

HEAT TRANSFER FROM INTERNALLY HEATED HEMISPHERICAL POOLS

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by

J.D. Gabor, P.G. Ellison, and J.C. Cassulo

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J. D. Gabor, P. G. Ellison, and J. C. Cassulo

Reactor Analysis and Safety Division  
ARGONNE NATIONAL LABORATORY  
9700 South Cass Avenue  
Argonne, IL 60439

## ABSTRACT

Experiments were conducted on heat transfer from internally heated  $ZnSO_4-H_2O$  pools to the walls of hemispherical containers. This experimental technique provides data for a heat transfer system that has to date been only theoretically treated. Three different sizes of copper hemispherical containers were used: 240, 280, and 320 mm in diameter. The pool container served both as a heat transfer surface and as an electrode. The opposing electrode was a copper disk, 50 mm in diameter located at the top of the pool in the center. The top surface of the pool was open to the atmosphere.

The heat transfer to the entire hemispherical surface was correlated by:

$$Nu = 0.55 Ra^{0.15} \quad 10^{10} < Ra < 6 \times 10^{11}$$

where the length parameter in both the Nusselt and Rayleigh numbers is the radius of curvature.

Experiments were also conducted in which the pool depth,  $H$ , was varied from  $0.5 R$  up to  $R$ . The data were correlated by

$$Nu = 0.55 Ra^{.15} (H/R)^{1.1} \quad 0.5 \leq H/R \leq 1.0$$
$$2 \times 10^{10} < Ra < 2 \times 10^{11}$$

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## NOMENCLATURE

$C_p$	specific heat
$g$	gravitational constant
$k$	thermal conductivity
$H$	pool depth
$Nu$	Nusselt number, $QR/k(T_p - T_s)$
$q$	volumetric heat generation rate
$Q$	heat flux
$R$	radius of curvature
$Ra$	Rayleigh number, $g\beta QR/\alpha vk$
$T_p$	maximum pool temperature
$T_s$	surface temperature

## Greek Symbols

$\alpha$	thermal diffusivity, $k/\rho C_p$
$\beta$	volumetric expansion coefficient
$\nu$	kinematic viscosity
$\rho$	density

## INTRODUCTION

The subject of heat transfer by natural convection from internally heated pools is of particular interest to nuclear reactor safety analysis, geophysics and chemical engineering. A variety of geometries for convective heat transfer from internally heated pools have been studied. However, experimental measurements on downward heat transfer in internally heated hemispherical pools have not been reported.

Min and Kulacki [1] measured heat transfer from the upper surface of a cylindrical fluid layer bounded from below by a segment of a sphere. The thermal boundary conditions were maintained for zero heat flux to the side wall and the curved bottom. They found that as the pool depth relative to its diameter increased, the heat transfer correlation approached that of a horizontal layer. Jahn and Reineke [2] theoretically and experimentally investigated free convection in rectangular and semicircular cavities. Gabor et al. [3] measured heat transfer to circular segments in the horizontal and downward directions.

Kulacki and Goldstein [4] and Kulacki and Emara [5] correlated upward and downward heat transfer in rectangular geometries. Martin [6], Murgatroyd and Watson [7], and Watson [8] analyzed free convective heat transfer in a vertical cylinder. Randall and Sesonske [9] investigated heat transfer in a horizontal cylinder.

These studies have been either for heat transfer to flat surfaces or to surfaces with a two-dimensional curvature. Mayinger et al. [10] addressed the problem of heat transfer to a three-dimensional curved surface with a numerical analysis of a heat transfer to the upper and lower surfaces of a hemispherical cavity. These calculations were for constant temperature surfaces. In this paper an experimental approach to this problem is presented.

## EXPERIMENTAL DESCRIPTION

Experiments were conducted with hemispherical pool containers of the type shown on Fig. 1. The pool containers were constructed from spun copper (2.4 mm thick). Three different sizes of hemispherical containers were used: 240 mm, 280 mm and 320 mm in diameter. Six equally spaced cooling coils of 9.5 mm diam. copper tubing were soldered to the back side of each of the containers. These coils were used to regulate the surface temperature of the hemispherical container and to measure heat fluxes to the container.

