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THE AXIAL POLARIZABILITY OF NUCLEONS AND NUCLEI

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ABSTRACT : The part of the static nuclear axial polarizability arising from the nucleonic excitations is derived from the low energy expansion of the πN amplitude. It is shown that the contribution of the Δ intermediate state, though dominant, does not saturate the nucleonic response. A similar effect, though more pronounced, is known to occur for the magnetic susceptibility.

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The recent and beautiful experiments on forward (p,n) reactions (Bainum et al 1980 ; Horen et al 1980), which have displayed collective Gamow-Teller states, have triggered a new interest in the Gamow-Teller transitions. As the magnetic transitions have indicated a lack of strength with respect to the sum rule (Knüpfer et al 1978 ; Papanicolas et al, to be published) suggesting a quenching of the magnetic moment of the nucleon inside the nuclear medium, it is interesting to see if this feature is general and if a similar quenching applies as well to the axial coupling constant g_A.

The first suggestion of the existence of a quenching of g_A came from the forward dispersion relations for the scattering of threshold pions on nuclei (Ericson 1971). In this approach the quenching was related to the shadowing of the total cross sections in the region of the 33 resonance. It showed the importance of the 33 resonance on the exchange effects of the axial coupling constant.

This approach based on dispersion relations was by nature restricted to a sum over all possible transitions and it was not clear whether the same quenching factor would apply to individual transitions. Nevertheless an analysis of the experimental data showed that an average quenching of g_A by $\approx 10\ \%$ was present for a series of transitions in light nuclei (Wilkinson 1973).

The next step applied instead to individual transitions: it was based on an analogy with solid state physics (Ericson et al 1973) . The nuclear medium was treated as polarizable, responding to the influence of an external spin isospin perturbation. Only the dipole dominant response was retained ; without explicit mention of the Δ it confirmed the importance of its role since this resonance dominates the axial dipole response.

The nucleonic response to such an excitation is linked to the p wave π nucleon amplitude, as illustrated by the following analogy (Ericson 1976) : a pionic wave represents a spin isospin perturbation since the πN coupling is of the form $\tau\ \vec{\sigma}.\vec{\nabla}$
The nucleonic response modifies the pion wave which becomes (for p wave pions of momentum $\vec{k}$)

$$\phi(\vec{x}) \sim e^{i\delta}\left[\cos\delta\ j_1(kx) + \sin\delta\ y_1(kx)\right]\cos\theta$$

where θ is the angle between $\vec{k}$ and $\vec{x}$, and δ the p wave phase shift. In the limit $\vec{k} \rightarrow 0$

$$\phi(\vec{x}) \sim \frac{1}{3}\left[\vec{k}.\vec{x} - \frac{3\delta}{k^3}\,\frac{\vec{k}.\vec{x}}{x^3}\right]$$

This expression ressembles to the modification of the electric potential V by a dielectric sphere (of electric susceptibility χ and radius R) embedded in a uniform field $\vec{E}$:

$$V(\vec{x}) = -\vec{E}.\vec{x} + \frac{4}{3}\pi R^3\,\frac{\vec{P}.\vec{x}}{x^3} = -\vec{E}.\vec{x} + \frac{4}{3}\pi R^3\chi\,\frac{\vec{E}.\vec{x}}{x^3}$$

where $\vec{P}$ is the polarization vector $\vec{P} = \chi\vec{E}$.

The axial nucleonic polarizability is therefore, as $k \rightarrow 0$: $a = \lim_{k\rightarrow 0} 3\delta/k^3 = -3c$ where c is the p wave scattering volume (In general a is related to the p wave amplitude f_p. An extrapolation of f_p in the unphysical region is then required to obtain $f_p\ (\omega = 0)$ and the static polarizability).

In the nuclear medium the nucleonic spin is surrounded by a polarization cloud of induced axial dipoles. If the medium has a granular structure as in a correlated medium there appear axial charges at the boundary of the correlation hole . This has the effect of renormalizing the axial coupling constant by a Lorentz-Lorenz factor, $1 + \alpha/3$, to first order in the unit volume polarizability α. The fact that p wave pions are attracted by the nuclear medium ($\alpha < 0$) implies a quenching of the axial coupling constant in heavy nuclei.

These ideas have been taken up and further pursued by a number of authors for the Gamow-Teller or the magnetic transitions (Rho 1974, Ohta and Wakamatsu 1974, Delorme et al 1976, Mukhopadhyay et al 1979). The factor $\frac{1}{3}$ appearing in the classical Lorentz-Lorenz effect, $(1 + \alpha/3)^{-1}$ is now replaced by its empirical value g' as appears in the Lorentz-Lorenz term of pion propagation* $(1 + g'\alpha)^{-1}$; it is also the Migdal parameter of the particle hole force which plays such an important role in the problem of pion condensation.

There is presently an evidence from other pieces of information (such as the position of unnatural parity states and the experiments on pp' scattering) that g' is large. Nevertheless it is interesting to extract an independant information from the quenching of the Gamow Teller or magnetic strength. The field is becoming mature enough to allow this possibility. This attempt has already been made (Oset and Rho 1979; Knüpfer et al 1980, Toki and Weise 1980) However the parameter g' always appears together with the polarizability α in the Lorentz Lorenz effect and the question is to what extent is this quantity known ?

Our ideas was to infer it from the p wave πN scattering while it has become popular to deduce it from an explicit model for the nucleonic axial polarizability, taken to originate in the virtual excitation of the Δ resonance alone. This model is attractive in the sense that it puts the nucleonic excitations (Δ-holes) and the nuclear excitations (N-holes) on the same grounds with both contributing to the nuclear polarizability. While this procedure gives the major part of the axial polarizability in a symmetric

*The identity of the g' value appearing in the vertex and propagator renormalizations is not rigorous ; it can be invalidated by the Pauli principle, see for instance Delorme et al (1976).

construction, the original approach in principle can encompass further physics. In the language of solid state physics the axial response divides into two parts : the "diamagnetic" response corresponding to highly excited states like the Δ and the "paramagnetic" response of low lying excitations of the particule hole type. The study of the "diamagnetic" part necessitates the knowledge of the off shell p wave scattering volume. If this amplitude is dominated by the Δ then the result should be the same as the Δ-hole model.

The real issue is therefore wether the inclusion of contributions other than the Δ significantly affect the p wave amplitude. This is the problem that will be treated in this letter. We will first discuss the axial response of a free nucleon and then the nucleonic part of the nuclear response for a system of non interacting nucleons.

Due to the spin and isospin of the nucleon the nucleonic polarizability is a tensor in space and isospin space. We discuss here the spin isospin average quantity related to the p wave scattering volumes α : $c_0 = \frac{1}{3}\left[\alpha_{11} + 2\alpha_{13} + 2\alpha_{31} + 4\alpha_{33}\right]$ with the notations $\alpha_{2T,2J}$ where T and J represent the total isospin and spin of the πN system. The combination c_0 is the one which appears in the Lorentz Lorenz effect, since all nucleons contribute coherently through this amplitude. We are concerned with its static value c_0 ($\omega = 0$). We will show below how it is derived from the πN amplitude $T = -A - i\gamma.q\,B$ The helicity non spin flip amplitude is :

$$C = A + \frac{\nu}{1 - t/4m_N^2} B$$

where $\nu = \dfrac{s - u}{4m_N} = \dfrac{q'.(p+p')}{2m_N}$

and q, q', p,p' are the pion and nucleon momenta.

This amplitude is expanded in ν and t. The nucleon pole term has a rapid energy variation in the unphysical region and it has to be separated out in order to perform the expansion. Note that the nucleonic excitations which build the polarizability enter in the non-Born part of the amplitude only. Defining

$$\tilde{C}^{+} = C^{+} - C^{+}_{pole}$$

with

$$C^{+}_{pole} = -\frac{g^2}{m_N}\,\frac{\nu^2}{(\nu^2-\nu_B^2)(1-\frac{t}{4m_N^2})}$$

where $\nu_B = -\frac{q.q'}{2m_N}$ and $g^2/4\pi = 14.6$

the quantity $\tilde{C}^{+}$ varies slowly as a function of ν and t and can be expanded around the symmetry crossing point $\nu = 0$.

$$\tilde{C}^{+}(\nu,t) = x_{00} + x_{10}\nu^2 + x_{01}t + x_{11}\nu^2 t + x_{12}\nu^2 t^2 + \ldots$$

The coefficients of this expansion are obtained from a fixed t dispersion relation (Höhler et al 1972) and a recent determination is given in the compilation of Nagels et al (1976). As pointed out by Höhler et al (1972) the error introduced by the extrapolation in the unphysical region is small since the integrals in ν are strongly convergent.

In the static situation, $\nu = (t-4m_\pi^2)/4m_N$ is small and we neglect all terms in ν^2 in the above expansion of $\tilde{C}^{+}$. The p wave amplitude is then the coefficient of $\cos\theta = (t-2m_\pi^2)/2m_\pi^2$ in the remaining expression. From the value(Nagels et al 1976) $x_{01} = 1.18 \pm 0.05$, one deduces the static polarizability $a(\omega = 0) = 4\pi C_0\ (\omega = 0) = 2.36\ m_\pi^{-3}$. This value is in complete agreement with Adler's determination (Adler, 1968).

As emphasized by Höhler et al (1972) the coefficients of this expansion are experimental in the sense that their errors are not larger than those of the physical scattering lengths and they can be used to test the validity of models for the πN scattering such as the Δ model. We are interested here in this confrontation for the coefficient x_{01} which provides the polarizability. In the Δ model its value depends on the $\pi N\Delta$ coupling constant ; with $f^2_{\pi N\Delta} / f^2_{\pi NN} = 4$ as deduced from the recoiless Chew Low model currently used in connection with pion condensation, one finds $x_{01} = 64\pi f^2_{\pi NN}/9\,\omega_r$ where ω_r is the pion energy at the 33 resonance. This corresponds to $a\,(\omega = 0) = 1.63\ m_\pi^{-3}$, sizably smaller thant the "experimental" value $2.36\ m_\pi^{-3}$. With recoil as was used by Delorme et al (1976) the agreement is somewhat improved : $a\,(\omega = 0) = 1.88\ m_\pi^{-3}$.

The deficiencies of the Δ model have been discussed by Höhler et al (1972) who pointed out the mediocre reproduction of the coefficient x_{01} by this model. Higher nucleon resonances can improve this coefficient as was shown by Chemtob and Rho in their work on exchange currents (Chemtob and Rho 1971). They used a phenomenological Lagrangian approach and they introduced beside the Δ_{33}, the N^*_{11} and N^*_{31}. In this way they obtained $a(\omega=0) = 1.91\ m_\pi^{-3}$ which they compared to Adler's value $2.36\ m_\pi^{-3}$. However the appeal of the Δ-hole model is largely lost with the addition of other resonances. Höhler and Stichel (1971) improved the Chew Low result with a substraction procedure which incorporates instead the high energy resonances in contributions of the t channel singularity . About 2/3 of the coefficient x_{01} comes from the Δ exchange and the remaining part is attributed to the influence of the T = J = 0 exchange in the t channel (Höhler et al 1972).

The situation in that respect is similar to the one that occurs for the magnetic susceptibility of the nucleon. The Δ resonance alone would lead to a large value of the magnetic susceptibility which would surpass the electric one (Bernabeu et al 1972). This dominance is contradicted by the experiments ; this is explained by a large influence of the t channel exchange which reduces the magnetic polarizability (Bernabeu and Tarrach 1977).

