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DYNAMICS OF HADRON-NUCLEUS INTERACTIONS

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## DYNAMICS OF HADRON-NUCLEUS INTERACTIONS

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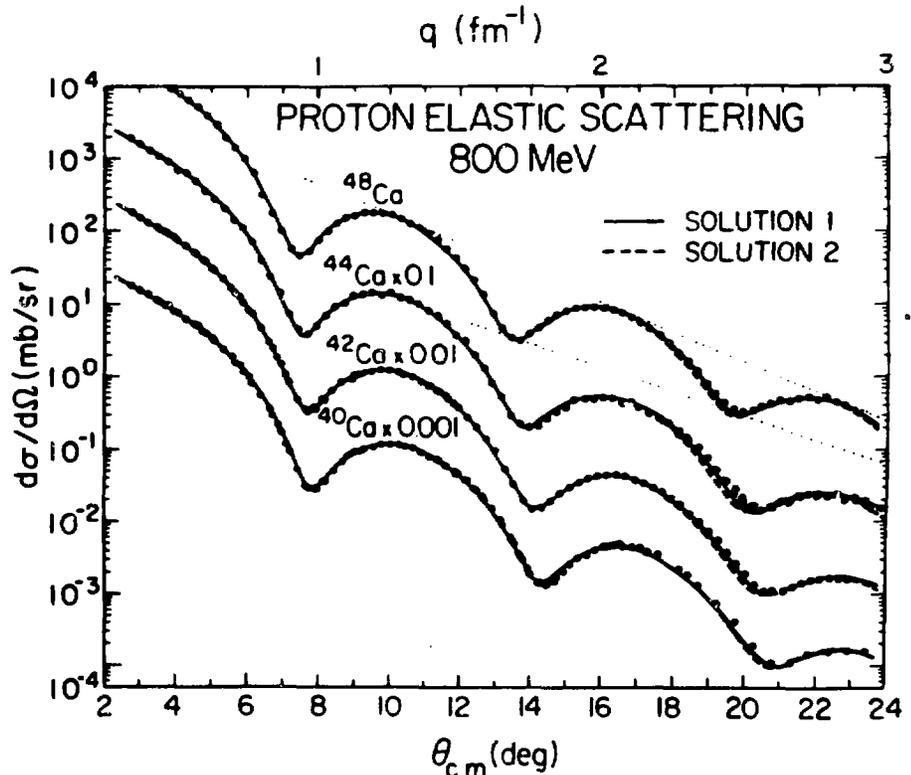
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**Abstract:** Recent progress in diffraction theory shows that proton-nucleus scattering at nonforward angles is dominated by the interference of waves from two or more "bright spots". Analytic formulas based on asymptotic theories of diffraction yield valuable new insights into the scattering and these formulas can be readily extended to illuminate the role of dynamical ingredients, i.e., the nucleon-nucleon amplitudes. The governing parameters of the diffraction and some direct connections between the observed cross sections and the input dynamics are reviewed. New information regarding the nucleon-nucleon parameters based on recent phase shift analyses show some systematic differences from the "effective" NN amplitudes which produce fits to proton-nucleus diffraction data. Recent progress in understanding the role of  $\Delta$ -isobars in proton-nucleus dynamics is reviewed.

## 1. Introduction

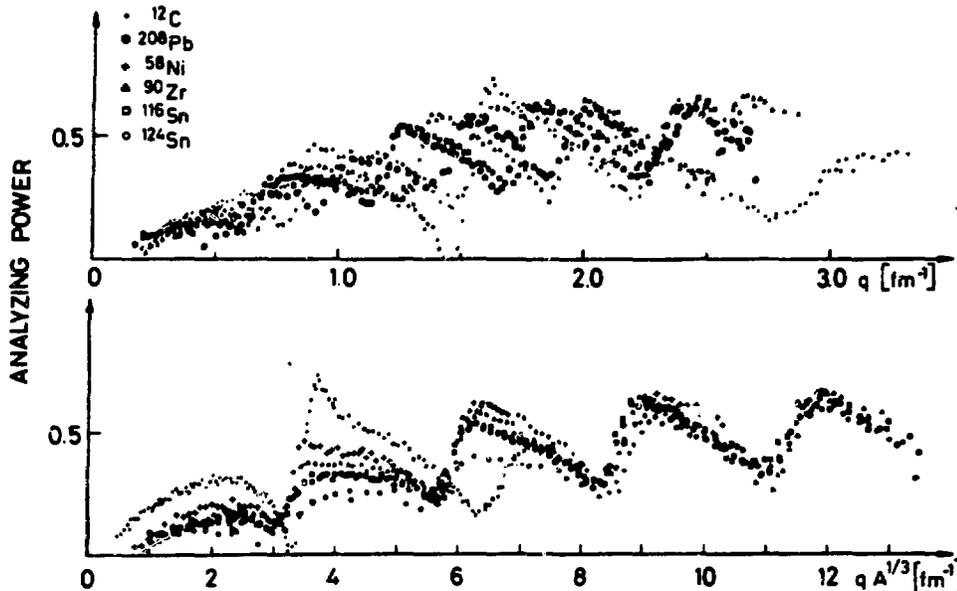
Beautiful diffraction patterns are produced by elastically scattering fast hadrons from nuclei. In recent years, experimental programs at Gatchina and Saclay<sup>1-17)</sup> using 1 GeV proton beams and at Los Alamos<sup>18-30)</sup> using a 0.8 GeV proton beam have concentrated on the accumulation of accurately detailed proton-nucleus diffraction data. The experiments include some exceptional examples of high resolution in both energy and angle together with high statistical accuracy. Figure 1 displays some recent diffraction angular distributions from the High Resolution Spectrometer at Los Alamos<sup>28)</sup>. The characteristic oscillations arise from interference of waves diffracting from an equivalent absorptive disk and there is an interferometric sensitivity to the radius. A second characteristic feature of the patterns is the approximate exponential decrease with  $q$  for nonforward angles as indicated by the dotted lines in fig. 1.

Fig. 1 Differential cross section data points ( $\bullet$ ) and KMT optical potential fits for elastic scattering of 800 MeV protons by Ca isotopes<sup>28)</sup>.



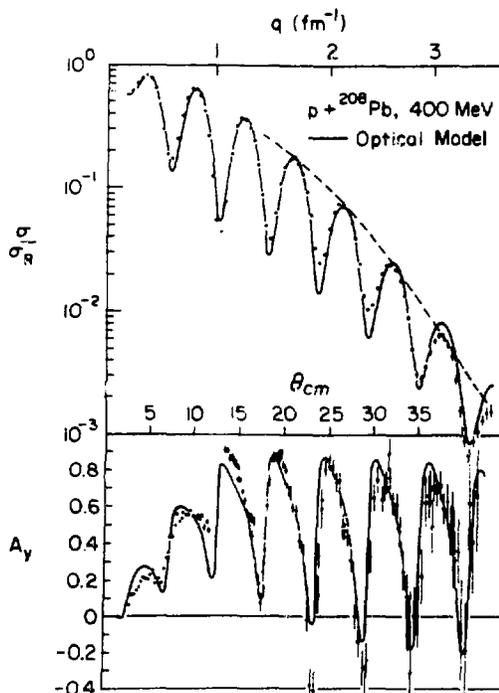
Polarization data also display some very simple features as shown in fig. 2, particularly when plotted versus  $q A^{1/3}$  to line up the diffractive oscillations.

Fig. 2 Analyzing power data for 800 MeV proton-nucleus elastic scattering.



Similar experimental results are now emerging from Los Alamos and Vancouver near 0.5 GeV and lower energies. Figure 3 shows 0.4 GeV proton scattering from  $^{208}\text{Pb}$ .<sup>31)</sup> Because the cross section data are shown as the ratio to Rutherford scattering, the characteristic exponential fall-off appears as a curve in this plot. In the foreseeable future, the diffraction scattering will be mapped out from 0.2 GeV (Indiana) through 1 GeV.

Fig. 3 Elastic cross section and analyzing power data for 400 MeV proton scattering by Pb.<sup>31)</sup>



At the higher energies, it has been established by many calculations that the proton data can be successfully interpreted using the simplest multiple scattering theories, i.e., Glauber's multiple diffraction theory<sup>32-56</sup>) or the Kerman, McManus, Thaler optical potential approach<sup>57-72</sup>). Generally these calculations incorporate simple assumptions about the dynamics, i.e., the NN amplitudes, because accurate NN information has not been available. One of the reasons for some confidence in the multiple scattering calculations is that a number of second-order effects, e.g., Pauli correlations, center-of-mass correlations, noneikonal effects, the influence of double spin flip terms in the NN interaction and the influence of the magnetic spin flip interaction have all been studied to some degree. Generally there are significant but minor effects due to second-order corrections. For quantitative accuracy one needs to include them in theoretical analyses.

Recent reviews by Alkhazov, Belostotsky and Vorobyov<sup>73</sup>), Whitten<sup>74</sup>), Ray<sup>75</sup>), Igo<sup>76</sup>), Thomas<sup>77</sup>) and Wallace<sup>78</sup>) have discussed many aspects of proton-nucleus scattering. In this article the emphasis is on dynamics. However, dynamical issues are concealed behind the predominant and basically simple diffraction structure for  $A \gtrsim 12$  nuclei. Thus a primary issue has become the separation of diffraction and dynamics and there are some genuine bright spots which can be reported in this area (sec. 2).

Progress in NN amplitude determinations is discussed in sec. 3 and a summary of recent work on the role of  $\Delta$ -isobars in proton-nucleus interactions is given in sec. 4.

## 2. Bright spots

Recent developments of asymptotic expressions for large momentum transfer have added considerable insight into proton-nucleus diffraction. In a very interesting paper, Amado, Dedonder and Lenz<sup>79</sup>) considered the diffraction produced by protons scattering from a Woods-Saxon density using the eikonal approximation. They showed that a pair of saddle points at complex values of the impact parameter account for most of the features of diffraction by nuclei for  $q \gtrsim 1 \text{ fm}^{-1}$ .

Unpublished work by Glauber and Bleszynski<sup>80</sup>) illustrates that saddle points are present for density forms other than the Woods-Saxon and that there can be multiple saddle points when strong Coulomb effects are present, for example, in heavy ion scattering. However, high energy proton-nucleus diffraction seems to involve just two saddle points. There are in effect two slits rather firmly lodged in the nuclear surface as indicated in fig. 4. Typically at 800 MeV, for angles of  $10^\circ$  to  $50^\circ$ , the scattering is dominated by the rays emerging from these bright spots. The nucleus is rather similar to a two-slit interferometer!

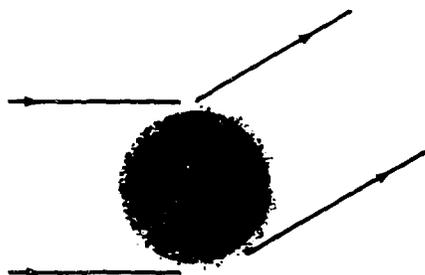


Fig. 4 Upper sketch indicates protons scattering near the edge of the nucleus. Lower sketch indicates slit-shaped bright spots which dominate large  $q$  elastic diffraction.



