TRN AU8105587

UM-P-81/51

SPACE PLASMAS 2

Dielectric Response of Particle-Antiparticle Plasmas in a Magnetic Field

N. E. Frankel, K.C. Hines and V. Kowalenko

School of Physics, University of Melbourne,

Parkville, Victoria 3052, AUSTRALIA

To be published in the Proceedings of "Symposium on Plasma Research, Theory and Experiment."

Ed. B. McNamara

SPRING COLLEGE ON FUSION PHYSICS

SPACE PLASMAS 2

Dielectric Response of Particle-Antiparticle Plasmas in a Magnetic Field

1. ABSTRACT

We have considered the longitudinal dielectric response of an ultra-degenerate relativistic plasma composed of electrons and positrons. We have used the relativistic Hartree self-consistent field method to investigate the dispersion relations and damping parameters of such a plasma in the presence of a magnetic field. These properties must be studied in the various regimes appropriate for a relativistic plasma as detailed by Tsytovich and Jancovici. Although it is hoped that this work will yield new insight into certain astrophysical phenomena (such as pulsars), it is interesting to note that laboratory electron-positron plasmas may be a thing of the immediate future as a result of suggested new experiments using an intense relativistic electron beam.

2. INTRODUCTION

In recent years it has become clear that relativistic plasma physics is an essential tool for astrophysics, quite apart from its increasing relevance for large scale fusion devices. Much of the work completed so far has been of an extremely formal, field theoretic nature, with the direct astrophysical applications being potential rather than actual. Alternatively, astrophysicists have studied the accretion rates for electrons and positrons from a basically heuristic viewpoint.

There has so far been very little work on the direct study of plasma response theory for pair systems. This paper is concerned with the development of such a response theory for an electron-positron plasma. It seems to us that the direct calculation of the modes of oscillation and of the damping parameters needed for astrophysical work can best be carried through by this technique.

For the pair plasma in the absence of a magnetic field the astrophysical motivation comes from the fact that the interior of dense white dwarf stars corresponds to conditions just slightly above pair threshold. This field-free electron-positron plasma has been studied by Jancovici [1], Tsytovich [2] and (using the response theory approach) Delsante and Frankel [3].

We present here the response theory for the pair plasma in the presence of a magnetic field. The mode structure in this more complicated case must be unravelled before the emission from pulsars can be completely understood. Although there have been many theories of the pulsar emission mechanism, as detailed in Manchester and Taylor [4] and Irvine [5], no complete investigation of the behaviour of the pulsar magnetosphere, consisting of dense relativistic electrons and positrons in magnetic fields of the order of 10^{12} G, is possible until the properties of the pair plasma as a problem in plasma physics have been elucidated. The first treatment of the magnetized pair plasma is due to Svetozarova and Tsytovich [6] and represents an extension of the work of reference [2].

As a final 'way out' astrophysical application of the physics of pair plasmas, mention may be made of the late lepton era of the early universe where the temperature (circa 10^{11} °K) is still above the pair threshold.

Electron-positron plasmas are also beginning to come into serious consideration in the laboratory. Tsytovich and Wharton [7] have discussed the possibility of creating a pair plasma by means of intense relativistic electron beams and they suggest a variety of experiments involving linear and nonlinear wave propagation which it would be important to carry out.

For the relativistic electron-positron plasma the classic papers are those of Tsytovich [2] and Jancovici [1]. Tsytovich has used a relativistic quantum propagator approach to develop both longitudinal and transverse response functions, although the transverse interaction has not been considered. Later Svetozarova and Tsytovich extended this work to include the presence of an external magnetic field.

Jancovici used a quasi-boson Hamiltonian and included not only the particle ring diagrams (taking account of the Coulomb interaction between particles) but photon ring diagrams as well. For the work of Tsytovich and ourselves the photons are included implicitly either in the field theory formalism or in the structure of the Dirac Hamiltonian. Jancovici's results tend not to hold for conditions such that the approximate results of Tsytovich are valid, and where the two sets of results do overlap, they do not always agree.

