with reference to the cartesian coordinate system x, y, can be written as:

Energy equation:

$$\rho\left(u\frac{\partial i}{\partial x} + v\frac{\partial i}{\partial y}\right) = \frac{\partial}{\partial x}\left(k_e\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_e\frac{\partial T}{\partial y}\right) \qquad (1)$$

where $i = C_pT$, and $\rho$, $C_p$, $k_e$, T, u and v are density, specific heat, effective heat conductivity, temperature, x-directional velocity, and y-directional velocity, respectively.

Continuity equation:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0. \qquad (2)$$

Momentum equations:

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = f_x + \frac{\partial}{\partial x}\left(\mu_e\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_e\frac{\partial u}{\partial y}\right) \qquad (3)$$

and

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = f_y + \frac{\partial}{\partial x}\left(\mu_e\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_e\frac{\partial v}{\partial y}\right), \qquad (4)$$

where $f_x$ and $f_y$ are the driving force terms and given by

$$f_x = -\frac{\partial p}{\partial x} + \rho g_x \beta (T - T_i) \tag{5}$$

and

$$f_y = -\frac{\partial p}{\partial y} + \rho g_y \beta (T - T_i), \tag{6}$$

where $\mu_e$, $\beta$ are the effective viscosity and the thermal expansion coefficients, and $g_x$ and $g_y$ are gravity force terms in the x and y directions, respectively. If the flow motion is caused by an external force such as pump torque, the driving force terms are represented by the pressure differential term only and the body force terms are generally neglected. However, if the buoyancy force is the main driving force or has the same order of magnitude as the external force, both the pressure differential term and the buoyancy force term should be kept in equations (5) and (6). For a vertical channel flow considered in the present study, $g_x = g$ and $g_y = 0$.

The viscosity term $\mu_e$ in the momentum equations and the thermal conductivity term $k_e$ in the energy equation are given by

$$\mu_e = \mu + \mu_t,$$

and $k_e = k + k_t$, respectively,

where $\mu_t$ and $k_t$ are the so-called turbulent viscosity and the turbulent thermal conductivity. The present study uses the two-equation turbulence model to calculate the turbulent viscosity $\mu_t$ and turbulent thermal conductivity $k_t$. The format of the two-equation model is not uniform for all physical problems. Further, the model varies from researcher to researcher. In general, when turbulence models or "closures" are used for calculating the turbulent transport phenomena, a length scale information of turbulence is required. This implies that explicit and implicit empirical assumptions have to be included in the model. Most of the existing turbulence models, therefore relate to homogeneous turbulence or isotropic turbulence. The limitations of such "closures" are well discussed in reference [12]. The present study employs the form proposed by Jones and Launder [9]. The following equations are normalized by inlet velocity U for velocities and by effective channel width D for length dimensions.

$$u\frac{\partial K}{\partial x} + v\frac{\partial K}{\partial y} = \frac{1}{Re}\left\{\frac{\partial}{\partial y}\left[\left(1 + \frac{\nu_t}{\sigma_K \nu}\right)\frac{\partial K}{\partial y}\right]\right\} + \frac{\partial}{\partial x}\left[\left(1 + \frac{\nu_t}{\sigma_K \nu}\right)\frac{\partial K}{\partial x}\right] + \frac{\nu_t U}{D}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] - \frac{D}{U}\varepsilon, \tag{7}$$

and

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{1}{Re}\left\{\frac{\partial}{\partial y}\left[\left(1 + \frac{\nu_t}{\sigma_\varepsilon \nu}\right)\frac{\partial \varepsilon}{\partial y}\right]\right\} + \frac{\partial}{\partial x}\left[\left(1 + \frac{\nu_t}{\sigma_\varepsilon \nu}\right)\frac{\partial \varepsilon}{\partial x}\right] + \frac{C_1 U}{D}\frac{\varepsilon}{K}\nu_t\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] - \frac{DC_2\varepsilon^2}{UK}, \tag{8}$$

where K and $\varepsilon$ are the turbulent kinetic energy and the dissipation rate, respectively. The turbulent viscosity $\nu_t$ is defined as

$$\nu_t = C_\mu K^2/\varepsilon. \tag{9}$$

$C_\mu$, $C_1$, $C_2$, $\sigma_K$, $\sigma_\varepsilon$ are 0.09, 1.44, 1.92, 1.0, and 1.3, respectively. Some researchers suggested that these constants are functions of the Reynolds number.