The pool container served both as a heat transfer surface and as an electrode.  $\text{ZnSO}_4\text{-H}_2\text{O}$  electrolyte (17.5 wt. %  $\text{ZnSO}_4$ ) was used as the heat generating liquid. The opposing electrode was a copper disk, 50 mm in diameter and 6.4 mm thick located at the center of the top surface of the pool.

Chromel-alumel thermocouples were used to measure the inlet and outlet temperatures of the cooling water. The cooling water flows were measured with Matheson #7630 rotometers. The temperatures on the container surface were measured by 13 thermocouples mounted at distributed points on the backside of the container. A Doric Trendicator Model 410K indicated the temperatures with a resolution of 0.1K. The outside of the container was covered by packing a 30 mm layer of WRP-X-AQ felt ( $\text{Al}_2\text{O}_3$  and  $\text{SiO}_2$  fibers) insulation (Refractory Products Company, Carpentersville, Illinois) to insulate the container and the cooling coils from the surroundings.

The experiments were conducted with the container wall maintained at a uniform temperature (within 1°K) by regulating the cooling water flows and inlet temperatures. The top surface of the pool was open to the atmosphere. Heat transfer measurements were made with all three containers filled to pool depths equal to the radius (hemispheres) and with ratios of pool depth to radius (H/R) of 0.5, 0.667, 0.813 and 1.0 in the 280 mm diam. container.