Following this pioneering work, the subject remained dormant for a number of years until interest in the phenomena mentioned earlier sparked a new surge of development during the last decade. In 1971 Bezzerides and Dubois [8] initiated a formal investigation of a covariant framework for treating ultrarelativistic plasmas. They employed a combination of non-equilibrium statistical mechanics and quantum electrodynamics. Although this approach is extremely rigorous, it does not lead readily to numerical results on the mode structure of the plasma.

Somewhat before our own work on relativistic plasmas, Bakshi, Cover and Kalman returned to the problem using much the same methods as Tsytovich. Their particular concern to begin with was the vacuum polarization in the presence of strong magnetic fields. They went on in a sequence of papers [9] to calculate the electromagnetic response of an electron gas and also of the vacuum in very strong magnetic fields.

Recently the Melbourne group has used a relativistic generalization of the Hartree self-consistent field method to calculate the longitudinal dielectric response of a relativistic ultra-degenerate electron plasma both with and without an external magnetic field. In the first paper of this series Delsante and Frankel [3] were able to obtain the response function itself and to exhibit the dispersion relations and damping characteristics of the plasma, both parallel and perpendicular to the magnetic field.

Here we have extended the calculations referred to above to include an additional positron component within the plasma. Although the numbers of electrons and positrons in the plasma are given input data rather than being determined by outside quantum electrodynamic processes with, say, an astrophysical origin, the plasma nevertheless remains quasineutral as a result of assumed positive and negative smoothed out backgrounds of charge with the appropriate magnitudes.

The treatment and results of the present work are entirely restricted to the longitudinal response function of the plasma. This restriction helps to keep the formalism for this paper relatively simple. It is our contention that the RPA - self-consistent field dielectric constant method represents a direct and physically transparent way of obtaining actual results for a relativistic plasma.

The same technique could be used to calculate the full dielectric tensor, so including the transverse components needed for a discussion of electromagnetic wave propagation. Since we have neglected the transverse effects in this paper, we are concerned to develop the response theory for calculating the real and imaginary parts of the dielectric constant, the dispersion relations and damping parameters in the various regimes appropriate for a relativistic electron-positron plasma. This programme has been developed first for the field-free plasma and then, using the appropriately altered matrix elements, for the more important case of the relativistic electron-positron plasma in an external magnetic field. Although our procedure is quite different, the results should agree with those obtained by Bakshi, Cover and Kalman [9].

3. FORMALISM

The second quantized form of the Hamiltonian is identical in form to that given by Delsante and Frankel [3], and the essence of the present problem consists in the implicit difference in interpretation of the following expression:

$$H(t) = \int d^3x\, \hat{\psi}^{\dagger}(x)\left[c\underset{\sim}{\alpha}\cdot\left(\underset{\sim}{p} - e\underset{\sim}{A}^{(0)}(\underset{\sim}{x})\right) + \beta mc^2\right]\hat{\psi}(x) + \frac{1}{2}\iint \frac{\rho(x)\rho(x')}{|\underset{\sim}{x} - \underset{\sim}{x}'|}\, d^3x\, d^3x' \quad \ldots (1)$$

In contrast to the earlier work, the field operator $\hat{\psi}$ (x) must be expanded in terms of both the positive energy and the negative energy eigenfunctions of the Dirac equation in order to include the positrons in the calculation. Thus

$$\hat{\psi}(\underline{x},t) = \sum_{\underline{p}}\sum_{s=1,2}\sqrt{\frac{mc^2}{\xi_p}}\left(b_{\underline{p}}^{(s)}u^{(s)}(\underline{p})\exp(i\underline{p}\cdot\underline{x}/\hbar - i\xi_{\underline{p}}t/\hbar) + d_{\underline{p}}^{(s)\dagger}v^{(s)}(\underline{p})\exp(-i\underline{p}\cdot\underline{x}/\hbar + i\xi_{\underline{p}}t/\hbar)\right) \quad \ldots (2)$$

in which the creation and annihilation operators (b's and d's) and Dirac eigenfunctions (u's and v's) conform to the notation of Sakurai [10].