If the Prandtl number is not far from unity, it can be simply assumed that $\nu_t = \alpha_t$. However, in some cases the analogy between the thermal transport and the mass transport cannot be applied. As expected, if the Prandtl number is too large or too small, the turbulent thermal diffusivity is not the same as the momentum diffusivity. For this case, particularly for the small Prandtl number case, researchers proposed a relationship between $\alpha_t$ and $\nu_t$ by introducing a multiplier which is a function of the Prandtl number, $\nu_t$, and $\nu$ [14,15]. However, if the profiles of velocity and temperature are distinctly different from each other because of the buoyancy force effect or some other physical effects, even the proposed multiplier cannot be applied. The present study calculates $\alpha_t$ by solving equations similar to the two-equation model given by equations (7,8). The following *equations similar to those in existing* studies [7,10,11], are used to determine $\alpha_t$:

$$u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} = \frac{1}{Pe}\left\{\frac{\partial}{\partial y}\left[\left(1 + \frac{\alpha_t}{\sigma_I \alpha}\right)\frac{\partial I}{\partial y}\right]\right\} + \frac{\partial}{\partial x}\left[\left(1 + \frac{\alpha_t}{\sigma_I \alpha}\right)\frac{\partial I}{\partial x}\right] + \frac{\alpha_t}{UD}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{D}{U}\eta, \tag{10}$$

and

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$$u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} = \frac{1}{Pe}\left\{\frac{\partial}{\partial y}\left[\left(1 + \frac{\alpha_t}{\sigma_\eta \alpha}\right)\frac{\partial \eta}{\partial y}\right] + \frac{\partial}{\partial x}\left[\left(1 + \frac{\alpha_t}{\sigma_\eta \alpha}\right)\frac{\partial \eta}{\partial x}\right]\right\} + \frac{C_3}{UD}\frac{\eta}{I}\alpha_t\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{DC_4}{UI}\eta^2, \quad (11)$$

where $\alpha_t$ is defined by

$$\alpha_t = C_K I^2/\eta, \text{ and} \quad (12)$$

$C_K$, $C_3$, $C_4$, $\sigma_I$, $\sigma_\eta$, are constants to be determined. I represents the turbulent energy flux and $\eta$ represents the turbulent energy dissipation rate. Because no data are available for the constant in equations (10-12), the same constants in the K-$\varepsilon$ equations are used for the I-$\eta$ equations. The basic philosophy underlying the "closures" or turbulence models for turbulent energy transport is very similar to that for turbulent momentum transport [10 to 12]. However, because the velocity information is an input to the energy equation, uncertainties included in the stress closure further accumulate in the turbulent energy calculation. Refinements should be made as more basic experimental data for thermal turbulence become available.

Because all governing equations are coupled for the natural circulation phenomenon, an iteration procedure should be used to obtain the solutions. The calculational method used for the present calculation is described in reference [16]. The essence of this calculational method is to apply the Galerkin principle to the governing equations with introduction of appropriate interpolation functions. After obtaining a set of matrix equations for each governing equation, the global matrix is formed by summing up individual matrices over all elements. The final solutions are obtained by solving the resultant matrix equations using the Gaussian elimination method.

## RESULTS AND DISCUSSION

There may exist several ways to improve the numerical stability and accuracy of the calculated results, for example, by modifying the proposed turbulence model to fit the physical problems of interest or by fitting the boundary conditions. In the present study, no particular measure was taken to improve the numerical results. Instead of using functional forms as used in earlier studies [6,8], previously given constant values were used in the present application of the two-equation turbulence model. Calculations were begun from the wall rather than using the turbulence model only for the turbulent core and applying the law of wall near the wall. At the wall, the turbulent kinetic energy and energy flux values were set to zero. Also the rate of dissipation for both momentum transfer and heat transfer were assumed to be zero.

The present study considers turbulent flow between parallel plates. The solution domain is divided into the large number of small but equal-sized elements (34 in the radial direction and 17 in the axial direction). For simplicity of calculation, a constant wall temperature boundary condition is selected so that the Grashof number can be controlled easily by chaning the wall temperature. Unless otherwise noted, y=0 indicates the hot wall and y=1 indicates the cold wall in the folloiwng Figures.