For what concerns the nucleonic polarizability, we have seen that the Δ model does not reproduce accurately its value. When it comes to its contribution to the nuclear polarizability does it offer a clear advantage ?

In the optics of incorporating the nucleonic excitations in a structure constant (axial polarizability) these excitations are supposed to be infinitely high and the nuclear polarizability has a local form, $\alpha(x) = a\,\rho(x)$ where ρ is the nuclear density.(This expression applies to a system of non interacting nucleons). How does this form compare with the one derived in the Δ-hole picture ? A similar local expression follows in this model if the closure approximation is performed over the intermediate excitations. But the energy of the 33 resonance is sufficiently high for this approximation to be a good one : we have checked that in the case of ^4He the closure approximation (neglecting the particule hole energies) introduces an error of less than ten percent in the Δ-hole model for the dominant multipoles of the static polarizability as shown in the Table I

TABLE I

J	L	$\Pi^J_L(\vec{q},\vec{q}\,')$ Δhole model	with closure
1	0	159.5	160
0,2	1	29.2	31.2
1,3	2	3.26	3.65
2;4	3	0.26	0.30

Multipoles of the nucleonic part of the axial polarizability of ^4He in the Δ-hole shell model and in the same model but with the closure approximation neglecting the particle hole energy. The static situation is considered i.e. $q_0 = q'_0 = 0$ and $\vec{q}^{\,2} = \vec{q}\,'^2 = -m_\pi^2$.

This table shows that it is justified to perform the closure approximation and consequently to use a structure constant (the nucleonic axial polarizability) to incorporate the excitations of the nucleon.

However the polarizability method inspired by the analogy with solid state physics has also its weakness : since one is in principle able to explore a large range of momenta, a full description of the polarizability is required, which a single structure constant cannot provide. The expansion of the πN amplitude is performed for pions on the mass shell and there is no prescription to take them off the mass shell i.e. to vary the momenta q and q' . In the Δ-hole model a form factor is introduced at the $\pi N\Delta$ vertex : $\Gamma(q^2)=(\Lambda^2-m_\pi^2)/(\Lambda^2-q^2)$ (usually with $\Lambda \approx 1$ GeV). One can introduce such a form factor in the description with a structure constant in an ad hoc way with the following ansatz for the nucleonic polarizability $a(\omega=0, \vec{q}^{\,2}) = a\,\Gamma^2(-\vec{q}^{\,2})$ with $a = 2.36\ m_\pi^{-3}$.
With the introduction of this form factor, the nuclear polarizability arising from the nucleonic excitations acquires a non local character.

At zero momentum transfer like in β decay, the form factor effect represents a very small correction $\Gamma^2(q^2=0) \approx 0.96$ and the fact that the pions are off the mass shell is unimportant. The main requirement is to have a good description of the static polarizability on the mass shell, which is best achieved by using the expansion of the πN amplitude. With such a procedure the interpretation of the quenching of the Gamow Teller or magnetic transitions in term of a Lorentz Lorenz renormalization results in a smaller value of g' than with the Δ-hole model.

At large momenta $q \approx 2$ or $3\ m_\pi$ i.e. in the region where the critical opalescence is searched for, the nuclear polarization is dominated by the nuclear excitations. The nucleonic part representing only $\approx \frac{1}{3}$ of the total at the normal nuclear density ρ_0 , the precise model for the nucleonic polarizability is less important . However this fraction in creases with the density and the model will influence the prediction of the critical density for pion condensation to occur.

In conclusion we have shown that the static axial nucleonic polarizability is not acurately described by the assumption of the total dominance of the Δ resonance, which underestimates its magnitude. A proper description requires other contributions which are automatically incorporated if the polarizability is deduced from the extrapolation of the πN amplitude in the unphysical region. This has to be kept in mind when the analysis of the Gamow-Teller or magnetic strength is performed: if it gives a value of g' significantly smaller than obtained from other sources, a likely explanation is that mechanisms other than the Lorentz Lorenz effect are contributing.

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