Amado, Dedonder and Lenz employ the standard Fourier-Bessel impact parameter representation for the diffraction integral;

$$f(q) = \frac{k}{i} \int_0^{\infty} db b J_0(qb) [S_F(b) - 1]. \quad (1)$$

The impact-parameter amplitude  $S_F(b)$  contains all the information about nuclear geometry, nucleon-nucleon amplitudes and electromagnetic effects. Glauber and Bleszynski employ a two-dimensional form where the x-axis is parallel to momentum transfer  $\vec{q}$ . The impact parameter amplitude  $S_F(\vec{b})$  need not be rotationally invariant in this case and the diffraction integral over the  $\vec{b}$ -plane is done in two steps:

$$f(q) = \frac{k}{2\pi i} \int_{-\infty}^{\infty} dx e^{iqx} S(x), \quad S(x) = \int_{-\infty}^{\infty} dy S_F(x, y). \quad (2)$$

In either approach,  $S_F$  is rather smoothly varying and the oscillations of  $J_0(qb)$  or  $\cos e^{iqx}$  produce dramatic cancellations when  $qR$  is large, where  $R$  is the nuclear radius. Figure 5 illustrates the situation for the diffraction integral (1).

Direct numerical evaluation of eq. (1) is a standard task in diffraction calculations, however this approach provides little insight regarding how the nuclear geometry and the dynamics (e.g., the NN amplitudes) affect the observed diffraction. Similar loss of insight holds for the partial wave calculations based on the optical model. The situation is dramatically improved by deforming the contour of integration into the complex impact parameter plane since there are points where rapid variations of  $S_F(b)$  can match the oscillations due to the Bessel function. This technique has a long tradition in scattering theory<sup>81)</sup> analysis

but has only recently been applied to the high energy proton scattering. Convergence requires that one first separate the Bessel function into two parts due to the asymptotic behavior for large values of  $qb$ :

$$J_0(qb) \sim \frac{1}{2} \left(\frac{2}{\pi qb}\right)^{1/2} [e^{i(qb-\pi/4)} + e^{-i(qb-\pi/4)}], \quad (3)$$

one of which is exponentially damped in the upper-half  $b$ -plane while the other has that property in the lower-half  $b$ -plane. Thus, two amplitudes are of interest at large  $q$ :

$$f^{(\pm)}(q) = \frac{k}{2i} \int_0^{\infty} db b \left(\frac{2}{\pi qb}\right)^{1/2} e^{\pm i(qb-\pi/4)} S_F(b). \quad (4)$$

Since  $q \neq 0$ , the 1 part of eq. (1) produces no contribution and therefore

$$f(q) = f^{(+)}(q) + f^{(-)}(q). \quad (5)$$

The observed cross section involves the interference

$$\sigma(q) = \sigma^{(+)}(q) + \sigma^{(-)}(q) + 2[\sigma^{(+)}(q)\sigma^{(-)}(q)]^{1/2} \cos[\phi(q)], \quad (6)$$

where  $\sigma^{(\pm)}(q) = |f^{(\pm)}(q)|^2$  and  $\phi(q)$  is the phase difference between  $f^{(+)}$  and  $f^{(-)}$ . The observation of Amado, Dedonder and Lenz is that a pair of complex stationary phase points, i.e., saddle points, dominate  $f^{(+)}$  and  $f^{(-)}$ . The integration con-

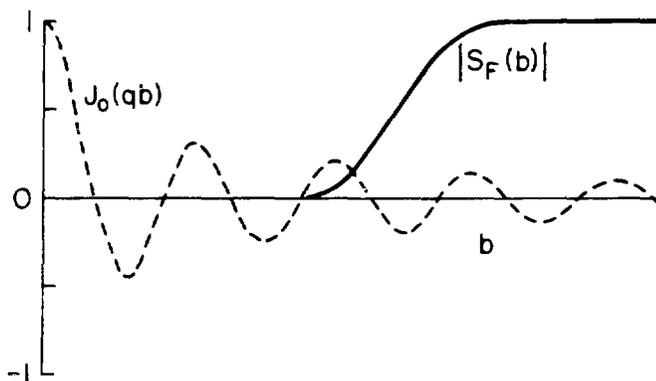


Fig. 5

tour for  $f^{(+)}$  is deformed into the upper-half  $b$ -plane to pass through the saddle point  $b_+$  nearest to the real axis and similarly the contour for  $f^{(-)}$  is deformed into the lower-half  $b$ -plane to pass through the nearest saddle point ( $b_-$ ) as illustrated in fig. 6. The saddle points are defined by the stationary phase requirement, i.e., the first derivative of the phase of the integrand must vanish;

$$\left. \frac{d}{db} \left[ \frac{1}{2} \ell_n b + \ell_n S_F(b) \right] \right|_{b=b_{\pm}} \pm iq = 0. \quad (7)$$

Consequently the solutions  $b_{\pm}$  of this relation depend on  $q$  and the geometrical and dynamical parameters which control the transmission function  $S_F(b)$ .

A second requirement which is needed to optimize the saddle point contributions is that the contours of integration must be chosen

to pass through the points  $b_{\pm}$  in the "steepest descent" direction which is specified by the phase of a complex length  $\ell_{\pm}$ . Near a saddle point the integrand in eq. (4) takes the form

$$\left( \frac{2b_{\pm}}{\pi q} \right)^{1/2} e^{i(qb_{\pm} - \pi/4)} S_F(b_{\pm}) e^{-\pi(b-b_{\pm})^2/\ell_{\pm}^2}, \quad (8)$$

where the only  $b$ -dependence retained is the Gaussian factor which governs the phase variation. Complex length parameters  $\ell_{\pm}$  are defined in terms of the second derivative of the complex phase at the saddle point and consequently they also depend on  $q$  and the other parameters.

The saddle point contributions to the scattering amplitudes are easily evaluated by carrying out the  $b$ -integration with the result,

$$f^{(\pm)}(q) = \frac{k}{2i} \ell_{\pm} S_F(b_{\pm}) \left( \frac{2b_{\pm}}{\pi q} \right)^{1/2} e^{\pm i[q \operatorname{Re} b_{\pm} - \pi/4]} e^{-q |\operatorname{Im} b_{\pm}|}. \quad (9)$$

Thus once the saddle points  $b_{\pm}$  are located, the bright spot amplitudes are calculated from eq. (9) and the interference pattern is calculated from eq. (6). The presence of dominant saddle points is actually quite common for proton-nucleus scattering and does not rely on specific analytic forms for  $S_F(b)$ . The asymptotic methods therefore provide a very useful simplification of the diffraction problem but without any loss of dynamical information. This is a decisive advantage over many other approaches.

The general features noted in the introduction indicate that  $\operatorname{Re}(b_{\pm})$  is approximately the nuclear radius and that  $\operatorname{Im} b_{\pm}$  is approximately constant and is related to the diffusivity of the optical density. Exactly these features were shown to prevail by Amado, Dedonder and Lenz for a minimal model based on a Woods-Saxon nuclear density,

$$\rho_m(r) = \frac{\rho_0}{1 + \exp((r-c)/a)}, \quad (10a)$$

where  $c$  is the half-density radius and  $a$  is the diffusivity.

In the eikonal approximation one has

$$S_F(b) = e^{-\gamma t(b)} \quad (10b)$$

$$t(b) = \int_{-\infty}^{\infty} dz \rho_m(\sqrt{z^2 + b^2}) \quad (10c)$$

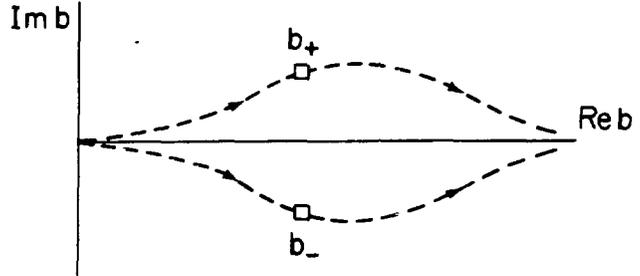


Fig. 6 The dashed lines indicate contours in the complex impact parameter plane which pass through complex saddle points at  $b_+$  and  $b_-$ .

where

$$\gamma = \frac{\sigma}{2} (1 - i\rho) \quad (10d)$$

is a constant involving the isospin average NN cross section,  $\sigma$ , and a real to imaginary ratio,  $\rho$ . The Coulomb interaction is not included at this level but that is easily remedied. Analytic solutions for the saddle points in this example are approximately given by

$$b_{\pm} = c \pm i\pi a - \frac{1}{2} \left[ \frac{\alpha^2 (c \pm i\pi a)}{q} \right]^{1/3} \exp(\pm i \frac{2\pi}{3}) \quad (11)$$

where  $\alpha = 2\pi\gamma a \rho_0$  is a dimensionless constant. In the special case of the Woods-Saxon density, eq. (11) shows that the dominant saddle points tend to fixed values  $c \pm i\pi a$  as  $q \rightarrow \infty$ . This is due to the existence of poles of the density at  $r = c \pm i\pi a$ . However, the existence of saddle points does not require such specific analytic properties of the density. The point is that the optical thickness function may be defined as a Fourier transform of a real form factor  $F(q)$  multiplied by the scalar NN amplitude  $A(q)$ , as follows:

$$\gamma t(b) \rightarrow \bar{A}(b) = 2 \int_0^{\infty} dq q J_0(qb) F(q) A(q). \quad (12)$$

From eq. (3) one sees that, for complex  $b$ , there is an exponentially growing contribution which leads to arbitrary values of  $t'(b)$ . Thus solutions to the saddle point condition (7) can generally be found.

When the analysis is carried through and Coulomb effects are inserted, the following asymptotic form for the diffraction cross section emerges<sup>79</sup>:

$$\sigma(q) = |A_0|^2 \frac{e^{\epsilon(qc)} q^{1/3}}{8/3} e^{-2\pi qa} \{e^{2\psi} + e^{-2\psi} + 2 \cos(2qc + \phi)\} \quad (13)$$

where  $A_0$  is a constant and  $\psi$  and  $\phi$  are slowly varying functions of  $q$ . This expression displays both the diffraction oscillations with period  $\approx c/\pi$  that arise directly from the nuclear half-density radius  $c$  and an exponential decay factor  $e^{-2\pi qa}$ . The factor in brackets oscillates between  $4\cosh^2\psi$  and  $4\sinh^2\psi$  envelopes, thus the function  $\psi$  describes filling of diffraction minima and it involves the Coulomb parameter ( $\eta = Ze^2/\hbar v$ ) and the ratio ( $\rho$ ) of the real to imaginary parts of the forward NN amplitude as follows:

$$\psi \approx \eta[-2\pi a/c] + \rho \left[ \frac{1}{2}(qc)^{1/3} (\pi a/\lambda)^{2/3} - .51(\pi a)^{3/2}/(\lambda c^{1/2}) \right]. \quad (14)$$

In this expression,  $\lambda = (\sigma\rho_0)^{-1}$  is the proton's mean free path. As  $\psi$  increases slowly with  $(qc)^{1/3}$ , the envelope of oscillations narrows as is illustrated by the convergence of the dotted lines in fig. 1. Recent large  $q$  data<sup>29</sup>) on  $^{12}\text{C}$  provide an example where the oscillations essentially disappear. These features can be understood in terms of one of the bright spot amplitudes becoming dimmer than the other by the factor  $e^{-\psi}$  due to a differing optical path length through the nuclear surface. The function  $\phi$  provides a slow variation of the period of oscillation due to the difference of optical paths of the two interfering rays:

$$\phi \approx \frac{3}{2}(qc)^{1/3} (\pi a/\lambda)^{2/3} + 5\pi/3 - .51(\pi a)^{3/2}/(\lambda c^{1/2}). \quad (15)$$

Figure 7 shows a comparison of the saddle point cross section formula with accurate numerical integration of the diffraction integral. The reproduction of the numerical calculations is very good except for the most forward angles.

