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THE ISOTOPE DENSITY INVERSE PROBLEM IN MULTIGROUP NEUTRON TRANSPORT

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PROBLEM ODWROTNY DLA GĘSTOŚCI IZOTOPOWYCH W WIELOGRUPOWYM TRANSPORCIE NEUTRONÓW

ОБРАТНАЯ ПРОБЛЕМА В МНОГОГРУППОВОЙ МОДЕЛИ ПЕРЕНОСА НЕЙТРОНОВ ДЛЯ ОБОЗНАЧАНИЯ ИЗОТОПИЧЕСКИХ ПЛОТНОСТИ СРЕДЫ

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Abstract

The inverse problem for stationary multigroup anisotropic neutron transport is discussed in order to search for isotope densities in multielement medium. The spatial- and angular-integrated form of neutron transport equation, in terms of the flux in a group - density of an element spatial correlation, leads to a set of integral functionals for the densities weighted by the group fluxes. