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HADRON STRUCTURE FUNCTIONS

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A B S T R A C T

The x dependence of hadron structure functions is investigated. If quarks can exist in very low mass states ($\mathcal{O}$(10 MeV) for d and u quarks) the pion structure function is predicted to behave like (1-x) and not $(1-x)^2$ in a x-region around 1. Relativistic and non-relativistic quark bound state pictures of hadrons are considered together with their relation with the Q^2 evolution of structure functions. Good agreement with data is in general obtained.

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Hadron structure functions are measured chiefly in deep inelastic scattering of leptons on nuclear matter (proton and neutron structure functions) (fig. 1a) and in lepton pair production in hadron-hadron collisions (so-called Drell-Yan mechanism; proton, neutron, pion, kaon structure functions) (fig. 1b). These

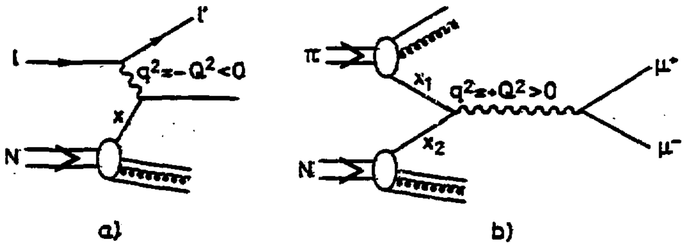

Fig. 1 a) Deep inelastic lepton-nucleon scattering, b) Muon pair production in pion-nucleon collision.

functions, $F(x,Q^2)$, depend on x, the fraction of the hadron longitudinal momentum (light cone variable) carried by the struck or annihilating quark (or antiquark) and on Q^2, which characterizes the scale at which the hadron structure is probed. Perturbative QCD tells something about the Q^2 dependence of those structure functions for large enough $Q^2 (Q^2 \geq 1\ \mathrm{GeV}^2)$. The purpose of this talk is to try to tell something about the x dependence of those structure functions.

Experimental results on nucleon structure functions[1] lead to $x[u_v^p(x) + d_v^p(x)] \sim 3.4\sqrt{x}(1-x)^3$ for $Q^2 \sim 20\ \mathrm{GeV}^2$, where u_v^p and d_v^p are respectively the u and d valence quark distributions inside a proton. For pion structure functions, experimental results are[2),3)] $x\bar{u}_v^{\pi^-}(x) = xd_v^{\pi^-}(x) \sim \sim 0.6\sqrt{x}(1-x)$, also for $Q^2 \sim 20\ \mathrm{GeV}^2$. In the case of the pion structure functions, many theorists have found that $\bar{u}_v^{\pi^-}(x) = d_v^{\pi^-}(x) \sim A(1-x)^2$ when x goes to 1, a prediction which appears to be in contradiction with experimental measurements. Let us discuss the theoretical arguments and how we can explain the disagreement with experiment.

First, there is a perturbative QCD argument[4),5),6)]. The π^- structure functions are measured via $\mu^+\mu^-$ production in π^-N collision. Let k be the 4-momentum of the $\bar{u}$ quark inside the π^-, which annihilates the u quark of the nucleon, and m the mass of the recoiling constituents of the pion. From kinematics, we get

$$k^2 = xm_\pi^2 - \frac{xm^2 + k_T^2}{1 - x}$$

where k_T is the $\bar{u}$ transverse momentum, with respect to the pion momentum. When x goes to 1

$$k^2 \sim - \frac{m^2 + k_T^2}{1 - x} \leq - \frac{m^2}{1 - x} \qquad (1)$$

implying that k^2 gets large and negative. If k^2 gets large the dominant contribution to the Drell-Yan mechanism comes from the diagram of fig. 2, where

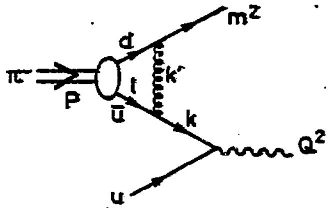

Fig. 2 One gluon exchange contribution to the pion structure function.