The expression for the charge density

$$\rho(\underline{x},t) = e\,\hat{\psi}^{\dagger}(\underline{x},t)\hat{\psi}(\underline{x},t) \quad \ldots (3)$$

may be Fourier transformed according to the formula

$$\rho(\underline{x},t) = \sum_{\underline{q}}\rho(\underline{q},t)e^{i\underline{q}\cdot\underline{x}} \quad \text{where } \rho(\underline{q},t) = \frac{1}{V}\int d^3x\, e^{-i\underline{q}\cdot\underline{x}}\rho(\underline{x},t) \quad \ldots (4)$$

The Hamiltonian of the system may now be recast in the form

$$H(t) = \sum_{\underline{p}}\sum_{s}|\xi_{\underline{p}}|\left(b_{\underline{p}}^{\dagger(s)}b_{\underline{p}}^{(s)} + d_{\underline{p}}^{\dagger(s)}d_{\underline{p}}^{(s)}\right) + H_{int} \quad \ldots (5)$$

where the interaction part of the Hamiltonian is

$$H_{int} = \frac{1}{2}\sum_{\underline{q}}\rho(\underline{q},t)\rho(-\underline{q},t)\,4\pi/q^2 . \quad \ldots (6)$$

4. THE DIELECTRIC CONSTANT

Using the physics of the previous section one is now able to follow the same procedure as detailed in Delsante and Frankel [3] . The main difference is that more terms must be carried to take account of the presence of the positron component in the plasma. The end result of all this is the general form for the full dielectric constant of the plasma. This result, which we now give, is correct whether or not there is an external magnetic field.

$$\epsilon(q,\omega) = 1 + \frac{4\pi e^2}{\hbar q^2 V}\sum_{\substack{\underline{p}\,\underline{p}'\\ s\,s'}}\left(n_{\underline{p}}^- - n_{\underline{p}'}^-\right)\frac{\langle u,\underline{p},s|e^{i\underline{q}\cdot\underline{x}}|u,\underline{p}',s'\rangle\langle u,\underline{p}',s'|e^{-i\underline{q}\cdot\underline{x}}|u,\underline{p},s\rangle}{\omega - (\xi_{\underline{p}} - \xi_{\underline{p}'})/\hbar + i\eta}$$

$$+ \frac{4\pi e^2}{\hbar q^2 V}\sum_{\substack{\underline{p},\underline{p}'\\ s\,s'}} -\left(n_{\underline{p}}^+ - n_{\underline{p}'}^+\right)\frac{\langle v,\underline{p},s|e^{i\underline{q}\cdot\underline{x}}|v,\underline{p},s\rangle\langle v,\underline{p}',s'|e^{-i\underline{q}\cdot\underline{x}}|v,\underline{p},s\rangle}{\omega - (\xi_{\underline{p}'} - \xi_{\underline{p}})/\hbar + i\eta}$$

$$+ \frac{4\pi e^2}{\hbar q^2 V}\sum_{\substack{\underline{p},\underline{p}'\\ s,s'}}\left(n_{\underline{p}}^- + n_{\underline{p}'}^+ - 1\right)\frac{\langle u,\underline{p},s|e^{i\underline{q}\cdot\underline{x}}|v,\underline{p}',s'\rangle\langle v,\underline{p}',s'|e^{-i\underline{q}\cdot\underline{x}}|u,\underline{p},s\rangle}{\omega - (\xi_{\underline{p}'} + \xi_{\underline{p}})/\hbar + i\eta}$$

$$+ \frac{4\pi e^2}{\hbar q^2 V}\sum -\left(n_{\underline{p}}^+ + n_{\underline{p}'}^- - 1\right)\frac{\langle v,\underline{p},s|e^{i\underline{q}\cdot\underline{x}}|u,\underline{p}',s'\rangle\langle u,\underline{p}',s'|e^{-i\underline{q}\cdot\underline{x}}|v,\underline{p},s\rangle}{\omega + (\xi_{\underline{p}'} + \xi_{\underline{p}})/\hbar + i\eta}$$