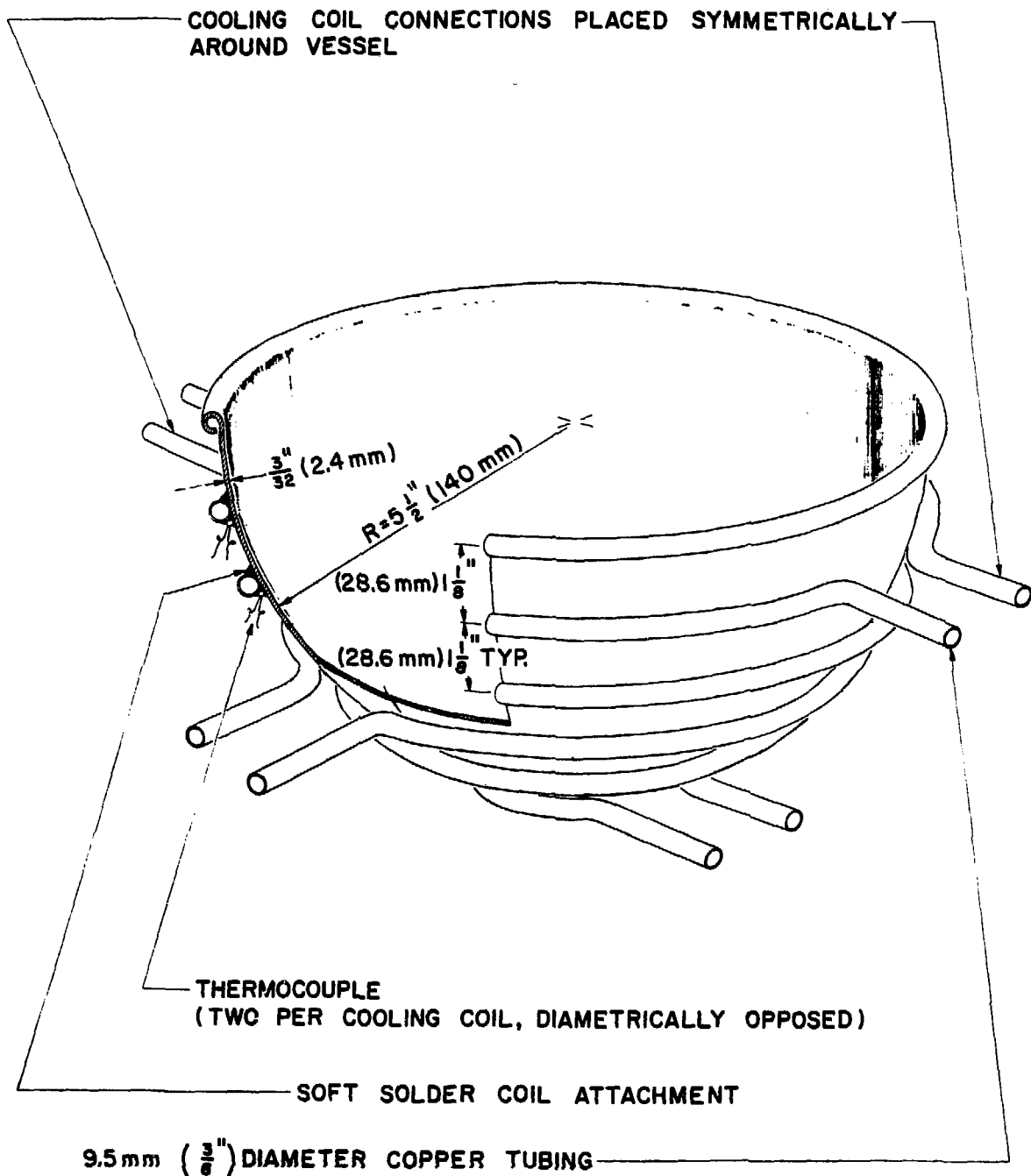


Fig. 1. Hemispherical Container for Internal Heat Generation Experiments.

This experimental technique for internal heat generation does not result in uniform heat generation throughout the pool since the electrical current density decreases radially from the center electrode to the wall. While these experiments do not precisely model analytical efforts based on uniform heat generation and nuclear applications, they do provide first of its kind information for this geometry. Min and Kulacki [1] also were constrained to operation with nonuniform heat generation with their cylindrical apparatus with spherical-segment bottom.

Electric resistance heating (which is the technique used for careful studies of convective heat transfer) even with geometries with equal sized electrodes which are uniformly spaced does not produce uniform internal heat generation because of variation in the electrical conductivity of the electrolyte with temperature. Peckover [11] addressed this problem for heat transfer to flat surfaces and related the heat transfer rates for uniform and nonuniform heat generation. Additional analytical work is needed on nonuniform heat generation which takes into account geometry and temperature effects to model practical experimentation.

## RESULTS

The data for each vessel size are tabulated in Tables 1, 2 and 3. These data are plotted on Fig. 2 in terms of Nusselt number as a function of Rayleigh number. The Nusselt number is defined in terms of the radius of curvature and the difference between the maximum temperature in the pool and the surface temperature:

$$Nu = QR/k(T_p - T_s)$$

The maximum pool temperature occurred at the center of the upper pool surface (near the upper electrode).

The Rayleigh number was also based on the radius of curvature:

$$Ra = g\beta qR^5/\alpha vk$$

The physical properties were determined from the average of the surface temperature and maximum pool temperature.

The heat transfer to the entire hemispherical surface was correlated by:

$$Nu = 0.55 Ra^{0.15} \quad 10^{10} < Ra < 6 \times 10^{11}$$

A numerical study by Mayinger et al., gave the following relationship for heat transfer to a hemispherical surface:

$$Nu = 0.55 Ra^{0.2} \quad 7 \times 10^6 \leq Ra \leq 5 \times 10^{14}$$

This correlation is plotted on Fig. 2 along with the current experimental data. The data lie below the curve obtained from the model of Mayinger et al.