m is the recoiling quark mass. The dominant part of this diagram corresponds to the virtualnesses ℓ^2 and $(P-\ell)^2$ remaining small (of order 0.1 GeV^2). In this case, the virtualness of the exchanged gluon is of order $k^2 : k'^2 \sim (1-\xi)k^2$, where ξ is the light cone variable relative to ℓ. For a pion, the most probable value of ξ is 1/2. Since the virtualness of the exchanged gluon is large, the diagram of fig. 2 is a short distance process and the quark-gluon coupling is $\alpha_s(k'^2)$. The computation of this diagram leads to $\bar{u}^{\pi^-}(x) \sim A(1-x)^2$, when x goes to 1. For the nucleon, similar diagrams lead to $u^p(x)$ and $d^p(x)$ behaving like $(1-x)^3$, when x goes to 1[4),7)]. One problem with these results has to do with the recoiling quark mass m [8)]. What is this mass? In the case of the pion, if it is the constituent mass ($\sim$ 340 MeV), (1) reads as $k^2 \leq -0.1$ GeV$^2/(1-x)$. In order to be able to apply perturbative QCD one needs $k'^2 \leq -1$ GeV$^2 \Rightarrow k^2 \leq -2$ GeV$^2 \Rightarrow x \geq 0.95$. Therefore, in this case, one can justify the $(1-x)^2$ behaviour only in the region $0.95 < x < 1$. Moreover, a quark has also a current mass, which is very small for d and u quarks ($\sim$ 8 MeV for d quark), implying that in a pion some quark state with very low mass can recoil. If m is of order 8 MeV, the x interval where perturbative QCD can be applied (giving the $(1-x)^2$ behaviour) becomes very very small ($x > 0.99\ldots$). In the limit $m = 0$, the $(1-x)^2$ behaviour never shows up because k^2 is never necessarily large and negative. In this case, the non-perturbative contribution of the pion bound state wave function, corresponding to small values of k^2, always dominates over the large k^2 perturbative QCD contributions of diagram of fig. 2, this even near $x = 1$ [8),9),10)]. In fact, when $m = 0$, and for a large class of bound state wave functions, we find, when x goes to 1, the following behaviours which depend only on kinematical constraints[8)]:

$$\bar{u}^{\pi^-}(x) = d^{\pi^-}(x) \sim A(1-x)$$

$$u^P(x) \text{ and } d^P(x) \sim B_{u,d}(1-x)^3 \ .$$

More generally, for a target with n quark constituents we find

$$\lim_{x\to 1} q_v(x) \sim (1-x)^{2n-3}$$

which directly reflects the number of constituents rather than the related exchanges of large k'^2 gluons.

There exist non-perturbative calculations of the pion structure functions, based on the solution of a Bethe-Salpeter equation for a quark-antiquark bound state with massless gluon exchange[11)]. The kernel of this equation corresponds in fact to the diagram of fig. 2. The result of such calculations exhibits also the $(1-x)^2$ behaviour:

$$\bar{u}_v^{\pi^-}(x) = d_v^{\pi^-}(x) \sim Ax^2(1-x)^2 \text{ for } 0 \le x \le 1 \ . \qquad (2)$$

This result is obtained in the framework of conventional quantum field theory, i.e. using $1/(\not{k} - m_q)$ quark propagator, with $m_q > m_\pi/2$, and $g^{\mu\nu}/k'^2$ gluon propagator. Those propagators are not in agreement with confinement because they lead to free quarks of mass m_q and free massless gluons. Therefore one can have some doubts on result (2).

We propose a quark propagator which is in agreement with quark confinement and gives a meaning to the small current mass of d and u quarks[8),9)]. Our proposed quark propagator is a cut starting at m_o, possibly related to the current quark mass, and does not have the perturbative QCD pole. However, although there is no pole on the real axis, the propagator spectral function is peaked at a mass m, possibly related to the constituent quark mass, with a width γ^2 of the order of the strong interaction scale Λ^2. Explicitly, if in a physical gauge (like an axial gauge), we write the quark propagator as

$$P(k) = \Pi(k^2)\not{k} + M(k^2), \qquad (3)$$

we can choose the absorptive part of $\Pi(k^2)$ as

$$\text{Abs}\ \Pi(k^2) \sim \frac{\pi^{-1}\ \gamma^2\ \theta(k^2 - m_o^2)}{(k^2 - m^2)^2 + \gamma^4} \qquad (4)$$