In equation (7) n^+ refers to the equilibrium Fermi-Dirac distribution function for the positrons and n^- for the electrons. The first term is the electron gas result of Delsante and Frankel. This would be the complete expression if the positrons were switched off. Likewise, the second term refers to the positrons alone. The third and fourth terms refer to spontaneous emission and absorption processes occurring in an electron-positron plasma. Tsytovich refers to these as the pair production terms. These mixed terms do not vanish when $n^+ = n^- = 0$. In order to overcome this problem the responce function has to be renormalized (renormalization of the vacuum), which has the effect of taking the real part of the vacuum or 1-terms to zero

We now consider explicit results for the field-free and magnetic field cases.

A External Field Zero.

In calculating the matrix elements for zero field we use the standard plane wave solutions of the Dirac equation. We quote them first for electrons of momentum p with spin up (s = 1) and spin down (s = 2):

$$u_1(\underline{p}) = \sqrt{\frac{\mathcal{E}_p + \mathcal{E}_0}{2\mathcal{E}_p}} \begin{bmatrix} 1 \\ 0 \\ p_z c/(\mathcal{E}_p + \mathcal{E}_0) \\ c(p_x + ip_y)/(\mathcal{E}_p + \mathcal{E}_0) \end{bmatrix}, \quad u_2(\underline{p}) = \sqrt{\frac{\mathcal{E}_p + \mathcal{E}_0}{2\mathcal{E}_p}} \begin{bmatrix} 0 \\ 1 \\ (p_x - ip_y)c/(\mathcal{E}_0 + \mathcal{E}_p) \\ -p_z c/(\mathcal{E}_p + \mathcal{E}_0) \end{bmatrix} \quad \ldots (8)$$

and then for the positrons, also for the two spin possibilities:

$$v_1(\underline{p}) = -\sqrt{\frac{\mathcal{E}_p + \mathcal{E}_0}{2\mathcal{E}_p}} \begin{bmatrix} (p_x - ip_y)c/(\mathcal{E}_p + \mathcal{E}_0) \\ -p_z c/(\mathcal{E}_p + \mathcal{E}_0) \\ 0 \\ 1 \end{bmatrix}, \quad v_2(\underline{p}) = \sqrt{\frac{\mathcal{E}_p + \mathcal{E}_0}{2\mathcal{E}_p}} \begin{bmatrix} p_z c/(\mathcal{E}_p + \mathcal{E}_0) \\ (p_x + ip_y)c/(\mathcal{E}_p + \mathcal{E}_0) \\ 1 \\ 0 \end{bmatrix} \quad \ldots (9)$$

With these solutions of the free-particle Dirac equation, the various matrix elements appearing in equation (7) can be evaluated as follows:

$$\langle u, \underline{p}, s \,|\, e^{-i\underline{q}\cdot\underline{x}} \,|\, u, \underline{p}', s' \rangle = u_s^\dagger(\underline{p})\, u_{s'}(\underline{p}')\, \delta_{\underline{p}', \underline{p}+\underline{q}}$$

$$\langle v, \underline{p}, s \,|\, e^{-i\underline{q}\cdot\underline{x}} \,|\, v, \underline{p}', s' \rangle = v_s^\dagger(\underline{p})\, v_{s'}(\underline{p}')\, \delta_{\underline{p}, \underline{p}'+\underline{q}}$$

$$\langle v, \underline{p}, s \,|\, e^{-i\underline{q}\cdot\underline{x}} \,|\, u, \underline{p}', s' \rangle = v_s^\dagger(\underline{p})\, u_{s'}(\underline{p}')\, \delta_{\underline{p}, -\underline{p}'+\underline{q}}$$

$$\langle u, \underline{p}, s \,|\, e^{-i\underline{q}\cdot\underline{x}} \,|\, v, \underline{p}', s' \rangle = u_s^\dagger(\underline{p})\, v_{s'}(\underline{p}')\, \delta_{-\underline{p}, \underline{p}'+\underline{q}} \quad \ldots (10)$$

This completes the work for the zero field case.