Experiments were conducted with the 280-mm container in which the pool depth, H, was varied from 0.5 R up to 1.0 R (see Table 4). For these experiments the data were correlated by:



Table 1. Heat Transfer Data from an Internally-heated Hemispherical Pool  
Vessel Diameter = 320 mm

Run No.	Power kW	Max Pool Temp. °C	Wall Temp. °C	Heat flux W/m <sup>2</sup>	Nu	Ra
1	.320	34.6	20.2	1747	31.92	$1.09 \times 10^{11}$
2	.150	31.1	20.9	957	24.76	$5.84 \times 10^{10}$
3	.440	38.5	22.6	2301	37.76	$1.52 \times 10^{11}$
4	.575	49.1	24.3	3031	31.42	$2.19 \times 10^{11}$
5	.702	67.0	25.6	3865	23.54	$3.19 \times 10^{11}$
6	.870	69.3	26.8	4712	27.88	$3.98 \times 10^{11}$
7	1.024	59.6	19.3	4634	29.47	$3.59 \times 10^{11}$
8	1.260	69.2	20.6	6511	33.93	$5.53 \times 10^{11}$
9	.150	29.7	18.0	1168	26.28	$6.90 \times 10^{10}$
10	.238	33.4	18.0	1561	26.76	$9.56 \times 10^{10}$
11	.352	38.0	19.0	2031	27.99	$1.30 \times 10^{11}$
12	.440	43.2	19.8	2200	24.42	$1.49 \times 10^{11}$
13	.506	50.0	25.4	3225	33.63	$2.37 \times 10^{11}$
14	.552	46.4	23.1	3265	36.21	$2.31 \times 10^{11}$
15	.676	59.2	24.4	3735	27.38	$2.92 \times 10^{11}$
16	.840	62.3	26.7	4514	32.16	$3.63 \times 10^{11}$
17	1.020	68.9	27.1	4555	27.09	$3.82 \times 10^{11}$

Table 2. Heat Transfer Data from an Internally-heated Hemispherical Pool  
Vessel Diameter = 280 mm

Run No.	Power kW	Max Pool Temp. °C	Wall Temp. °C	Heat flux W/m <sup>2</sup>	Nu	Ea
42	.0814	27.3	18.6	917	24.18	$3.23 \times 10^{10}$
45	.058	23.6	17.1	566	19.92	$1.90 \times 10^{10}$
46	.0912	27.8	17.8	813	18.56	$2.87 \times 10^{10}$
47	.123	28.8	18.0	996	21.02	$3.56 \times 10^{10}$
48	.150	31.8	18.5	1246	21.22	$4.57 \times 10^{10}$
49	.470	53.7	20.1	3424	22.52	$1.56 \times 10^{11}$
50	.480	50.2	20.6	2907	21.74	$1.29 \times 10^{11}$
51	.112	28.1	17.9	1093	24.26	$3.85 \times 10^{10}$
52	.132	30.4	18.2	1326	24.61	$4.79 \times 10^{10}$
53	.160	33.7	18.3	1479	21.71	$5.52 \times 10^{10}$
54	.177	36.8	18.6	1748	21.62	$6.73 \times 10^{10}$
56	.123	30.2	18.3	1055	20.17	$3.83 \times 10^{10}$
57	.163	34.0	18.8	1171	17.36	$4.42 \times 10^{10}$
58	.194	35.3	19.0	1777	24.50	$6.82 \times 10^{10}$
59	.144	33.5	18.9	1280	19.72	$4.81 \times 10^{10}$
60	.104	31.4	18.6	1117	19.76	$4.09 \times 10^{10}$

Table 3. Heat Transfer Data from an Internally-heated Hemispherical Pool  
 Vessel Diameter = 240 mm