with $\gamma^2 = \mathcal{O}(\Lambda^2)$. For d and u quarks, m_o and m will be respectively of order 10 and 336 MeV, whereas for s quark they will be of order 150 and 540 MeV. Moreover, we can choose the large k^2 behaviour of our propagator to be given by perturbative QCD. Since it does not possess a real pole, this propagator is in agreement with confinement. Using this quark propagator spectral function, together with some realistic bound state wave functions, we find the following results for hadron structure functions: assuming $m_o = 0$ for d and u quarks ($m_o = \mathcal{O}$(10 MeV) changes the result only for $0.99 < x < 1$), we find $\bar{u}_v^{\pi^-}(x)$ and $d_v^{\pi^-}(x)$ roughly proportional to $(1-x)$ for $0.45 < x < 1$. On the other hand, using $m_o = 150$ MeV for s quark, we find that $\bar{u}_v^{K^-}(x)$ is roughly a decreasing linear function of x for $0.4 < x < 0.85$ and behaves like $(1-x)^2$ for $0.85 < x < 1$. For the nucleon, we find that $u_v^N(x)$ and $d_v^N(x)$ behaves like $(1-x)^3$ when x goes to 1. Therefore, the pion structure functions that we obtain are in much better agreement with experiment than the ones obtained assuming a finite quark mass (of order 300 MeV for d and u quarks).

It is possible to apply a Drell-Yan-West[12)] relation to connect the x near 1 behaviour of the pion structure functions with the resonance channels appearing in deep inelastic scattering of charged lepton on pion[13)]: $\gamma(Q^2)\pi \to \rho, A_1, A_2, \ldots$ (the elastic channel $\gamma(Q^2)\pi \to \pi$ contributes only to the longitudinal structure functions, not to the transverse ones). As a result, the $(1-x)$ behaviour of the transverse pion structure functions implies a $1/[Q^2]^{3/2}$ behaviour for the $F^{\gamma}_{\pi\rho}(Q^2)$ form factor (the $\gamma(Q^2)\pi\rho$ coupling is defined as $\epsilon_{\mu\nu\tau\sigma}q^\mu\epsilon_\gamma^\nu r^\tau\epsilon_\rho^\sigma F^\gamma_{\pi\rho}(Q^2)$ where q and r are respectively the photon and ρ momenta and ϵ_γ and ϵ_ρ the photon and ρ polarization vectors). This behaviour leads to a $e^+e^- \to \pi\rho$ cross-section decreasing as $1/s^3$ at large s (e^+e^- invariant mass $\sqrt{s} \geq 1$-2 GeV).

The quark propagator (3), (4) has a complex pole in the second sheet of the k^2 plane. Can we say something about the related singularities of the quark-antiquark bound state wave function? In conventional quantum field theory, if the quark-antiquark interaction takes place via exchanges of massless gluons, the bound state wave function has the same kind of singularity as the quark propagator. This may also be true in QCD if we assume that the gluon propagator has a cut starting at $k'^2 = 0$. In this case, the bound state wave function possesses also the complex pole of the quark propagator. Moreover, up to an overall phase, the bound state wave function should be real. This is a consequence of unitarity, connected to the fact that, even if the quark and antiquark propagators have cuts starting at $m_0^2 = \mathcal{O}((10\ \mathrm{MeV})^2)$, the pion propagator should possess only a pole at $m_\pi^2 = 0.02\ \mathrm{GeV}^2$ and a cut starting at $9m_\pi^2$. Therefore, a mechanism must exist, which in the case of color singlet bound states makes the wave function real (this is also related to the stability of the pion in the framework of QCD). This reality can be obtained if we assume, for example, that the bound state wave function possesses also the pole which is complex conjugate of the one of the quark propagator. In this case, an approximation of the quark-antiquark bound state wave function would be

$$\psi(k^2) \sim \frac{A}{(k^2 - m^2)^2 + \gamma^4}$$

Assuming that this kind of wave function is also valid for a heavy quark-antiquark bound state (e.g. J/ψ) and making a non-relativistic approximation, we deduce the following quark-antiquark potential[14)]