B Non-Zero Magnetic Field.

The only modification required in the previous work when there is an external magnetic field concerns the matrix elements. The new matrix elements may be readily evaluated (see e.g. Johnson and Lippmann [11]). We designate the matrix elements in the order in which they occur in equation (10) by I_1, I_2, I_3 and I_4 respectively. When a magnetic field is present these matrix elements become:

$$I_1 = \frac{1}{2} NN' \delta_{p_x', p_x + q_x} \delta_{p_z', p_z + q_z} \Big\{ \delta_{\sigma,\sigma'} \Big[(\xi_r \xi_{r'} + p_z p_z') M_{nn'} + p_- p_-' M_{n-\sigma, n'-\sigma'} \Big] + \delta_{\sigma,-\sigma'} \Big[i p_-' p_z \, M_{n, n'-\sigma'} - i p_z' p_- \, M_{n-\sigma, n'} \Big] \Big\}$$

where

$$N^2 = \frac{1}{\varepsilon_\ell \xi_r}, \quad N'^2 = \frac{1}{\varepsilon_{\ell'} \xi_{r'}}, \quad \xi_r = \varepsilon_\ell + m,$$

$$p_- = \left[(2n - \sigma + 1) eB \right]^{1/2}, \quad p_-' = \left[(2n' - \sigma' + 1) eB \right]^{1/2}$$

and

$$M_{nn'}(p_x, q) = \int_{-\infty}^{\infty} G^n(y - y_0) \, G^{n'}(y - y_0') \, e^{-i q_y y} \, dy \qquad \ldots(11)$$

with similar expressions for the other three I's which we have insufficient space to quote here.

In equation (11)

$$G^n(y) = N_n H_n\left[(m\omega_c)^{1/2} y \right] \exp\left(-\frac{m\omega_c y^2}{2} \right), \quad N_n = \left(\frac{m\omega_c}{\pi} \right)^{1/4} \left(2^n n! \right)^{-1/2}$$

and H is a Hermite polynomial. The single-particle energy is

$$\varepsilon_\ell = \varepsilon_{n, p_z, \sigma} = \left(m^2 + p_z^2 + (2n - \sigma + 1) eB \right)^{1/2} \qquad \ldots(12)$$

The matrix elements calculated in this way may now be introduced into equation (7) to yield the longitudinal dielectric response function. In the next section we go on to consider the mode structure of the plasma both with and without the external field.

5. THE MODES

A External Field Zero.

As this paper is concerned mainly with the magnetic field case, there is no space to give the full dispersion relation for zero field. We shall later present the full response function for the magnetic field case. Here we shall be content to look at the dispersion and damping in a few limiting cases of interest.

We consider first the region $\omega \simeq q$. By solving the dispersion relation

$$\mathrm{Re}\,\epsilon(q,\omega)\big|_{\omega=q} = 0 \qquad \ldots(13)$$

we get the solution

$$\omega^2 = \omega_\ell^2 \sim \frac{4e^2}{\pi}\mathcal{E}_{p_F}^2 \ln\left|\frac{p_F+\mathcal{E}_{p_F}}{m}\right|,\ p_F/m \gg 1,\ \mathcal{E}_{p_F} = \sqrt{p_F^2+m^2}. \qquad \ldots(14)$$

Now by introducing $\omega = \omega_\ell + \Delta\omega$ and $q = \omega_\ell + \Delta q$, i.e. perturbing about the solution in q and ω space, the imaginary part of the dielectric function may be obtained from the complete expression for $\epsilon(q,\omega)$. This leads to the following expression for the damping constant:

$$\gamma \simeq -\frac{\pi}{4}\frac{\omega_\ell}{(p_F/m)(\mathcal{E}_{p_F}/m)}\left(1 - \frac{2}{3}\frac{m^4}{(q^2-\omega^2)^2}\frac{\omega_\ell^3}{(p_F/m)(\mathcal{E}_{p_F}/m)\,\mathcal{E}_{p_F}^2}\right) \qquad \ldots(15)$$

where we have used the general expression

$$\gamma = -\mathrm{Im}\,\epsilon(q,\omega)\Big/\frac{\partial}{\partial\omega}\mathrm{Re}\,\epsilon(q,\omega) \qquad \ldots(16)$$