Some care is needed before concluding that the exponential fall-off of the cross section necessarily implies the exponential fall-off of the density. Glauber and Bleszynski (fig. 8) have shown that even for a Gaussian density there is an approximate exponential fall-off of the cross section for a finite range of  $q$ . It is the signature of saddle points which move slowly with  $q$ , the movement being controlled by  $q^{-2/3}$  for the Woods-Saxon case of eq. (11) and by  $\ln(q)$  for the Gaussian case.

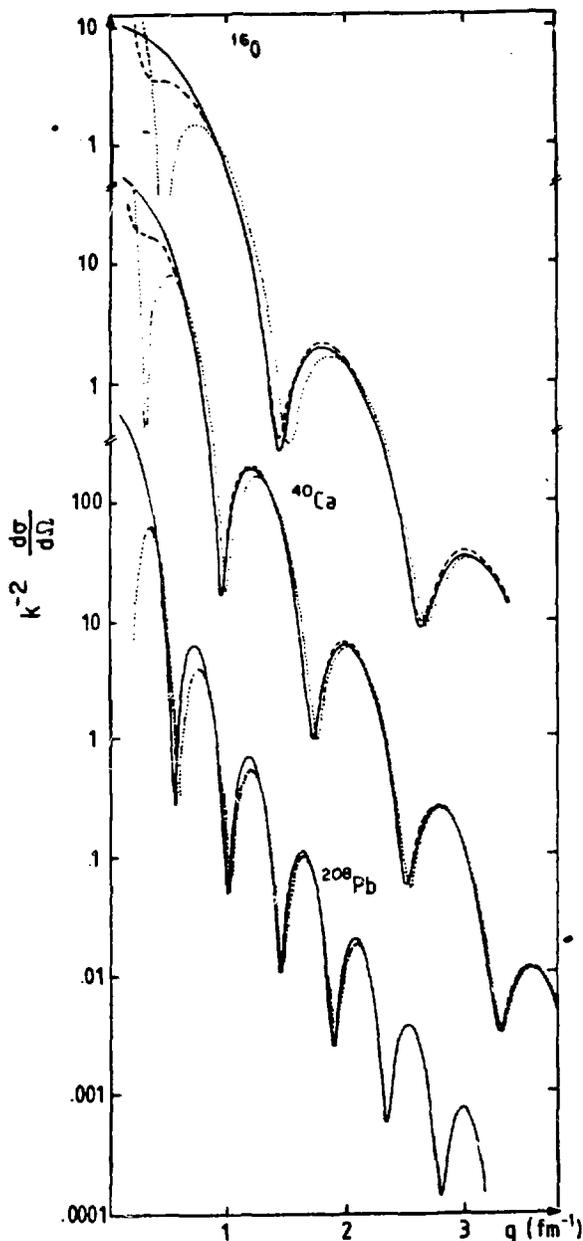
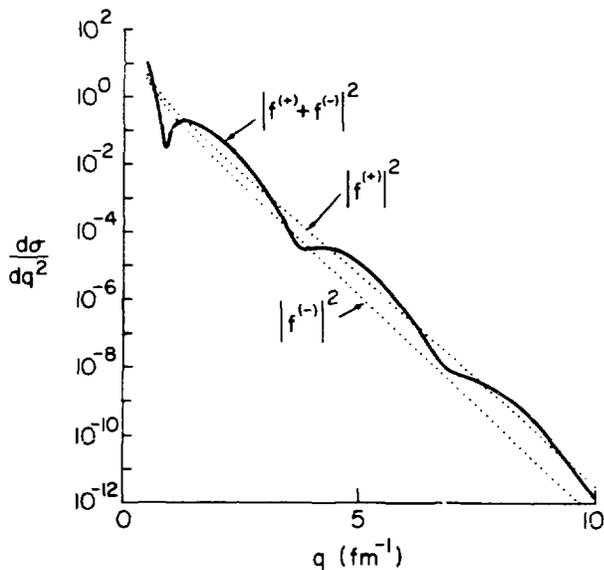


Fig. 7 Numerical result <sup>79)</sup> based on the diffraction integral of eq. (1). — is compared to the simplest asymptotic formula result ..... and to an improved asymptotic formula result -----.

Fig. 8 Differential cross section <sup>80)</sup> based on a Gaussian thickness function  $t(b)$  as in eq. (10b) but calculated in the asymptotic approximation to eqs. (2).



Although derivation of the asymptotic formulas (13)-(15) involves quite a few approximations, the results are reasonably accurate and they provide valuable insight to the diffraction for  $q \gtrsim 1 \text{ fm}^{-1}$ . For the first time one has a direct connection between the cross section and the nuclear geometry and NN amplitude parameters. Alternative methods must be employed for small  $q$  and there has been recent work relevant to that regime also. Germond and Johnson<sup>82)</sup>, and in a more general approach, Frahn<sup>83)</sup>, have considered analytic approaches to diffraction using nonasymptotic methods. There is considerable overlap in the angular range of validity of the asymptotic and nonasymptotic methods. However, analytic results in nonasymptotic diffraction theory generally rely on specific analytic forms for the impact parameter amplitude  $S_F(b)$ .<sup>84,85)</sup> The importance and flexibility of the asymptotic methods described above lies in the fact that the dynamical information need not be lost by forcing  $S_F(b)$  to take some convenient

analytic form. This point is illustrated by the following example which shows how the range of the NN amplitude may be incorporated.

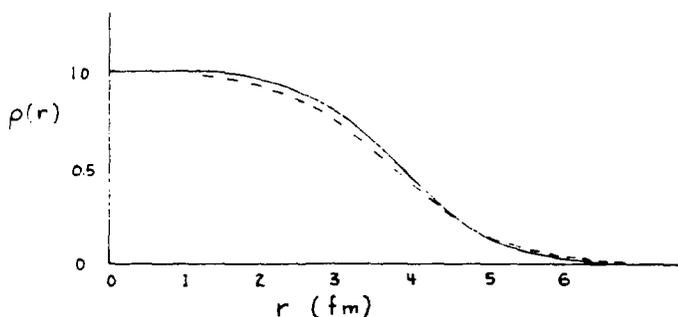
As is well known in Glauber theory<sup>86</sup>), the optical density involves the range  $\beta_A$  of the NN amplitude which has been taken to be zero in the approximation of eq. (10). Employing a Gaussian NN amplitude  $A(q) = A_0 e^{-\beta_A q^2}$  in eq. (12), and assuming a Woods-Saxon density as in eq. (10), the main effect of the NN range parameter is to cause a larger root-mean-square radius for the optical density than for the nuclear density via the relation  $r_{opt}^2 = r_{nuc}^2 + 6\beta_A$ . An approximate way to incorporate this effect in the spirit of analytic approximations is to ask for the new Woods-Saxon parameters  $c_{opt}$  and  $a_{opt}$  for the optical density which preserve the volume integral and incorporate the shift of rms radius. Generally the effects are small and they can be approximated to first order in  $\beta_A$  by the following relations:

$$c_{opt} = c - 2\beta_A / (c + \frac{7}{15} \frac{\pi^2 a^2}{c}) \quad (16a)$$

$$a_{opt}^2 = a^2 - 6\beta_A / \pi^2 \quad (16b)$$

Figure 9 compares the optical density for  $^{40}\text{Ca}$  with the nuclear density.

Fig. 9 Solid line shows a Woods-Saxon nuclear density for  $^{40}\text{Ca}$  and dashed line shows the equivalent Woods-Saxon optical density based on eqs. (16) for  $\beta_A = 0.2 \text{ fm}^2$ .



It is a simple matter to extend the analytic results (13)-(15) to include the influence of the  $\beta_A$  parameter by using  $c_{opt}$  and  $a_{opt}$  in place of the nuclear parameter  $c$  and  $a$  of eq. (10a). This seemingly trivial incorporation of the dynamical parameter  $\beta_A$  into the analytic relations would not be possible unless there was a direct connection between the cross section and the optical density. The small shifts between the optical and nuclear parameters illustrated in fig. 9 are, in fact, important. A principal source of uncertainty in the extraction of neutron radii from high energy proton scattering arises from the uncertainty in the NN range parameter  $\beta_A$  for pp and pn amplitudes. The relevant relation between rms radii for the optical density and the proton and neutron matter distributions is<sup>78</sup>)

$$r_{opt}^2 = \frac{Z}{A}(r_p^2 + 6\beta_{pp}) + \frac{N}{A}(r_n^2 + 6\beta_{pn}). \quad (17)$$

Proton scattering data essentially fix the rms optical radius  $r_{opt}$  to about  $\pm 0.02 \text{ fm}$  and electron scattering data fix the rms proton radius  $r_p$ . Thus, the imprecisely known quantities are the rms neutron radius  $r_n$  and the isospin average NN range parameter,

$$\bar{\beta} = \frac{Z}{A} \beta_{pp} + \frac{N}{A} \beta_{pn}. \quad (18)$$

Since the relation is linear, uncertainties in  $\bar{\beta}$  are directly translated into uncertainties in  $r_n$ ,

$$\delta r_n = -\frac{6\delta\bar{\beta}}{r_n} \left(\frac{A}{2N}\right). \quad (19)$$

In fig. 1, the proton scattering data from  $^{40}\text{Ca}$  is shown together with two equally good KMT optical potential fits labelled solution 1 and solution 2 based on NN range parameters differing by  $\delta\bar{\beta} = 0.1 \text{ fm}^2$ . The difference in rms neutron radii from solutions 1 and 2 may be estimated from eq. (19) to be  $\delta r_n = 0.17 \text{ fm}$ , whereas the careful optical model calculations<sup>28)</sup> yield  $\delta r_n = 0.20 \text{ fm}$ . Thus, the simple estimate is not misleading and one sees that the uncertainty  $\delta\bar{\beta}$  must be considerably reduced before one can extract absolute neutron radii from proton diffraction data to much better than  $\pm 0.1 \text{ fm}$  accuracy.