$$V(r) \sim B \cot \lambda r ,$$

where λ is of the order of the strong interaction scale (~ 200 MeV). This potential behaves as $1/r$ when r goes to zero, in good agreement with perturbative QCD. It is a confining potential which blows up at $r = \pi/\lambda \sim 4$ fermi. This last point should not be taken too seriously since we believe that our potential is meaningless for $r \geq 3.5$ fermi, a distance at which the D^+D^- threshold will occur. It is not surprising that we do not get a linear potential at large r, as in the case of the gluonic string, because first our quarks and antiquarks are not standard particles, second we have not used precise information on the gluon propagator (we use the fact that it has a cut starting at $k'^2 = 0$, but not the fact that it may behave as $1/k'^4$ in this region).

Going back to our results on pion, kaon and nucleon bound state structure functions, we can ask the following question: at which scale, i.e. at which Q^2, should we compare them with experiment? In particular, so far, our analysis does not include gluon radiation type diagrams (fig. 3) which makes the hadron struc-

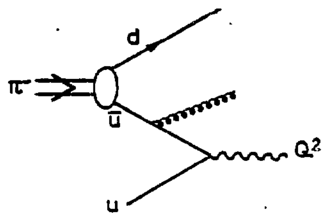

Fig. 3 Gluon radiation contribution to Drell-Yan mechanism.

ture functions vary with Q^2. Some people would say that the bound state structure functions correspond to the experimental ones at the scale for which perturbative QCD starts to be valid, i.e. for $Q^2 \sim 2$-$4\ GeV^2$. If so, we predict a pion structure function behaving like $(1-x)$ in a x-region near 1, for $Q^2 \sim 3\ GeV^2$. In addition, if we include diagrams like the one of fig. 3, in a leading log approximation, the predicted pion structure function behaves like $(1-x)^{1.3}$ for $Q^2 \sim 20\ GeV^2$, not in such a good agreement with experiment $[(1-x)]$. But it is possible that, because of non leading log terms of perturbative QCD, the $(1-x)$ behaviour does not change in a sensible way when Q^2 goes from $3\ GeV^2$ to $20\ GeV^2$. More dramatic is the fact that in our bound state structure functions almost 100% of the hadron momentum is carried by quarks and antiquarks and none is carried by gluons. This is in contradiction with the experimental result that for $Q^2 \sim 3\ GeV^2$, almost 50% of the hadron momentum is carried by gluons. One way out of this difficulty is to introduce gluons by hand, assuming that for $Q^2 \sim 3\ GeV^2$ the probability of observing a hadron as a pure bound state of its valence quarks and antiquarks, without gluons and sea quark-antiquark pairs, is much less than 1 (in ref. 10 this probability is estimated to be of order 0.2 to 0.25 in the case of the pion). But in order to have agreement with experiment we have also to assume that the contributions of the states containing gluons and sea pairs, in addition to valence quarks and antiquarks, do not modify in a sensible way the $(1-x)^p$ behaviour corresponding to pure valence quark and antiquark state ($p = 1$ for pion, $p = 3$ for nucleon). Another way of solving the difficulty of gluons, which may be equivalent to the previous one, is to assume that in the leading twist approximation the structure functions associated to pure valence quark and antiquark bound states correspond to a very small momentum scale $Q_o^2 (\sim 0.1\ GeV^2)$. Then, via perturbative and non-perturbative QCD, they evolve with Q^2, up to $Q^2 = 3\ GeV^2$ or $20\ GeV^2$. In this way we certainly generate gluons, but we have also to assume (or to check) that in doing so the $(1-x)^p$ behaviour corresponding to the pure valence quark and antiquark bound state is not modified (a property which is not satisfied by perturbative QCD in its leading log approximation).

There is a way of taking account of the Q^2 evolution of hadron structure functions in the framework of perturbative QCD without knowing explicitly what this evolution and the starting scale Q_o^2 are. It is to make a comparison between different quark distributions[8),15)]. In the leading twist approximation, the quantum fluctuations relative to a constituent inside a hadron do not depend on the spectator constituents. In particular, if we consider the moments of a non-singlet quark distribution

$$M_{q_{NS}}(n,Q^2) = \int_o^1 x^{n-1} q_{NS}(x,Q^2)dx \ ,$$

in the leading twist approximation and to all orders of perturbative QCD the ratio $M_{q_{NS}}(n,Q^2)/M_{q_{NS}}(n,Q_o^2)$ does not depend on the hadron which is considered. Moreover, if the quark mass squared is negligible as compared to Q_o^2 and Q^2, this ratio does not depend on the flavour of the quark q_{NS}. But this ratio does depend on the process which is probing the structure of the hadron (deep inelastic, lepton pair production, ...). Therefore, one can write

$$M_{u_v}\pi^-(n,Q^2) = M_{u_v}p(n,Q^2)\,\frac{M_{u_v}\pi^-(n,Q_o^2)}{M_{u_v}p(n,Q_o^2)}\ , \qquad (5)$$

a relation which is true for example for deep inelastic scattering. Taking $M_{u_v}p(n,Q^2)$ from experiment[1] and the ratio $M_{u_v}\pi^-(n,Q_o^2)/M_{u_v}p(n,Q_o^2)$ from our relativistic bound state model, we predict $M_{u_v}\pi^-(n,Q^2)$ for deep inelastic scattering. Including the corrections necessary to go from deep inelastic scattering to lepton pair production[16),8)] we get the same quantity relative to lepton pair production and then $x\bar{u}_v^{\pi^-}(x,Q^2)$ by an inverse Mellin transformation. Our results[8)] are shown on fig. 4 together with data of the CERN NA3 experiment[3)]. Agreement

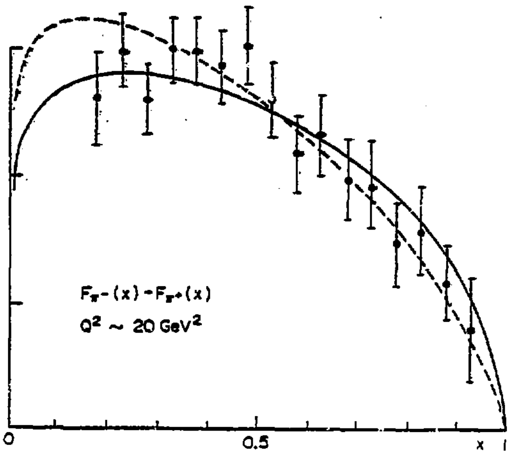

Fig. 4 Comparison of data[3)] on $F_{\pi^-}(x) - F_{\pi^+}(x)$ and theoretical predictions of our relativistic bound state model[8)]. The solid and dashed curves correspond respectively to spin and no spin correlation between the quark and the antiquark inside the pion. Normalization is arbitrary.

between the two is good. Let us note that our analysis does not include possible higher twist effects near $x = 1$. A relation identical to (5), in which we replace $\bar{u}_v^{\pi^-}$ and u_v^p respectively by $\bar{u}_v^{K^-}$ and $\bar{u}_v^{\pi^-}$, can be written for lepton pair production structure functions. Taking $M_{\bar{u}_v}^{\pi^-}(n,Q^2)$ from experiment[3] and the ratio $M_{\bar{u}_v}^{K^-}(n,Q_o^2)/M_{\bar{u}_v}^{\pi^-}(n,Q_o^2)$ from our relativistic bound state model we predict $M_{\bar{u}_v}^{K^-}(n,Q^2)$ and consequently $x\bar{u}_v^{K^-}(x,Q^2)$. The dashed curve of fig. 5 is the prediction of our relativistic bound state model[9] for the ratio $\bar{u}_v^{K^-}(x,Q^2 = 20\ GeV^2)/\bar{u}_v^{\pi^-}(x,Q^2 = 20\ GeV^2)$. The data points are from the CERN NA3 experiment[17]. Agreement between the two is not bad. The dashed curve corres-

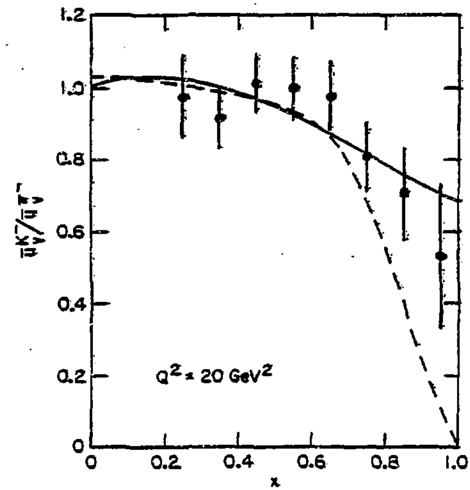

Fig. 5 The ratio of K^-/π^- lepton pair production structure functions versus x, for $Q^2 \sim 20\ GeV^2$. Data points are from the CERN NA3 experiment[17]. The dashed and solid curves are respectively the predictions of our relativistic and non-relativistic bound state models[9),15)].

ponds to the following values of the parameters: $(m_o,m) = (0,336\ MeV)$ for d and u quarks and $= (150\ MeV, 540\ MeV)$ for s quark. Note that our result is very sensitive to those parameters. So, most probably, an adjustement of the parameters can give a better agreement with data. Note also that this curve has been obtained assuming no spin correlation between the struck quark and the recoiling one. If our relativistic bound state model has something to do with reality, comparison with experiment can give information on the quark (d,u,s,...) propagators in the non-perturbative region. In particular, the kaon structure functions can tell us about the s quark propagator.

In quite a different manner, it is possible to make a non-relativistic approximation to compute $q_v(x,Q_0^2)$. In such an approximation, hadrons are considered as non-relativistic bound states of two or three particles[15),18),19)]. The distributions $q_v(x,Q_0^2)$ obtained are peaked at $x_0 = \mu/(m+\mu)$ with a width of order $1/RM$. μ, m and M are respectively the masses of the struck quark or antiquark, the recoiling constituents and the hadron; R^2 is the mass square radius of the hadron. For nucleon, pion and kaon, x_0 equals respectively 0.33, 0.5 and 0.38 (this last figure corresponds to the $\bar{u}_v$ distribution inside K^- when using $\mu = m_{\bar{u}} = 336$ MeV and $m = m_s = 540$ MeV) and $1/RM$ equals respectively 0.26, 2.4 and 0.80. The resulting distributions $q_v(x,Q_0^2)$ are quite different from those experimentally measured at $Q^2 \sim 20\ GeV^2$ (fig. 6). This means that a large amount of perturbative QCD corrections are needed to obtain $q_v(x,Q^2 = 20\ GeV^2)$. This is a feasible possibility which in fact is realized when using the leading log approximation for the perturbative QCD corrections[15),18)]. The shapes of the

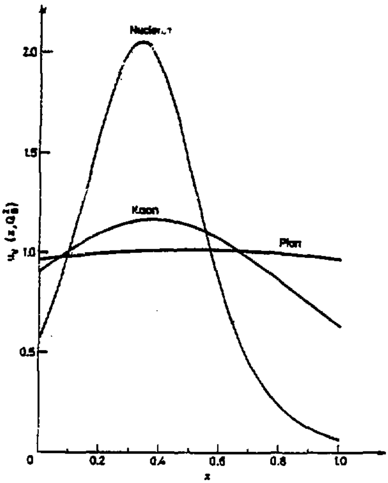

Fig. 6 The three non-relativistic bound state distributions $u_v(x,Q_0^2)$ as functions of x, for nucleon, π^+ and K^+.

distributions $q_v(x,Q_o^2)$ are quite different from the experimental ones, but there are two main features which are not modified by the perturbative QCD corrections and are in fact experimentally observed: i) the nucleon distribution is much more peaked and concentrated at small x than the pion one, ii) the kaon distribution drops faster than the pion one at large x. Using formula (5) in the same way as we did in the case of our relativistic bound state model, we predict[15] the lepton pair production structure function $x\bar{u}_v^{\pi^-}(x,Q^2)$ in the framework of this non-relativistic bound state model (fig. 7). Agreement with CERN NA3 data[3] is good.