Run No.	Power kW	Max Pool Temp. °C	Wall Temp. °C	Heat flux W/m <sup>2</sup>	Nu	Ra
18	.352	48.1	21.2	3828	27.53	$8.69 \times 10^{10}$
19	.414	56.2	22.8	4630	26.50	$1.12 \times 10^{11}$
20	.500	56.5	23.9	5275	30.83	$1.28 \times 10^{11}$
21	.160	35.3	19.5	2054	25.48	$4.13 \times 10^{10}$
22	.238	40.2	18.6	2846	25.63	$5.96 \times 10^{10}$
23	.320	48.6	19.1	3571	23.36	$8.01 \times 10^{10}$
24	.1815	37.2	19.7	2406	26.80	$4.90 \times 10^{10}$
25	.273	44.7	20.0	2921	22.95	$6.35 \times 10^{10}$
26	.2565	43.9	19.6	3127	24.99	$6.75 \times 10^{10}$
27	.336	49.3	20.9	3841	26.04	$8.73 \times 10^{10}$
29	.1668	37.0	26.2	1415	25.36	$3.05 \times 10^{10}$
30	.272	43.1	26.8	2148	25.50	$4.92 \times 10^{10}$
31	.400	54.2	25.9	2920	19.66	$7.26 \times 10^{10}$
32	.0872	35.7	22.8	1588	23.90	$3.28 \times 10^{10}$
34	.054	25.9	18.4	686	18.14	$1.17 \times 10^{10}$
36	.0968	31.7	23.6	609	14.77	$1.24 \times 10^{10}$

Table 4. Heat Transfer Data for Varying Pool Depths  
 Vessel Diameter = 280 mm

Run No.	Power, W	H/R	Max Pool Temp. °C	Wall Temp. °C	Heat flux W/m <sup>2</sup>	Nu	Ra × 10 <sup>-10</sup>
70	66.0	.813	25.3	19.1	660	16.1	2.25
71	105.6	.813	28.8	19.6	1056	19.5	4.25
72	92.4	.813	27.3	19.1	924	19.8	5.51
73	61.2	.813	30.6	19.5	612	11.7	3.17
74	88.0	.813	31.0	19.7	880	14.3	3.93
75	36.4	.667	23.7	19.3	443	14.8	1.93
76	61.2	.667	26.7	19.7	745	18.4	3.64
77	80.0	.667	27.6	20.1	974	16.9	3.48
78	101.2	.667	31.8	20.3	1233	15.5	5.50
79	127.2	.667	33.2	20.8	1549	18.1	7.46
80	81.0	.500	33.3	19.9	1315	11.6	7.77
81	104.0	.500	37.0	20.9	1689	12.0	10.54
82	132.0	.500	40.6	21.3	2144	13.3	14.8
83	106.0	.500	38.4	21.3	1721	9.1	8.23
84	82.8	.500	37.5	21.2	1348	10.7	10.28