The situation is somewhat brighter for isotopic shifts because the NN range uncertainty cancels out to a degree. Figure 10 shows the ratio of  $^{48}\text{Ca}$  and  $^{40}\text{Ca}$  cross sections dramatizing the sensitivity to isotopic differences. Based on some simple assumptions, the analytic expressions of eqs. (13)-(15) can be used to understand the basic features seen in fig. 10. For nuclear targets 1 and 2, the cross section ratio is approximately

$$\frac{\sigma_1}{\sigma_2} \approx C e^{2\pi q \Delta a} \left( 1 - \frac{\sin(q\Delta c) \sin(2q\bar{c} + \bar{\phi})}{\sin^2(qc_2 + \frac{1}{2}\phi_2) + \sinh^2 \psi_2} \right) \quad (20)$$

where  $\Delta a = a_1 - a_2$ ,  $\Delta c = c_1 - c_2$ ,  $\bar{c} = \frac{1}{2}(c_1 + c_2)$  and  $\bar{\phi} = \frac{1}{2}(\phi_1 + \phi_2)$  with  $\phi$  and  $\psi$  defined as in eqs. (14) and (15) for each nucleus.

The cross section ratio involves tangent-like oscillations due to  $\sin(2q\bar{c} + \bar{\phi}) / \sin^2(qc_2 + 1/2\phi_2) \approx 2/\tan(q\bar{c} + 1/2\bar{\phi})$ . The Coulomb interaction and the real to imaginary ratio  $\rho$  of the NN amplitude cause  $\psi_2$  to be nonzero and this controls the amplitude of these oscillations. The tangent-like oscillations are centered on an exponentially increasing curve which is rather directly related to the diffusivity difference  $\Delta a$  of the two-matter distributions. Detailed optical model fits to these data by L. Ray, et al.<sup>28)</sup> are shown by the solid and dashed lines for two NN amplitude sets and the conclusion is that  $r_n(^{48}\text{Ca}) - r_n(^{40}\text{Ca}) = 0.16 \text{ fm} \pm .05 \text{ fm}$ . Figure 11 shows neutron density differences deduced for 800 MeV proton scattering from calcium isotopes compared with density matrix expansion (DME) theoretical predictions.<sup>87)</sup> Again the chief contribution to the uncertainty is from the NN amplitudes. The solution 2 NN amplitudes of Ray and the available phase shift analyses of 800 MeV NN scattering yield the range parameter  $\beta_A \approx .2 \text{ fm}^2$ . However, this value leads to values of the neutron radius about 0.10 fm smaller than expected from DME theories. The solution 1 NN amplitudes of Ray employ  $\beta_A \approx .1 \text{ fm}^2$  and this value has been successful in fitting proton scattering data on a variety of nuclei with neutron radii closer to those expected from the DME theory.

Nucleon-nucleon amplitudes are generally considered to be better known in the 200 MeV-500 MeV energy range. The ambiguities encountered at 800 MeV have motivated experiments in the 400-500 MeV range where phase shifts are available and more conclusive tests of the multiple scattering theories are possible.

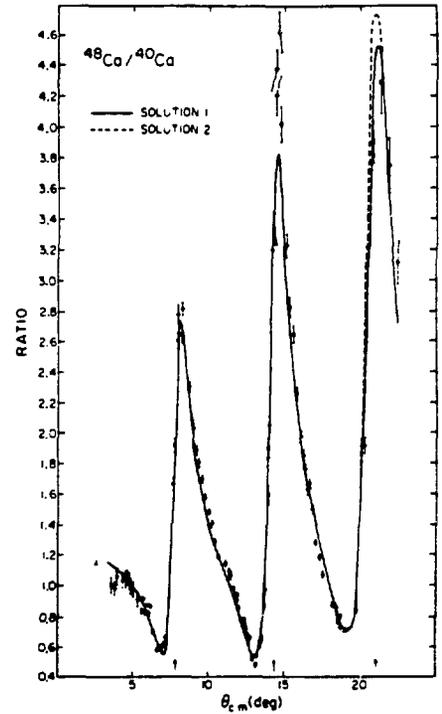


Fig. 10 Ratio of elastic differential cross sections for  $^{48}\text{Ca}$  and  $^{40}\text{Ca}$  from fig. 1.

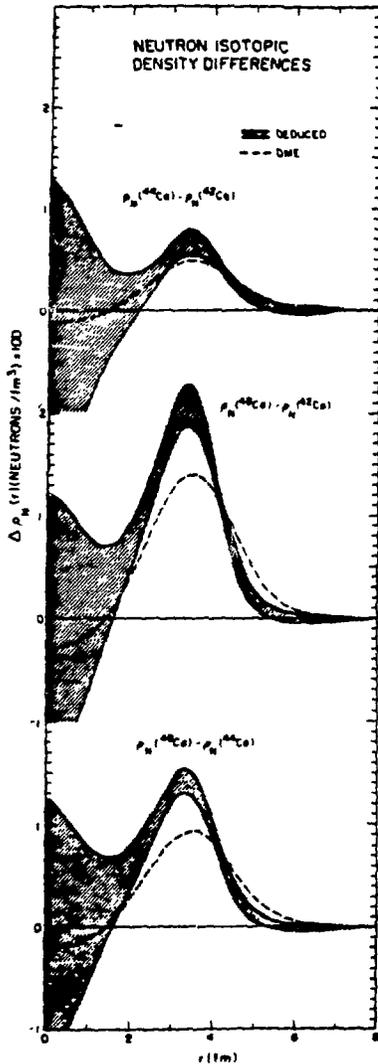


Fig. 11 Isotopic neutron density differences deduced by 800 MeV proton scattering are shown by shaded areas. Density matrix expansion results (DME) are shown by dashed lines.

### 3. Spin

A general form for the spin 1/2-spin 0 scattering amplitude is  $F(q) + \vec{\sigma} \cdot \hat{n} G(q)$ , where

$$F(q) = \frac{k}{i} \int_0^\infty db b J_0(qb) \left[ \frac{S_F^{(+)}(b) + S_F^{(-)}(b)}{2} \right] \quad (21a)$$

$$G(q) = k \int_0^\infty db b J_1(qb) \left[ \frac{S_F^{(+)}(b) - S_F^{(-)}(b)}{2} \right] \quad (21b)$$

The two Fourier-Bessel amplitudes  $S_F^{(\pm)}$  involve different geometries due to the spin dependence of NN amplitudes:

$$f(q)/(2ik) = A(q) + i\vec{\sigma} \cdot \vec{q} \times \hat{z} C(q) \quad (22)$$

For an uncorrelated nuclear density, the following simple form emerges for  $Z$  protons and  $N$  neutrons<sup>73</sup>):

$$S_F^{(\pm)}(b) = [i - \bar{A}_p(b) \pm \bar{C}'_p(b)]^Z [i - \bar{A}_n(b) \pm \bar{C}'_n(b)]^N \quad (23)$$

where  $\bar{A}_p(b)$  and  $\bar{A}_n(b)$  are defined as in eq. (12) for protons and neutrons. Spin functions  $\bar{C}_p(b)$  and  $\bar{C}_n(b)$  are defined in exactly the same fashion except that the spin flip amplitude  $C(q)$  from eq. (22) replaces the scalar amplitude  $A(q)$ . In eq. (23) the primes denote derivatives with respect to impact parameter.

For large  $q$  the scattering amplitudes may be approximated based on the asymptotic forms for the Bessel functions. The result is most simply expressed in terms of amplitudes  $F(\pm)$  for spin-up and spin-down scattering:

$$F(\pm) \equiv F \pm G = \frac{k}{i} \left( \frac{2}{\pi q} \right)^{1/2} \int_0^\infty db b^{1/2} \left( e^{\pm i(qb - \pi/4)} S_F^{(+)}(b) + e^{\mp i(qb - \pi/4)} S_F^{(-)}(b) \right). \quad (24)$$

In addition to the usual polarization parameter  $P$ , Glauber and Osland<sup>47)</sup> have pointed out a second parameter  $Q$  related to spin rotation. These two parameters are the real and imaginary parts of the scalar-spin flip interferences:

$$P + iQ = \frac{|F^{(+)}|^2 - |F^{(-)}|^2 + 2i|F^{(+)}||F^{(-)}| \sin \phi_{+-}}{|F^{(+)}|^2 + |F^{(-)}|^2} \quad (25)$$

where  $\phi_{+-}$  is the relative phase of  $F^{(+)}$  and  $F^{(-)}$ . Thus  $P$  determines the cross section difference for spin-up and spin-down scattering and  $Q$  determines the relative phase of spin-up and spin-down amplitudes.

In a simple but instructive limit where only the first-order spin-flip correction is retained and the flip amplitude  $C(q)$  is proportional to the scalar amplitude  $A(q)$ , the observables  $P$  and  $Q$  are smooth functions of  $q$  determined entirely by the forward NN amplitudes  $A_0$  and  $C_0$  in almost the same fashion as for NN scattering<sup>50,51)</sup>:

$$P + iQ \approx \frac{2q A_0 [iC_0]^*}{|A_0|^2 + q^2 |C_0|^2}. \quad (26)$$

The polarization data summarized in fig. 2 have tangent-like oscillations superimposed on a smooth background essentially determined by eq. (26). The explanation of the oscillations has been given by Amado, McNeil and Sparrow<sup>88)</sup> based on applying the saddle point method to the spin amplitudes to develop analytic formulas. When the spin terms  $\bar{C}'(b)$  in eq. (23) can be treated as a perturbation of the dominant scalar terms involving  $\bar{A}(b)$ , Amado, McNeil and Sparrow argue that the saddle points determined in eq. (11) are simply shifted. For the part of eq. (24) involving  $S_F^{(+)}(b) \approx \exp(-\bar{A}(b) + \bar{C}'(b))$ , where  $\bar{A}$  and  $\bar{C}$  are isospin-averaged quantities, there are two saddle points given by

$$b_{\pm}^{(+)} = b_{\pm} + w \left( 1 + \frac{\delta_{\pm}}{c \pm i\pi a} \left( 1 - 3 \left[ \frac{q(c \pm i\pi a)}{-\alpha} \right]^{2/3} \right) \right). \quad (27)$$

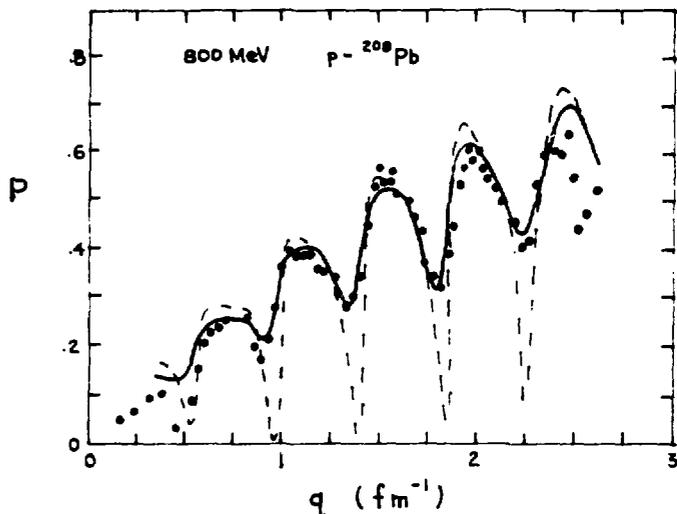
Similarly for the part of eq. (24) involving  $S_F^{(-)}(b) \approx \exp(-\bar{A}(b) - \bar{C}'(b))$  there are two saddle points,  $b_{\pm}^{(-)}$ , given also by eq. (27) but with the sign of  $w$  reversed. The complex length parameter  $\delta_{\pm}$  represents the geometry difference between  $\bar{A}(b)$  and  $\bar{C}(b)$  functions. Using eq. (16), this geometry difference may be directly related to the NN amplitude range parameters  $\beta_A$  and  $\beta_C$  as follows:

$$\delta_{\pm} = (\beta_C - \beta_A) \left( \frac{2}{c + \frac{7}{15}(\pi^2 a^2/c)} \pm \frac{3i}{\pi a} \right). \quad (28)$$

Analytic results for the spin-up and spin-down scattering amplitudes of eq. (24) follow in a straightforward manner.