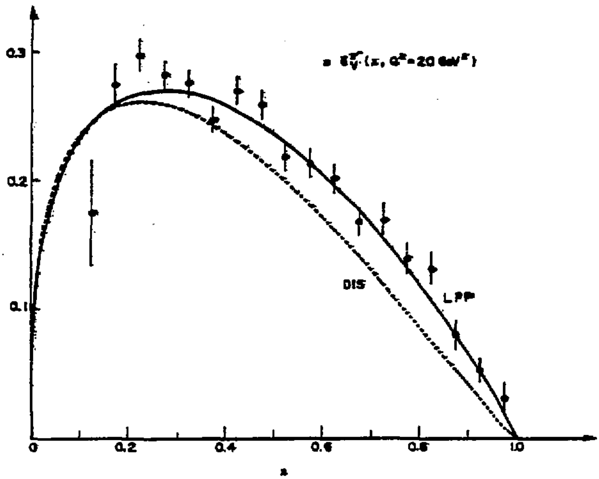

Fig. 7 Comparison between predictions of our non-relativistic bound state model[15] on $x\bar{u}_v^{\pi^-}(x,Q^2 = 20\ \mathrm{GeV}^2)$ as a function of x, and data of CERN NA3 experiment[3] on $F_{\pi^-}(x)$. Normalization of data is arbitrary. The solid curve corresponds to lepton pair production (LPP) and is the relevant one for comparison with data; the dashed curve corresponds to deep inelastic scattering (DIS). Note that data include some sea contributions at small x ($x < 0.3$).

Similarly, this non-relativistic bound state model predicts a ratio[15] $\bar{u}_v^{K^-}(x,Q^2 = 20\ \mathrm{GeV}^2)/\bar{u}_v^{\pi^-}(x,Q^2 = 20\ \mathrm{GeV}^2)$, shown as the solid curve on fig. 5. Agreement with data is also quite good.

Even if agreement with experimental data is good, some criticisms of the non-relativistic approximation have to be made. The main one is that it is difficult to consider pion and kaon as non-relativistic bound states. The second criticism is that we do not get an agreement between the non-relativistic bound state picture and the relativistic one. The solution of this contradiction may be the following: the relativistic bound state picture could correspond to the experimental situation at a scale $Q^2 \sim 2\text{-}4\ GeV^2$ (note that in this case, we have to add gluons and sea quark-antiquark pairs by hand). On the other hand, the non-relativistic bound state picture could be associated to a very small scale ($Q_0^2 \sim 0.1\ GeV^2$). At this scale, a large amount of unknown perturbative and non-perturbative QCD corrections ($1/Q^2$ terms) makes the comparison of the non-relativistic structure functions with experiment meaningless. However, if we let those structure functions evolve with Q^2 via perturbative QCD (mainly leading log terms) the $1/Q^2$ terms disappear at a scale $Q^2 \geq 2\text{-}4\ GeV^2$. At this last scale, the quark structure of hadrons shows up and comparison between the model and experiment becomes meaningful. At Q_0^2 the model assumes that there are no glue or sea quark-antiquark pairs. At $Q^2 \geq 2\text{-}4\ GeV^2$ it gives the right amount of gluons and sea pairs[20),18)]. This last point is in fact the only justification for considering this model, because it is difficult to understand how we can control perturbative QCD (and moreover the leading log approximation) for $Q^2 \leq 1\ GeV^2$.

Conclusions are, first of all, that if quarks can exist in very low mass states ($\mathcal{O}$(10 MeV) for d and u) there is no reason to obtain a $(1-x)^2$ behaviour for the pion structure function in a x-region around 1. In fact, in the framework of our relativistic bound state model, we get a $(1-x)$ behaviour which is in agreement with what is experimentally observed. However, our relativistic bound state picture remains to be included in a model taking account of the Q^2 evolution of structure functions. In particular, the 50% of gluons observed at $Q^2 \sim 2\text{-}4\ GeV^2$ is not understood. This last point can be understood in a non-relativistic bound state picture used together with a perturbative QCD Q^2 evolution of structure functions (mainly leading log approximation). But this model needs justification, both for the use of a non-relativistic bound state approximation and of perturbative QCD for $Q^2 \leq 1\ GeV^2$. Agreement between the relativistic and non-relativistic bound state pictures is also needed. Both pictures are in general in good agreement with experimental data, particularly the naive non-relativistic prediction on $\bar{u}^{K^-}/\bar{u}^{\pi^-}$.

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