Again, by solving $\mathrm{Re}\,\epsilon(\omega,\omega+\Delta q) = 0$, we obtain:

$$\frac{\Delta q}{\omega} \simeq \frac{\omega^2-\omega_\ell^2}{\omega_\ell^2},\ \omega_\ell^2 \overset{p_F/m\gg 1}{\sim} \frac{4e^2}{\pi}\frac{p_F\,\mathcal{E}_{p_F}^3}{m^2},\ \ q^2-\omega^2 = 2\alpha\,\omega_\ell\,\Delta q,\ \ \alpha = 2\ln\left|\frac{p_F+\mathcal{E}_{p_F}}{m}\right|\Big/\left(\frac{p_F\,\mathcal{E}_{p_F}}{m^2}\right) \qquad \ldots(17)$$

for the dispersion relation and the result

$$\gamma \simeq -\frac{e^2\mathcal{E}_{p_F}^2}{\omega^2}\left(1-\frac{m^4}{6\omega^2\mathcal{E}_{p_F}^2}\frac{\omega_\ell^4}{(\omega^2-\omega_\ell^2)^2}\right)\Big/\frac{8e^2}{\pi\,\omega^3}\frac{p_F\,\mathcal{E}_{p_F}^3}{2m^2} \qquad \ldots(18)$$

for the damping constant.

These results are in general agreement with the work of Tsytovich [2]. The conclusion is that the damping in both cases is extremely weak.

There is an additional regime where the damping can be found. Tsytovich refers to this as pair-production damping. It only occurs in the ultra-relativistic region and requires that the dispersion curve for the longitudinal oscillations must intersect the curve $\omega^2 = 4m^2 + q^2$. To find whether a solution does indeed exist for this condition on ω, we introduce $\omega^2 = 4m^2 + q^2$ into the general response function and take the small q limit to obtain

$$\mathrm{Re}\,\epsilon(q,\omega)\big|_{\omega^2=4m^2+q^2} \overset{\text{small } q}{\sim} 1 + \frac{2e^2}{\pi}\left\{-\frac{1}{3}\ln\left|\frac{p_F+\mathcal{E}_{p_F}}{m}\right| - \frac{1}{6}\frac{p_F\,\mathcal{E}_{p_F}}{m^2}\right\} \qquad \ldots(19)$$

Noting that p_F is the Fermi momentum, we can set the above expression equal to zero to obtain the density at which the real part of the response function will vanish. These densities are such that $p_F/m > \sqrt{3\pi/e^2}$. Let q_0 and ω_0 represent the solution to the dispersion relation where $\omega_0^2 = 4m^2 + q_0^2$. Standard procedures may now be used to obtain the damping constant for this regime:

$$\gamma \simeq -\frac{3\pi m}{\frac{q_0}{m}\ln\left|\frac{m}{2p_F}\right|}\sqrt{\frac{\Delta q}{q_0}},\quad q_0 \gg p_F \gg m \qquad \ldots(20)$$

B Non-Zero Magnetic Field.

We begin this section by giving first the real part of the response function correct to order q^2:

$$\mathrm{Re}\,\epsilon(q_3,\omega) \simeq 1 + \frac{e^2 B}{4\pi}\Bigg[\frac{8}{\omega^2}\frac{\sigma}{\sqrt{\sigma^2+m^2}} + \frac{16 m^2}{\omega^3(\omega^2-4m^2)^{1/2}} \ln\left|\frac{\epsilon_F\sqrt{\omega^2-4m^2}+\sigma\omega}{\epsilon_F\sqrt{\omega^2-4m^2}-\sigma\omega}\right|$$

$$+ \frac{24 q_3^2}{m^2\omega^4}\left(\frac{\sigma}{\sqrt{\sigma^2+m^2}} - \frac{2}{3}\frac{\sigma^3}{(\sigma^2+m^2)^{3/2}} + \frac{\sigma^5}{(\sigma^2+m^2)^{5/2}}\right) + \frac{24 q_3^2}{m^2\omega^4}\left(\frac{\sigma}{(\sigma^2+m^2)^{1/2}} - \frac{1}{3}\frac{\sigma^3}{(\sigma^2+m^2)^{3/2}}\right) + \frac{160 q_3^2 \sigma m^2}{\omega^6(\sigma^2+m^2)^{1/2}}$$

$$+ \frac{384 m^4 q_3^2}{\omega^7(\omega^2-4m^2)^{3/2}} \ln\left|\frac{\epsilon_F\sqrt{\omega^2-4m^2}+\sigma\omega}{\epsilon_F\sqrt{\omega^2-4m^2}-\sigma\omega}\right| + \frac{64 m^4 q_3^2}{\omega^7(\omega^2-4m^2)^{1/2}} \ln\left|\frac{\epsilon_F\sqrt{\omega^2-4m^2}+\sigma\omega}{\epsilon_F\sqrt{\omega^2-4m^2}-\sigma\omega}\right| - \frac{512 m^4 q_3^2 \sigma \epsilon_F}{\omega^6(\omega^2-4m^2)(\omega^2-4\epsilon_F^2)}$$

$$+ \frac{8 q_3^2 \sigma^3}{\omega^4(\sigma^2+m^2)^{3/2}} - \frac{320 m^2 q_3^2}{\omega^6}\frac{\sigma}{\sqrt{\sigma^2+m^2}} + \left(\frac{-72 m^2 q_3^2}{\omega^5(\omega^2-4m^2)^{1/2}} - \frac{8 m^2 q_3^2}{\omega^3(\omega^2-4m^2)^{1/2}} + \frac{240(\omega^2-4m^2)^{1/2} m^2 q_3^2}{\omega^7}\right)\times$$

$$\times \ln\left|\frac{\epsilon_F\sqrt{\omega^2-4m^2}+\sigma\omega}{\epsilon_F\sqrt{\omega^2-4m^2}-\sigma\omega}\right| - \frac{64\,\sigma\epsilon_F m^2 q_3^2}{\omega^4(\omega^2-4\epsilon_F^2)(\omega^2-4m^2)} + \frac{640\,\sigma\epsilon_F m^2 q_3^2}{\omega^6(\omega^2-4\epsilon_F^2)} + \frac{128 m^2 q_3^2 \epsilon_F \sigma}{\omega^4(\omega^2-4\epsilon_F^2)^2}\Bigg]$$

$$- \frac{1}{\omega^2}\Bigg\{\frac{4e^2B}{3\pi}\Bigg(\sum_{n=0}^{\infty} a_n\left(\frac{3}{2} + \frac{\omega^2}{4(m^2+2neB)} + \frac{3}{2}\frac{q_3^2}{\omega^2}\right) - \frac{3}{4}\sum_{n=n_\omega+1}^{\infty} a_n \frac{4(m^2+2neB)}{\omega(m^2+2neB-\frac{\omega^2}{4})^{1/2}}\times$$

$$\left[\tan^{-1}\left(\frac{\omega}{2(m^2+2neB-\omega^2/4)^{1/2}}\right) + \left(\frac{3}{2}\frac{q_3^2}{\omega^2} - \frac{q_3^2}{8(m^2+2neB-\omega^2/4)}\right)\tan^{-1}\left(\frac{\omega}{2(m^2+2neB-\omega^2/4)^{1/2}}\right)\right.$$