The dashed line in fig. 12 shows a calculation of the  $p$ -<sup>208</sup>Pb polarization based on these results, but omitting the important Coulomb effects. Two complex length parameters govern the polarization. The first is  $w = C_0/A_0$  which has been taken to be  $w = -0.16 - i0.20$  fm based on suitably averaging the pp and pn amplitudes obtained from Arndt's phase shifts<sup>89)</sup>. The second complex length,

Fig. 12 Asymptotic approximation to polarization for elastic scattering of 800 MeV protons by  $^{208}\text{Pb}$ . Experimental data are shown by dots. Solid line shows prediction of data-to-data relation.



$\delta_{\pm} = 0.08 \pm i0.15$  fm, has been fit to the polarization data. The parameter  $w$  governs the rise to the polarization that is similar to NN scattering, eq. (26), while the geometry difference parameter  $\delta_{\pm}$  is responsible for the tangent-like oscillations. The lengths  $\delta_{\pm}$  should also be related to the NN amplitude parameters as in eq. (28)

keeping in mind that the range parameters  $\beta_C$  and  $\beta_A$  are generally complex. This alters the analysis of reference because  $\delta_{+} \neq \delta_{-}^*$ . A recent phase shift analysis at 0.8 GeV leads to a prediction  $\delta_{+} = -.173 - i.001$  fm and  $\delta_{-} = .161 + i.065$  fm, both of which are far from the effective values obtained by fitting the  $^{208}\text{Pb}$  data.

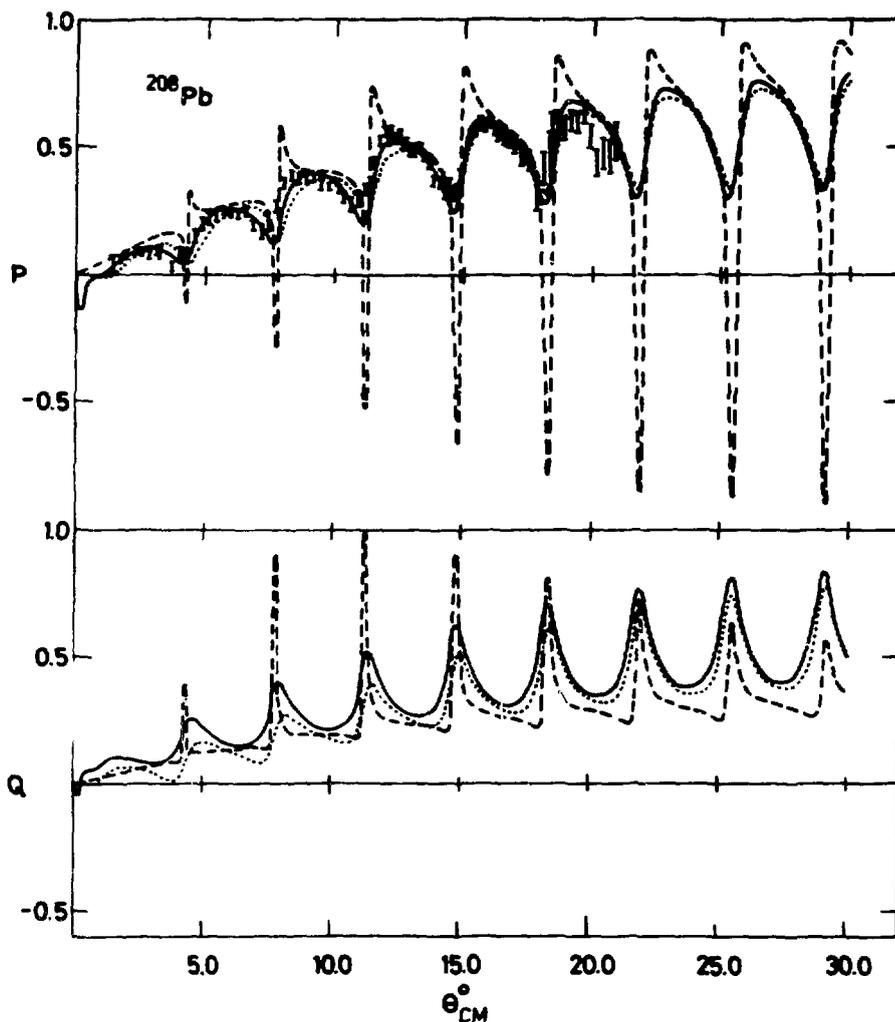
Amado, McNeil and Sparrow<sup>88</sup>) develop simple explanations of the characteristic tangent-like oscillations and they also point out a contribution proportional to  $1/\sigma$ , where  $\sigma$  is the elastic cross section, that seems to be new to polarization phenomenology. They show that the geometry difference between scalar  $\bar{A}(b)$  and spin flip  $\bar{C}(b)$  functions is crucial to understanding the spin measurements. The polarization is also expressed in terms of the elastic scattering cross section in a "data-to-data" relation. Although this approach loses contact with the underlying dynamics, it does provide a much better fit to the polarization as indicated by the solid line in fig. 12.

Using nonasymptotic methods, Fäldt and Ingemarsson<sup>52</sup>) have also developed approximate expressions for the polarization observables reaching some of the same conclusions. Fäldt and Ingemarsson<sup>53</sup>) have incorporated both Coulomb and magnetic effects in their analysis of  $p\text{-}^{208}\text{Pb}$  polarization as indicated in fig. 13. The Coulomb effects fill the dramatic minima and the magnetic effects are also important as was pointed out earlier by Osland and Glauber<sup>46</sup>). A recent KMT optical potential calculation by L. Ray also indicates the importance of magnetic spin flip effects. By comparison of figs. 12 and 13, one sees that the "data-to-data" relation in fig. 12 has mainly restored the missing Coulomb effects.

The above review has focused on the simple insights to high energy proton scattering which follow from the development of direct relations between the optical densities and the observed cross sections and polarizations. Additional insights should be mentioned as well--such as the development of simple relations between elastic and inelastic cross sections based on approximate models for the inelastic transition density<sup>90-91</sup>). Since the scattering is dominated by the "bright spots", many of the features of inelastic scattering can be quite simply explained using the Tassie model<sup>92</sup>) for collective transition density in the nuclear surface. The dynamical issues in proton scattering are considerably clarified by the asymptotic methods. Proton scattering measurements have an interferometric sensitivity to certain combinations of nuclear length parameters and NN amplitude parameters. Although firm conclusions cannot be drawn at present, there is clearly a trend towards finding that the free NN amplitudes and standard nuclear geometry parameters do not yield good fits to the data, but that effective NN amplitude parameters can be found that do fit the data. Preliminary

analyses of the first measurements of the spin rotation parameter  $Q$  at 0.5 GeV also support this trend.

Fig. 13 Polarization ( $P$ ) and spin rotation ( $Q$ ) for elastic scattering of 800 MeV protons by  $^{208}\text{Pb}$ . Dashed line omits Coulomb effects and dotted line omits magnetic effects.



#### 4. NN amplitudes

The most obvious uncertainties in theoretical analyses of proton scattering continue to be the NN amplitudes, however, recent progress in NN scattering has been impressive. Much of the interest has been driven by the possibility that dibaryons might be the explanation of the quite unexpected and large spin dependence discovered in Argonne National Laboratory measurements of  $\Delta\sigma_L$  and  $\Delta\sigma_T$ . A very nice review of the spin-dependent cross section measurements has recently been given by H. Spinka<sup>93</sup>). However, the opening of the  $\Delta$ -isobar production channel in NN scattering provides a simpler explanation of the basic features of the data<sup>94-96</sup>). Phase shift analyses for energies above 300 MeV are now available from three sources<sup>97-99</sup>) although the data base, particularly for np scattering, is not yet very good. In multiple scattering calculations simple assumptions about NN amplitudes are often employed or, alternatively, "effective" NN amplitude parameters are self-consistently deduced by optimizing fits to selected proton-nucleus data. In this section, some information regarding the "free" and "effective" NN amplitudes is reviewed. A more extensive review with tables of NN amplitudes is contained in ref. 78. A recent paper by Love and Franey<sup>100</sup>) provides simple forms for the NN  $t$  matrix to 800 MeV for use in distorted wave calculations.

A parameterization of the NN amplitude that is convenient for multiple scattering calculations is

$$(2ik)^{-1} f(q) = A(q) + i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \hat{z} C(q) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 B(q) + \vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q} D(q) + \sigma_{1z} \sigma_{2z} E(q) \quad (29)$$

where  $A(q) = A_0 e^{-\beta_A q^2}$ ,  $C(q) = C_0 e^{-\beta_C q^2}$ , etc., and  $\vec{q}$  is the momentum transfer and  $\hat{z}$  is the average momentum direction. The double spin flip amplitudes B, D and E do not play a significant role in elastic scattering by spin-zero nuclei, however they can be quite important in selected inelastic transitions. At forward angles, convenient forms for these amplitudes omitting Coulomb and magnetic spin flip effects are<sup>78)</sup>:

$$\begin{aligned} A_0 &= (8\pi)^{-1} \epsilon(1-i) \\ C_0 &= w A_0 \\ B_0 &= -(16\pi)^{-1} \Delta\sigma_T (1-i\epsilon_T) \\ E_0 &= -(16\pi)^{-1} [\Delta\sigma_L (1-i\epsilon_L) - \Delta\sigma_T (1-i\epsilon_T)] \end{aligned} \quad (30)$$

and

$$D_0 = -\frac{ig^2/(4\pi)}{4k_2 \sqrt{s_2} m_\pi^2}$$

The double spin flip amplitudes  $B_0$  and  $E_0$  are directly related to the cross section differences  $\Delta\sigma_T$  and  $\Delta\sigma_L$  in transverse and longitudinal spin states. Dispersion calculations in pure spin channels<sup>101)</sup> have been used to calculate the associated real to imaginary ratios  $\rho_T$  and  $\rho_L$  which tend to be large (typically 3 to 4). The D amplitude is dominated by one pion exchange at small q.