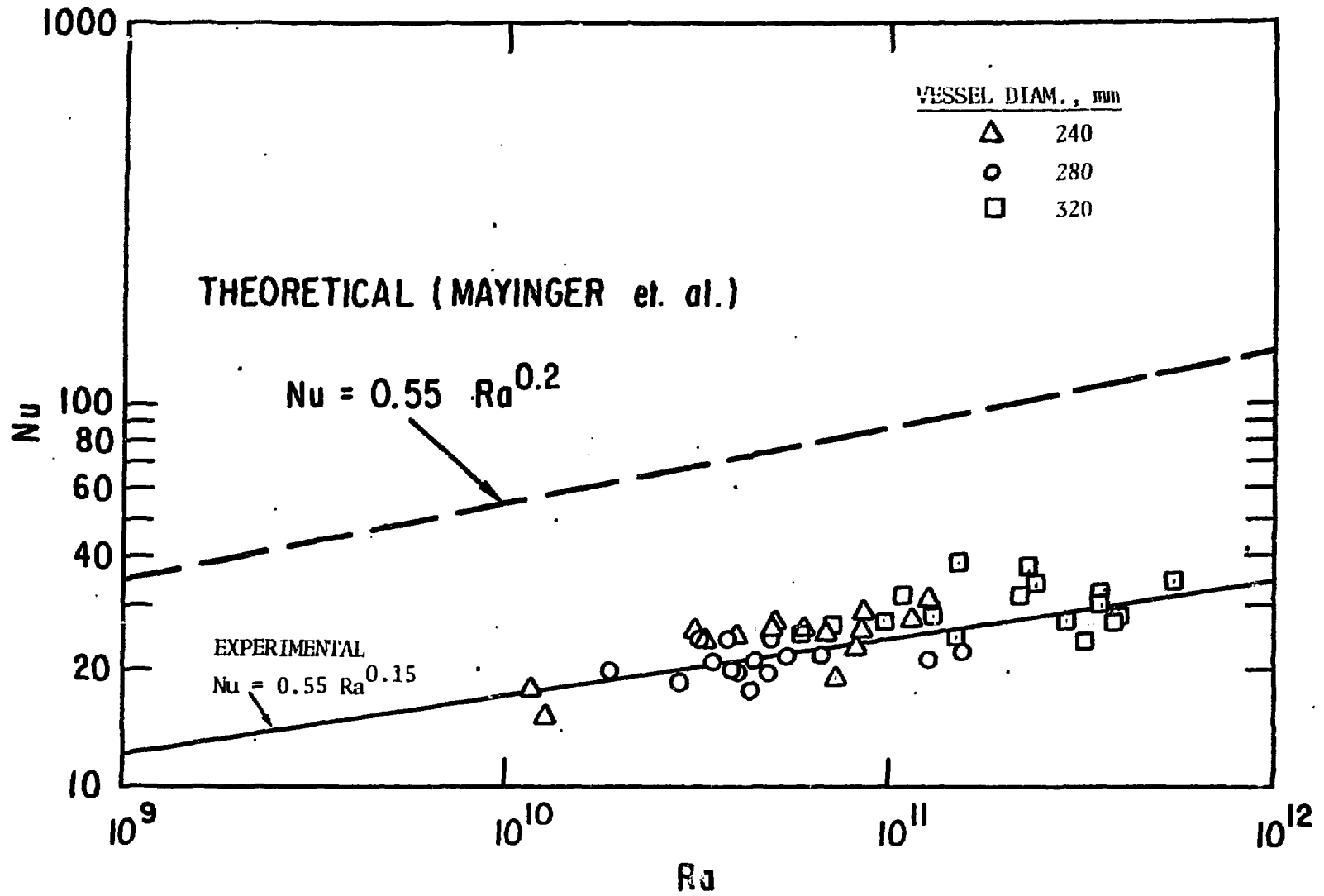


Fig. 2. Heat Transfer from a Heat-Generating Pool to Hemispherical Containers.

$$\text{Nu} = 0.55 \text{ Ra}^{0.15} (\text{H}/\text{R})^{1.1}$$

$$2 \times 10^{10} < \text{Ra} < 2 \times 10^{11}$$

$$0.5 \leq \text{H}/\text{R} \leq 1.0$$

The length parameter in both the Nusselt and Rayleigh numbers is again the radius of curvature (R). These data are plotted on Fig. 3.