In Figure 1, velocity profiles for the fully developed natural circulation turbulent flow at various Grashof numbers (BF = Grashof number/Reynolds number$^2$) are depicted. The Reynolds number and the Prandtl number used in these calculations are 10000 and 0.0112, respectively. The velocity profile is apparently different from that of laminar natural circulation flow. Compared with the laminar natural circulation flow case [4,16], the asymmetrical behavior of the velocity profile observed in laminar flow cases is almost unnoticeable in the present results. It is not clear whether this difference is due to the active mixing behavior of turbulent flow or to a numerical error. Also notice that the velocity profile is rather smooth compared with the steep change from the laminar sublayer to the turbulent core as shown in previous results for pipes [17]. If the velocity profiles of parallel plates should be the same as that in cylindrical pipes, the results of the present study can be improved by applying the two-equation model only at the turbulent core region or adjusting the constant values used in the turbulence model. Considering that Jones and Launder [9] reported a fairly good agreement between the experimental data and calculated results for pipes using the same turbulence model, some of these changes may improve the solution. However, this type of fix is not recommended without justification supported by experimental data. Figures 2 and 3 show the calculated kinetic energy K for momentum transfer and the energy flux I for heat transfer, respectively. The general appearance

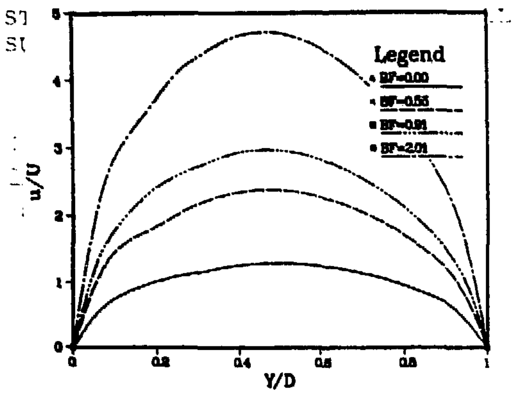


Figure 1. Effects of the Grashof Number on Velocity Profiles of Turbulent Natural Circulation Flow.

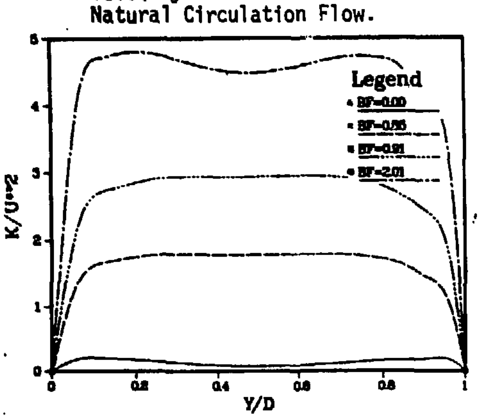


Figure 2. Effects of the Grashof Number on Turbulent Kinetic Energy of Natural Circulation Flow.

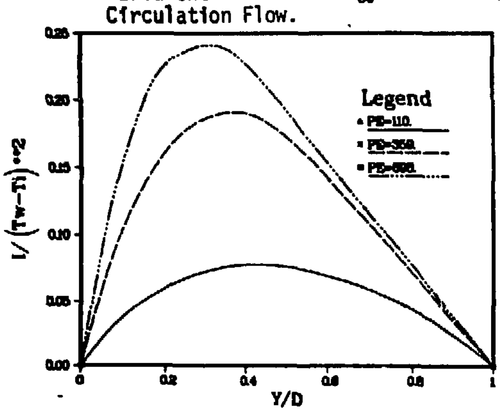


Figure 3. Effects of the Peclet Number on Turbulent Energy Flux I at Gr = 5.23*10**7.

of the kinetic energy for momentum transfer is in accord with the experimental data [17]. Note that the turbulent kinetic energy is normalized by the reference velocity U rather than the local average velocity (i.e., bulk velocity). Because of the significant acceleration by the buoyancy force, the turbulent kinetic energy under the buoyancy force effect is much larger than that of the forced convection only case. Due to the lack of experimental data for parallel geometries, any quantitative comparison should be delayed until further experimental data become available. Figure 4 shows heat transfer coefficients for the combined natural circulation and forced convection case. The purpose of this calculation was to investigate the effects of the Grashof number and the Peclet number on heat transfer coefficients. For all cases an approximate functional relationship is found to exist between the Grashof number and the Nusselt number. The relationship between the Grashof number and the Nusselt number can be expressed in the following form:

$$(Nu - Nu_f) \propto Gr^{0.4} \qquad (13)$$

where $Nu_f$ is the Nusselt number for the forced convection only case. On the other hand, as shown in Figure 4, the effect of the Prandtl number (or Peclet number) on the heat transfer coefficient is not as obvious as that of the Grashof number. There does not seem to exist a universal form of functional relationship between the Prandtl number and the Nusselt number as observed in the Grashof number variation case or the forced convection only case discussed below. The Grashof numbers used in this calculation are chosen