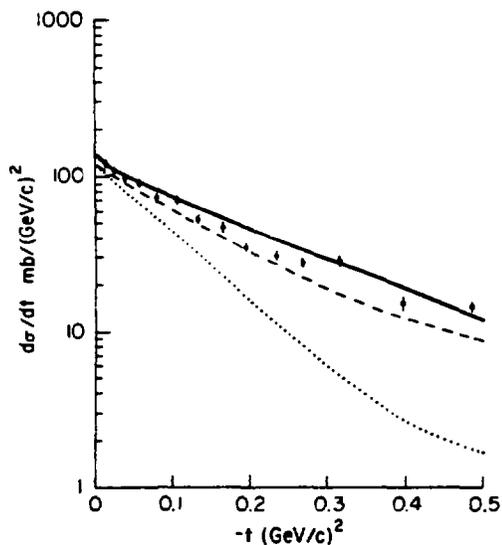
Due to the sizeable double spin flip terms, it is not possible to extract the more important scalar (A) and spin flip (C) amplitudes without a complete set of measurements.

Figure 14 shows a typical example where phase shift analysis<sup>99)</sup> has been used to determine the five complex amplitudes. The differential cross section is built up by the incoherent sum,

$$\frac{d\sigma}{dt} = 4\pi(|A|^2 + 2q^2|C|^2 + |B|^2 + |B+q^2D|^2 + |B+E|^2), \quad (31)$$

and the polarization is given by

$$P = 8\pi \operatorname{Re}[(A+B)(iqC)^*]/(d\sigma/dt). \quad (32)$$



The slope parameter of the differential cross section fit according to  $\exp(-2\beta_0 q^2)$  is  $\beta_0 \approx .1 \text{ fm}^2$  while the slope parameter of just the scalar amplitude is about twice as big:  $\beta_A \approx .2 \text{ fm}^2$ . The absence of phase shift amplitudes has often left no alternative to the simple assumption  $\beta_A \approx \beta_0$  in many proton scattering analyses,

Fig. 14 Proton-proton differential cross section at 1 GeV based on eq. (31) for Hoshizaki phase shifts —. Dashed line omits double spin flip amplitudes B,D,E. Dotted line omits all spin amplitudes.

however, it now becomes clear that substantial differences must be expected. As discussed above, the difference between  $\beta_0$  and  $\beta_A$  is very relevant to neutron radius determination. Figure 15 illustrates that the double spin flip B amplitude in eq. (32) plays a significant role in determining the polarization. One of the implications is that the spin flip range parameter  $\beta_c$  can only be reliably extracted when all the NN amplitudes are known.

Three recent phase shift analyses have been used in ref. 78 to extract the Gaussian NN parameters. For the scalar and spin flip amplitudes, the Gaussian approximation is generally accurate for  $q \lesssim 2 \text{ fm}^{-1}$  provided the slope parameters are complex. Table I lists some NN amplitude parameters at 500 MeV.

The Bugg, et al. phase shifts<sup>97)</sup> are based on extensive measurements at TRIUMF at 515 MeV. The Hoshizaki<sup>99)</sup> and the Arndt<sup>98)</sup> phase shift and solutions used in this comparison did not include all the data. Thus, the table represents the status of NN amplitudes near 500 MeV as of about the beginning of 1980 and reasonable but far from precise agreement of the three phase shifts is apparent. One surprise that emerges from the phase shift amplitudes is the need for large imaginary parts to the slope parameters  $\beta_A$  and  $\beta_c$ . As may be seen from eq. (28) this implies substantial geometric differences between the scalar and spin flip optical densities at 500 MeV.

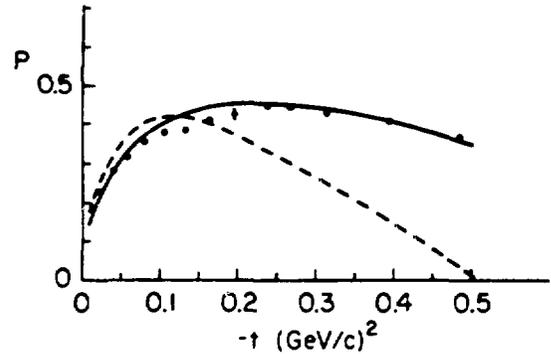


Fig. 15 Proton-proton polarization at 1 GeV based on eq. (32) for Hoshizaki phase shifts ———. Dashed line omits double spin flip B amplitude.

Table I: Comparison of 500 MeV phase shift amplitudes

	$A_0 (\text{GeV}/c)^{-2}$	$\beta_A (\text{GeV}/c)^{-2}$	$C_0 (\text{GeV}/c)^{-3}$	$\beta_c (\text{GeV}/c)^{-2}$
Bugg, et al. <sup>97)</sup> (.515 GeV)	3.24 - i0.74	4.27 - i3.93	-1.43 - i6.04	1.66 + i0.0
Arndt <sup>98)</sup> (.508 GeV)	3.22 - i1.41	5.35 - i6.78	-1.39 - i6.70	2.55 + i0.33
Hoshizaki <sup>99)</sup> (.508 GeV)	3.08 - i1.31	4.39 - i5.20	-1.76 - i6.55	3.01 + i0.68

An important point regarding the role of NN amplitudes in proton scattering is that they generally enter as in eq. (12) multiplied by a nuclear form factor. Because the form factor decreases rapidly with  $q$ , the NN amplitudes are primarily influential at small  $q \lesssim 2 \text{ fm}^{-1}$ . This means that accurate NN amplitudes at forward angles  $\theta_{\text{cm}} \lesssim 40^\circ$  are needed as input to proton-nucleus analyses, although for light nuclei the NN amplitudes at larger  $q$  also are important. One of the best ways to obtain pp amplitudes at small angles is to use a proton spectrometer as has been done in a recent Los Alamos experiment.<sup>102)</sup>

Table II lists some representative pp amplitude parameters which have been used in analyses of 1 GeV proton-nucleus data including polarizations. Chaumeaux, Layly and Schaeffer (CLS)<sup>63)</sup> and also Alkhazov, Belostotsky and Vorobyov (ABV)<sup>73)</sup> have obtained fits for heavy nuclei using relatively simple assumptions about NN parameters. An analysis of 1 GeV p-<sup>4</sup>He scattering by Wallace and Alexander (AWH)<sup>103)</sup> was based on pp amplitude from Hoshizaki's phase shift analysis,

however, the pn spin flip amplitudes were fit to the  $^4\text{He}$  data. Finally, the list includes amplitude parameters deduced by McNeil<sup>104</sup>) in an analysis of 1 GeV p-d scattering. Thus, each set of amplitudes in the table provides a good fit to one or more nuclei.

	$A_D(\text{GeV}/c)^{-2}$	$B_A(\text{GeV}/c)^{-2}$	$C_C(\text{GeV}/c)^{-3}$	$B_C(\text{GeV}/c)^{-2}$
Chaumeaux-Layly-Schaeffer (CLS)	$4.88 + i0.0$	$2.50 + i0.0$	$0.0 - i3.78$	$4.45 + i0.0$
Alkhazov-Belostotsky-Vorobyov (ABV)	$4.84 + i0.24$	$2.76 + i0.0$	$1.60 - i3.20$	$8.6 + i0.0$
Alexander-Wallace-Hoshizaki (AWH)	$4.79 + i0.88$	$4.39 - i0.24$	$-3.02 - i5.01$	$3.43 + i0.0$
McNeil (M)	$4.88 - i0.5$	$2.65 + i0.0$	$2.47 - i4.97$	$3.8 + i0.0$

Figures 16 and 17 compare the predictions for 1 GeV p-d scattering based on each set of NN parameters in this table<sup>104</sup>).

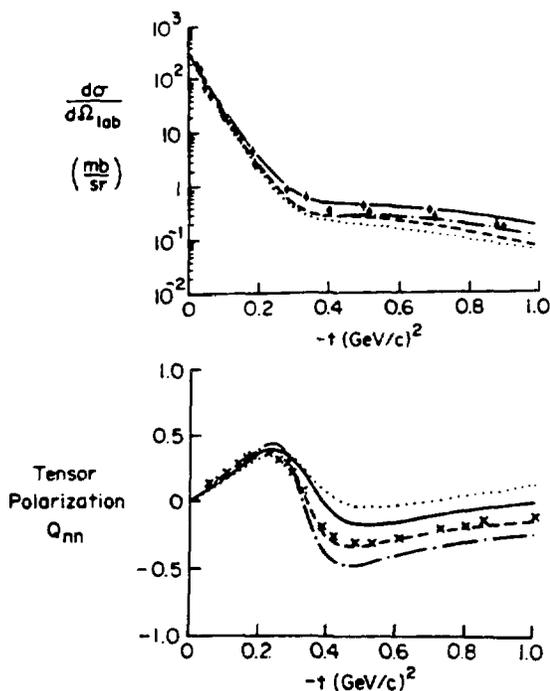


Fig. 16 Differential cross section and tensor polarization  $Q_{nn}$  data for 1 GeV p-d elastic scattering<sup>105</sup>) are compared with predictions based on NN amplitudes of Table II:<sup>104</sup>)

— M, - · - · - CLS, - - - - ABV, · · · · AWH.

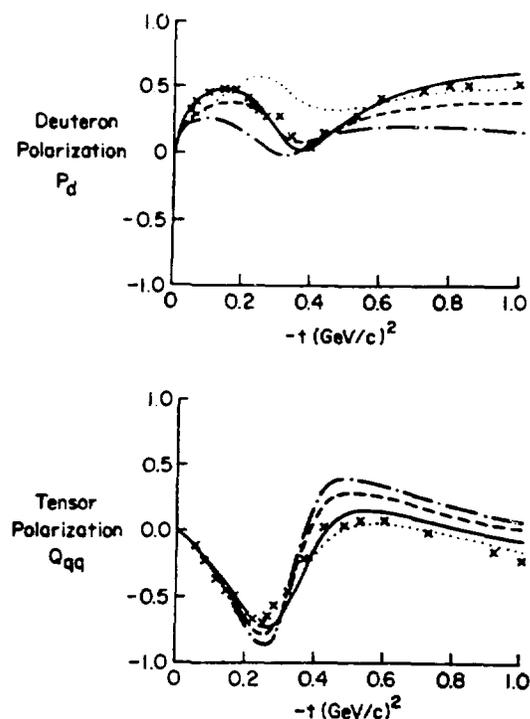


Fig. 17 Tensor polarization data for 1 GeV p-d elastic scattering. Lines have some meaning as in fig. 16.