$$\left. - \frac{q_3^2}{4\omega(m^2+2neB-\omega^2/4)^{1/2}}\right] - \frac{3}{4}\sum_{n=0}^{n_\omega}\frac{m^2+2neB}{\frac{\omega}{2}\left(\frac{\omega^2}{4}-(m^2+2neB)\right)^{1/2}}\Bigg[\ln\left|\frac{\frac{\omega}{2}-\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}{\frac{\omega}{2}+\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}\right|$$

$$+ \left(\frac{3}{2}\frac{q_3^2}{\omega^2} + \frac{q_3^2}{8\left[\frac{\omega^2}{4}-(m^2+2neB)\right]}\right)\times \ln\left|\frac{\frac{\omega}{2}-\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}{\frac{\omega}{2}+\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}\right|$$

$$- \frac{q_3^2}{2\omega}\frac{\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}{m^2+2neB} + \frac{q_3^2}{8\left[\frac{\omega^2}{4}-(m^2+2neB)\right]^{1/2}}\frac{\omega}{(m^2+2neB)}\Bigg]\Bigg)\Bigg\}$$

$$- \frac{e^2}{3\pi}\left[\frac{1}{2}b + \ln b + \psi\left(\frac{1}{b}\right)\right] \quad , \sigma \approx p_F \qquad \ldots(21)$$

As in the field-free case treated earlier, a renormalization procedure has been used to obtain this result. As has been pointed out by Cover, Kalman and Bakshi [9], the vacuum cannot be omitted in determining the real part of the response function when a magnetic field is present. In obtaining equation (21) we have used the renormalization procedure recommended by the above authors.

6. DISCUSSION

Analytical studies of equation (21) for the response function are in progress. Although the expression is extremely complicated, it is hoped that the mode structure for certain regimes can nevertheless be extracted. Bakshi, Cover and Kalman [9] have already investigated the modes to lowest order for the plasma, i.e. the $q_z = 0$ limit. Their work should be a useful guide in dealing with our own result (correct to order q^2).

It should be emphasized that everything done so far refers to the situation where $\underset{\sim}{q}$ is parallel to the magnetic field. Thus the general procedures of sections 3 and 4 need to be repeated for $\underset{\sim}{q}$ perpendicular to the external magnetic field. This will prove more difficult since although the particle contribution (without the l-terms) seems tractable, we are uncertain about the procedure for determining the vacuum contribution in this case. A search for these modes must await the solution of this renormalization procedure either by ourselves or others.

Finally, for the investigation of electromagnetic wave propagation in the plasma we must include the effect of the transverse photons. It is our intention to work out the full dielectric tensor using the plasma conductivity and to include some relevant quantum electrodynamic effects. Some formal results for the propagating modes have been given by a number of Russian authors whom we shall reference in a later paper.

7. REFERENCES

[1] JANCOVICI, B., Nuovo Cimento 25 (1962), 428.

[2] TSYTOVICH, V.N., Sov.Phys.JETP 13 (1961), 1249.

[3] DELSANTE,A.E.,FRANKEL,N.E.,Ann.Phys.(N.Y.) 125 (1980), 135.

[4] MANCHESTER,R.N.,TAYLOR,J.H.,Pulsars,Freeman, 1977.

[5] IRVINE, J.M., Neutron Stars, O.U.P., 1978.

[6] SVETOZAROVA,G.I., TSYTOVICH,V.N.,Izv.Vyssh.Ucheb. Zaved. Radiofiz. 5 (1962), 658.

[7] TSYTOVICH,V.N.,WHARTON,C.B., Comm.P.Phys.4(1978)91.

[8] BEZZERIDES,B., DUBOIS,D.F.,Ann.Phys.(N.Y.) 70(1972)10.

[9] BAKSHI,P.,COVER,R.A., KALMAN,G.,Phys.Rev.Lett.33(1974) 1113; Phys.Rev.D 14 (1976)2532; Phys.Rev.D20(1979)3015.

[10] SAKURAI,J.J., Advanced Quantum Mechanics, Addison-Wesley (1967).

[11] JOHNSON,M.H.,LIPPMANN,B.A., Phys. Rev.7[illegible] (19[illegible]9),8[illegible]3.