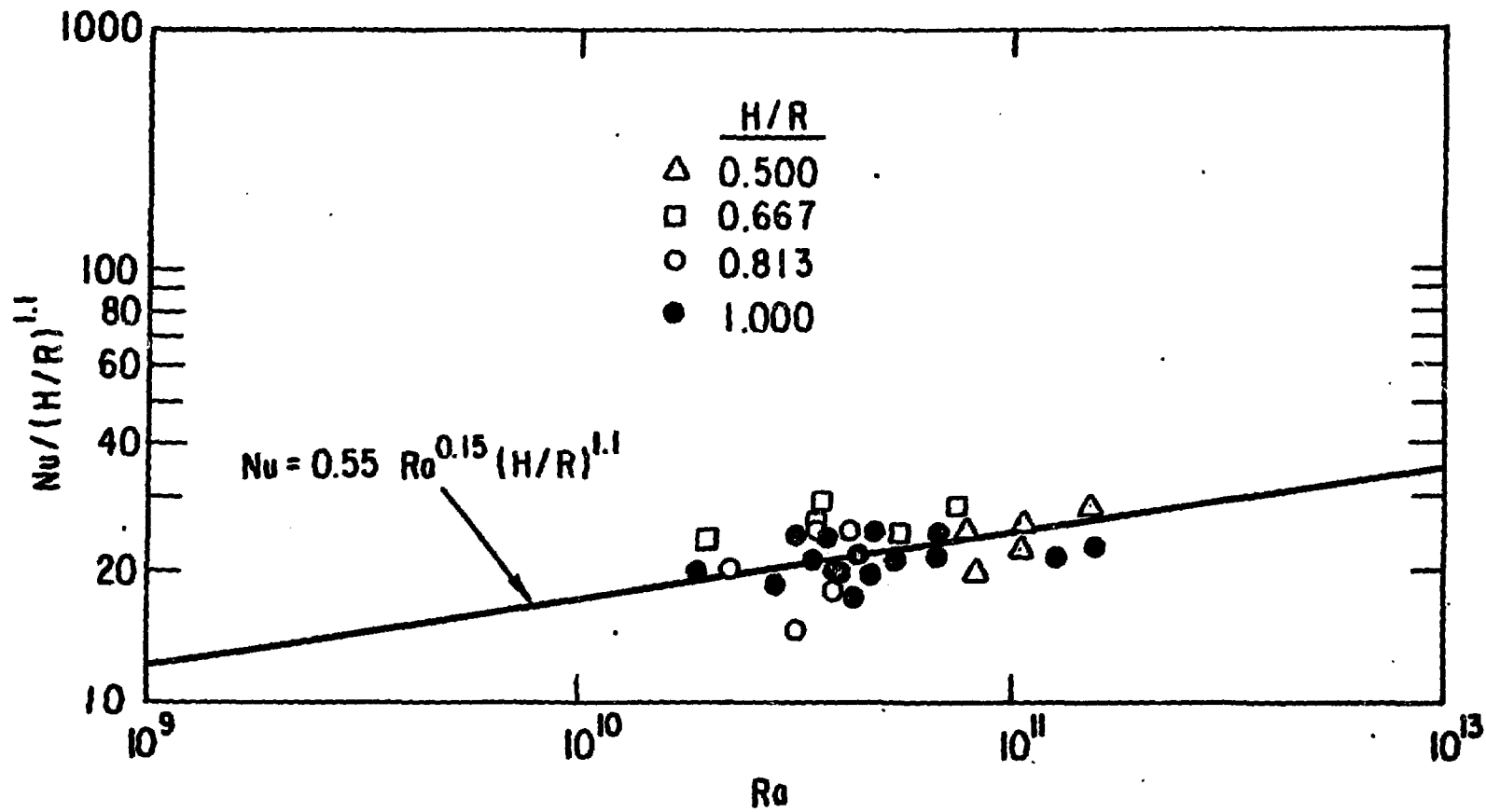


Fig. 3. Effect of Pool Depth on Heat Transfer.

## EXPERIMENTAL ERROR

Experimental error consists of errors in measurement (temperatures, power input, cooling water flow rates, and pool depth) and uncertainties in thermo-physical values. It is estimated that the total uncertainty in the Nusselt and Rayleigh numbers is in the order of 10% (cf. error estimates of Min and Kulacki [17]).



## DISCUSSION

The comparison of the data and the Mayinger et al. correlation should be considered in view of the differences of the experimental and theoretical models. The Mayinger et al. model is based on uniform heat generation throughout the pool whereas this was not achieved experimentally. In addition, the assumptions made in their model should also be considered which would lead to lower Nusselt numbers than would be experimentally observed. The model uses the concepts from forced convection which are inconsistent with free convection where there is no mean shear stress [12] and assumes that the effective thermal turbulent conductivity which applies in the core extends to the wall. At the wall the thermal conductivity approaches that of molecular conduction. This would lead to the prediction of higher heat transfer rates.

The heat transfer rates to the spherical segments were about one-half that given by correlations by Mayinger [10] and Gabor et al. [3] for downward heat transfer to circular segments for equivalent Rayleigh numbers.

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