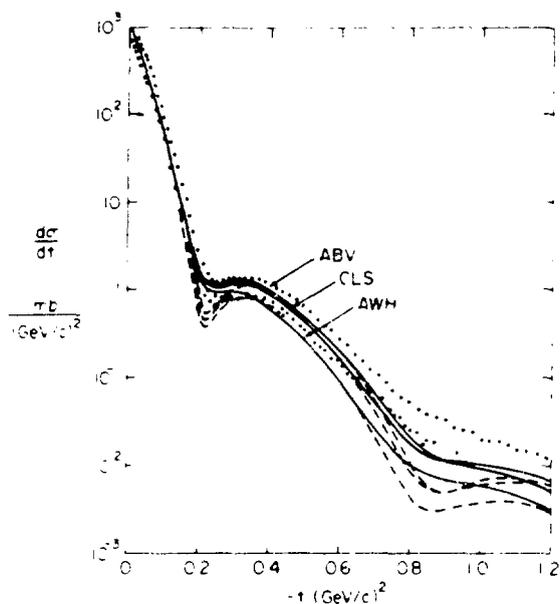


Fig. 18 Differential cross section data (upper points refs. 15, 7; lower points refs. 107, 108.) with predictions based on Gaussian NN amplitude parameters of Table II. Dashed line shows Glauber calculation while solid lines include intermediate isobar effects.

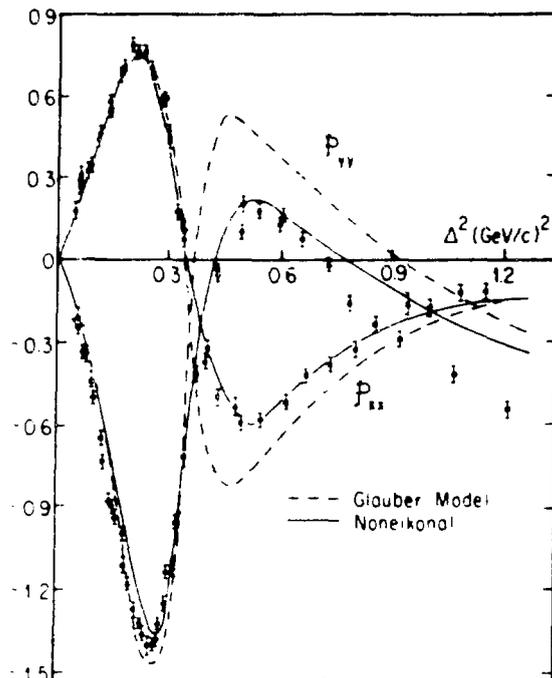


Fig. 19 1 GeV p-d elastic scattering tensor polarization predictions with and without noneikonal effects<sup>105</sup>).

Figure 18 shows similar predictions for 1 GeV p-<sup>4</sup>He scattering<sup>78</sup>). One point of these comparisons is that no one set of NN parameters has yet been successful in fitting both light and heavy nucleus scattering data. The second point is that light ion scattering is very sensitive to the NN amplitude parameters and therefore provides a very important testing ground for the proton-nucleus dynamics. An analysis by Alberi, et al.<sup>105</sup>), fig. 19, has recently pointed out that noneikonal effects are important in calculations of p-d spin-dependent observables.

Multiple scattering theory predictions are generally in qualitative agreement with proton-nucleus data, however, the differences in details can be large. It remains unsettled whether the "free" NN amplitudes become significantly modified in the nucleus, however the non-negligible spin effects suggest that the interaction should be modified by Pauli effects. In light ion scattering, there is a significant modification of a kinematical nature due to the NN amplitude being in the Breit frame rather than the NN center-of-mass frame<sup>117,78</sup>). Also, there is a significant contribution to 1 GeV p-<sup>4</sup>He scattering due to isobar intermediate states as is evidenced by the difference of solid and dashed curves in fig. 18.

## 5. $\Delta$ -isobars

The proton-nucleus reaction mechanism at intermediate energies is generally assumed to be dominated by a sequence of quasi-free NN scatterings. Above the inelastic threshold, this means that there is a substantial  $\Delta$ -production probability with subsequent decay of the  $\Delta$  to  $\pi N$  states. Recent isospin analyses by VerWest and Arndt<sup>109</sup>) suggest that all of the free NN  $\rightarrow$  NN $\pi$  reactions can be understood in terms of an intermediate  $N\Delta$  state in the energy region 0.5 to 1.0 GeV.

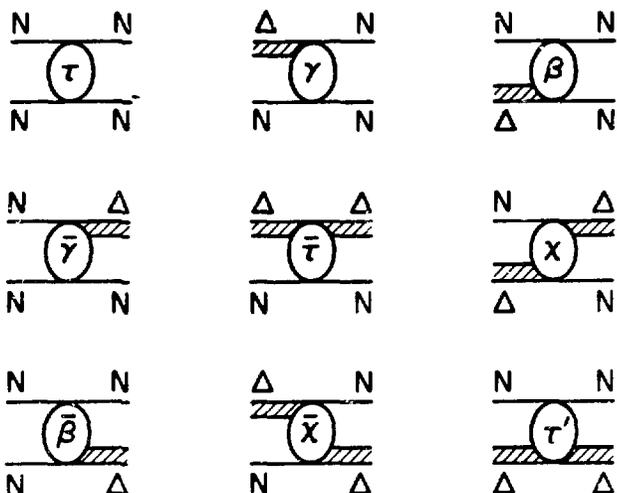


Fig. 20 Coupled-channel  $t$ -matrix elements for NN,  $\Delta N$  and  $N\Delta$  channels.

A dynamical model of NN scattering based on coupled NN,  $N\Delta$  and  $\Delta N$  channels is illustrated in fig. 20. In this model the nucleon-nucleon  $t$  matrix is represented by a  $3 \times 3$  matrix in which the usual NN  $t$  matrix appears as the upper left entry. A static model of the isobar is assumed<sup>110,111</sup>) and the three-dimensional dynamical equations are chosen to incorporate approximate cancellations between crossed and uncrossed Feynman diagrams. A detailed

discussion of this model is given in ref. 78 and therefore the present review will focus on qualitative aspects of the theory. Related work on a Hamiltonian theory of  $\pi$ - $N$ - $\Delta$  systems is also given in a recent paper by Betz and Lee<sup>112</sup>).

The distinction between  $\Delta N$  and  $N\Delta$  channels is important in proton-nucleus collisions since in the  $\Delta N$  case, the  $\Delta$ -isobar carries the projectile proton's momentum and in the  $N\Delta$  case, a nuclear  $\Delta$ -excitation is made. The nuclear  $\Delta$ -excitations are, of course, closely related to the isobar-hole states produced in pion scattering.

A multiple scattering theory based on the coupled-channel NN dynamics follows in a straightforward fashion in terms of a Watson-type expansion for the  $3 \times 3$   $\hat{T}$  matrix:

$$\hat{T} = \sum_i \hat{t}_i + \sum_i \sum_{j \neq i} \hat{t}_i \hat{G} \hat{t}_j + \dots \quad (33)$$

where each two-body operator,  $\hat{t}_i$ , is a  $3 \times 3$  matrix as in fig. 20. The  $3 \times 3$  matrix  $\hat{G}$  is diagonal with elements describing the propagation of a nucleon or a  $\Delta$ , depending on which channel carries the projectile proton's momentum, in the presence of a nuclear or isobaric excitation of the residual system.

In this formalism, the usual  $T$  matrix for proton-nucleus scattering must be projected from  $\hat{T}$  by demanding that the initial and final states have no isobars present and this causes the single scattering terms on eq. (33) to involve just the NN  $t$  matrix as in the usual Watson multiple scattering expansion. The double scattering terms contain new contributions due to three-body isobar intermediate state effects that cannot be included in the free NN  $t$  matrix. Originally discussed in relation to inelastic shadowing corrections for total proton-nucleus cross sections at high energy<sup>113-115</sup>), these intermediate isobar effects were observed by Ikeda<sup>116</sup>) to yield a significant correction to  $p$ - $^4\text{He}$  angular distributions in the 1 GeV energy region. Figure 21a illustrates one intermediate isobar contribution. Although the formalism suggests that fig. 21a be viewed as a  $\Delta$ -excitation created on nucleon  $j$  and destroyed on nucleon  $i$ , an alternative and completely equivalent view is that of fig. 21b in which the incident proton is viewed as scattering in resonant fashion from a  $T=1$  meson current being exchanged by nucleons  $i$  and  $j$ . Thus, the intermediate  $\Delta$ -states are related to meson exchange currents. A second type of intermediate isobar state is indicated in fig. 21c, or equivalently in fig. 21d, in which a nuclear  $\Delta$ -excitation is created and later destroyed. Because a spin-isospin correlation is needed for either process a or c to contribute to proton elastic scattering, the two tend to be equivalent even though process c involves nucleon exchange in the intermediate state.

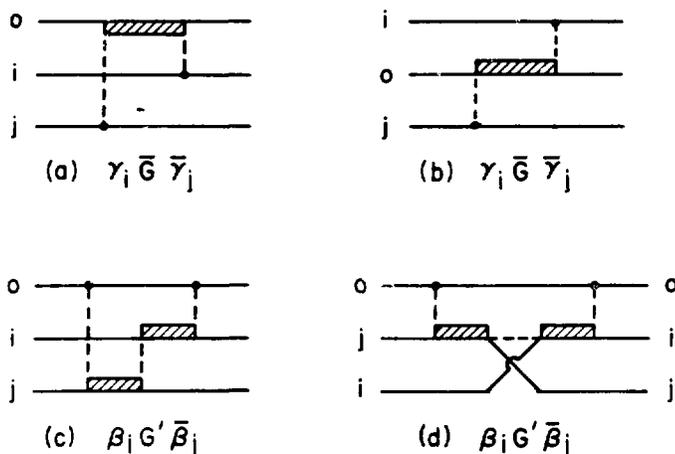


Fig. 21 Intermediate isobar states involving the projectile (0) and two target nucleons (i and j). See text for discussion.

Although the NN amplitudes are not well known at 1 GeV and even less is known about the  $NN \rightarrow N\Delta$  and  $N\Delta \rightarrow N\Delta$  amplitudes involved in the coupled-channel multiple scattering theory, calculations based on reasonable assumptions have indicated a significant intermediate isobar contribution to 1 GeV  $p\text{-}^4\text{He}$  scattering. Figure

22 shows the result of calculations by Wallace and Alexander<sup>103</sup>) who find that the effect of intermediate isobar states is to fill the interference minimum which

generally shows up in multiple diffraction calculations without intermediate isobar effects. The possibility that multiple scattering of the  $\Delta$ -isobar could reduce the intermediate isobar contribution of fig. 21a has been pointed out by Auger, Lazard and Lombard<sup>118</sup>) in a contribution to this conference. Because of amplitude uncertainties, particularly with respect to the phases of the  $NN \rightarrow N\Delta$  amplitude and the magnitude and phase of the  $N\Delta \rightarrow N\Delta$  amplitude, the evidence for intermediate isobar contributions to  $p\text{-}^4\text{He}$  remains inconclusive but interesting. Although the intermediate isobar effect is thought to be small in heavy nuclei for  $N=Z$ , it does provide a specific correction in the  $T=1$  NN channel which has yet to be considered in analyses of isotopic shifts of neutron radii. Another, perhaps more important, type of intermediate isobar effect arises due to the two-body  $N\Delta$  intermediate states in the NN t matrix.