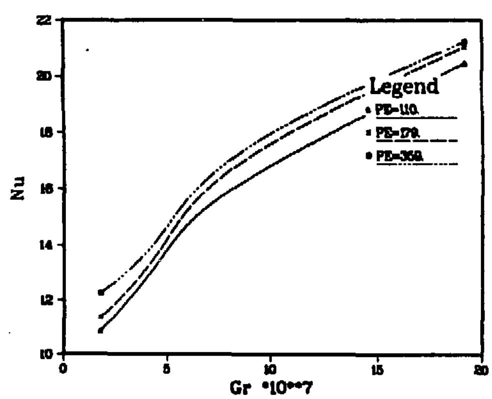


Figure 4. Grashof Number Effects on Nusselt Numbers at Different Pe Numbers.

arbitrarily so that the buoyancy effect is large enough to affect the flow behavior in spite of the high Reynolds number. The Prandtl number used in these calculations is ~0.01. In an actual reactor application, the Grashof number may be smaller than the number used in the calculation.

For the purpose of checking the proposed turbulence model, heat transfer coefficients were calculated for a fully developed forced convection heat transfer problem with a constant wall temperature boundary condition. Five different Peclet number cases (Pe = 110, 199, 359, 697, and 1794) are considered. In spite of the crude approximation, the results are in good agreement with the previous study [15]. The present study gives $N_u$ = 5.01, 5.25, 6.73, 8.20, 13.38, while Dwyer's correlation $N_u = 4.0 + 0.025\ Pe^{0.8}$ gives 5.07, 5.59, 6.77, 8.70, and 14.02 for the Peclet numbers listed above. Note that the boundary condition used in this calculation is the constant but unequal wall temperature condition. In Figure 5, the calculated Nusselt number for a developing temperature field problem is depicted along with the results of an existing study [13,14]. In this calculation, the velocity profile is assumed to be flat and the turbulent transport properly is obtained from equations (10-12) in order to compare with the existing studies. Compared with the earlier study [13], the present study results in higher values of the Nusselt number at the smaller Peclet number cases. However, at the large Peclet number case, these results yield ~~a smaller~~ Nusselt number. In the earlier calculation [13], Deissler's correlation for momentum diffusivity was used assuming that momentum diffusivity and thermal diffusivity are the same. The results of reference 14 were obtained by using the eddy diffusivity ratios of Azer and Chao [18]. Note that some of these referenced data were verified by the experimental data.

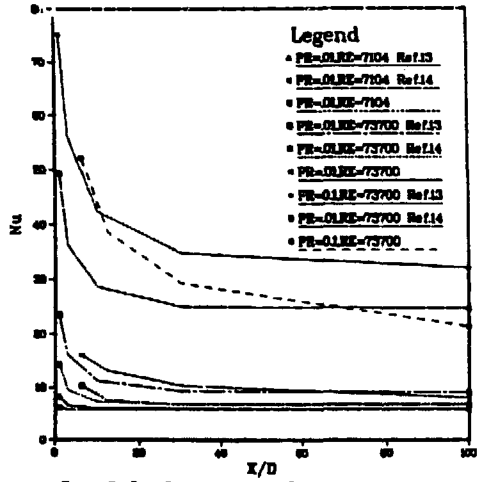


Figure 5. Calculated Nusselt Numbers for the Turbulent Thermal Entry Length Problem.

## CONCLUSIONS

The coupled energy and momentum equations of the fully developed turbulent natural circulation flow and the low Prandtl number forced convection heat transfer problem are solved using the two-equation turbulence model. In general, this turbulence model seems to work for the physical problems considered and the results obtained are in good accord with the results of the existing studies. However, any quantitative comparison is not warranted because of insufficient experimental and analytical work for turbulent natural circulation flow. For the verification of the calculational methods and the results, further experimental and analytical work is recommended.

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