Because the intermediate isobar states are reached by a  $T=1$ ,  $J=1$  exchange, the Pauli principle must modify the intermediate  $\Delta N$  of the NN t matrix in the nuclear medium. Also, there are well-known spreading effects for the nuclear-isobar states<sup>119,120</sup>) which are excited as intermediate states of the NN t matrix. Other isobar related effects of potential importance arise from higher-order terms in the multiple scattering expansion. Figure 23 illustrates some third-order terms,

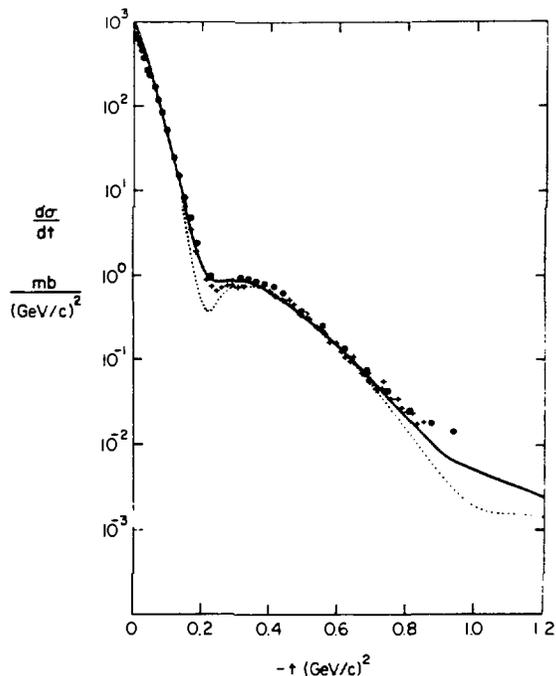


Fig. 22 Effect of intermediate states in 1 GeV  $p\text{-}^4\text{He}$  differential cross section. Dotted line shows Glauber calculation omitting isobar intermediate states. Kinematic transformation of the NN amplitudes<sup>117,78</sup>) fills the second minimum near  $-t = 0.9$  which is present in fig. 18.

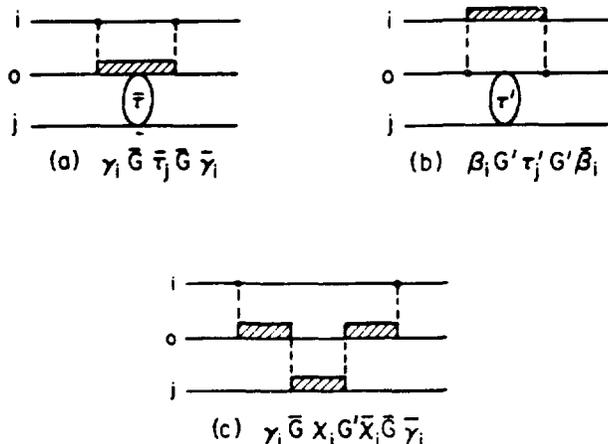


Fig. 23 Local field corrections due to interactions which modify the intermediate states of the NN  $t$  matrix due to the presence of a third nucleon.

called local field corrections, which act to modify the NN  $t$  matrix in the nuclear medium. In fig. 23a the intermediate isobar state in the scattering of the projectile (labelled 0) and nucleon  $i$  is modified by scattering of the isobar by a third nucleon (labelled  $j$ ). Figure 23b is a similar process except that the intermediate state nucleon scatters from a third nucleon.

Finally, fig. 23c indicates a more complicated local field correction due to  $\Delta$ -exchange in the intermediate states. All of these nuclear medium corrections to the NN  $t$  matrix can be organized into an effective NN  $t$  matrix in the nucleus in principle<sup>121</sup>), however a prerequisite to meaningful calculations of the effective  $t$  matrix is a satisfactory model of the free NN  $t$  matrix based on the coupled-channels approach.

Direct signals for the importance of isobars in proton-nucleus scattering are obtained in inclusive  $(p, p')$  experiments<sup>122, 123</sup>) which extend to large momentum loss. Figure 24 shows recent 800 MeV data for protons inelastically scattered by  $^2\text{H}$  and  $^{12}\text{C}$ . The  $\Delta$ -production shows up as a broad peak at large energy loss in

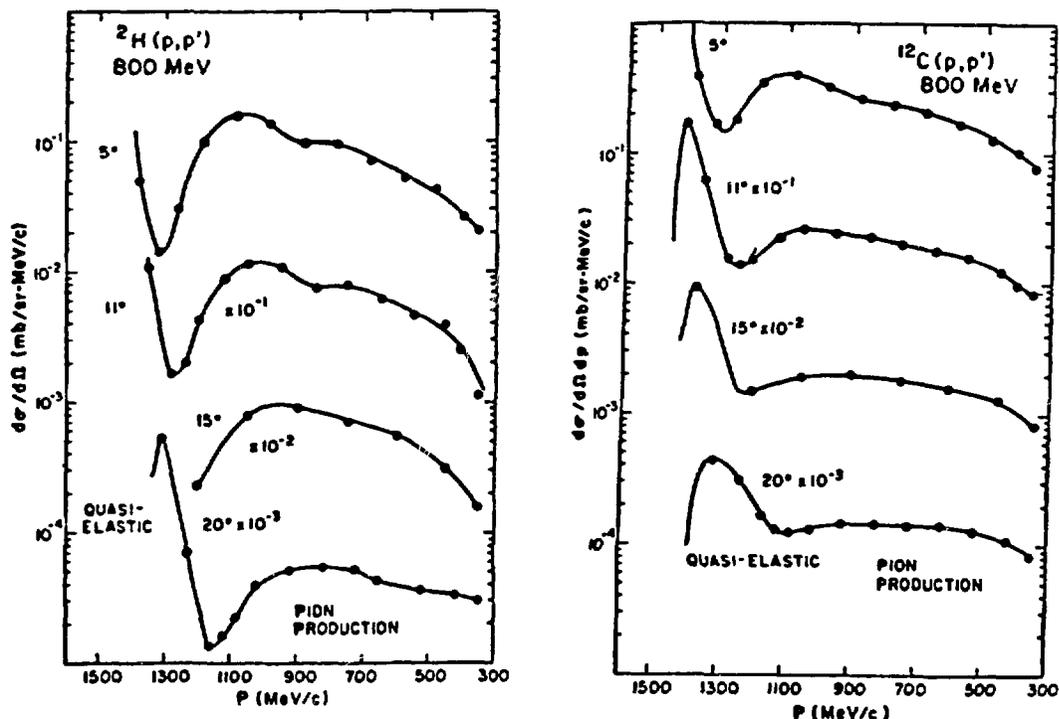


Fig. 24 Inclusive proton scattering spectra for  $^2\text{H}$  and  $^{12}\text{C}$  targets. The beam momentum is 1460 MeV/c. Regions labelled pion production involve quasi-free  $\Delta$ -production.

both spectra. The quasi-elastic NN scattering shows up as a sharper peak at smaller energy loss. The proton- $^{12}\text{C}$  reaction cross section inferred from such spectra indicates about one-third is due to quasi-free NN  $\rightarrow$  NN initiated processes and two-thirds is due to quasi-free NN  $\rightarrow$  N $\Delta$  initiated processes<sup>123,124</sup>). A very recent calculation by Krimm, Klar and Pirner<sup>125</sup>) reports success in describing the  $^{12}\text{C}$  (p,p') spectra based on using the eikonal approximation and a quasi-free multiple scattering theory related to the one discussed above.

Specific nuclear reactions can be chosen which favor the observation of isobars. A recent example is provided by the  $p+^3\text{He} \rightarrow x+^3\text{H}$  reaction where 850 MeV protons are incident on  $^3\text{He}$  and the triton is observed in the final state at forward angles<sup>126</sup>). Figure 25 shows the missing mass observed for the partner to  $^3\text{H}$  and there is a clear enhancement at the  $\Delta$ -isobar mass.

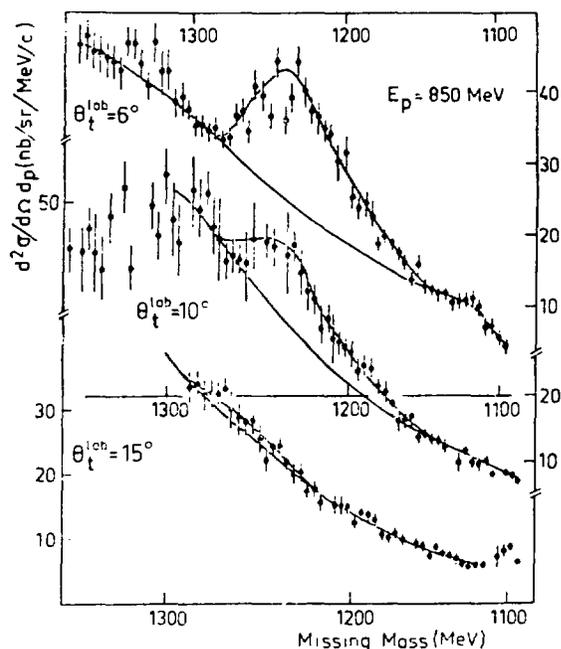


Fig. 25 Missing mass spectrum for  $p+^3\text{He} \rightarrow ^3\text{H}+x$  at three triton angles<sup>126</sup>).

## 6. Summary

Although asymptotic methods of analysis are very old, their application to high energy hadron nucleus diffraction is quite new. The bright spots which have been found to dominate proton-nucleus diffraction at nonforward angles provide both insight and simplification to theoretical analyses as well as a valuable guide to what information can be extracted from proton scattering. The development of phenomenological "data-to-data" relations for inelastic scattering to collective states suggests an underlying simplicity that mainly depends on the asymptotic normalization of the transition density at large  $r$ . To extract more detailed information seems to require the observation of significant differences between the inelastic scattering and the data-to-data predictions based on elastic scattering.

The development of direct connections between simple features of the data and the underlying geometric and dynamical ingredients of multiple diffraction theory is important. It is the basic feature which distinguishes the asymptotic theories from alternative approaches where the contact with the underlying dynamics is not made explicit. It is evident that effective NN amplitude parameters which are able to fit proton-nucleus data can be most easily deduced using the asymptotic theories. However, there are more parameters than can be determined from proton scattering by spin-zero nuclei. More difficult experiments involving the scattering of polarized protons from polarized light ions are required to complete a systematic program of measurements. As has been emphasized, these experiments provide a very fundamental testing ground for proton-nucleus dynamics. A much sharper confrontation between the "free" NN amplitudes and the "effective" amplitudes is likely by the time of the next conference in this series.

The role of isobars in proton-nucleus interactions begins to become visible in quasi-free scattering and in specific nuclear reactions. At present, the role of isobars is under intense investigation in nucleon-nucleon dynamics. Progress in nucleon-nucleon and proton-nucleus research is intertwined in this regard; however, there are clear theoretical reasons for suspecting the isobar intermediate-states of the NN interaction to be modified in the nucleus. A theory of multiple scattering which explicitly displays the isobar production channel has been developed.

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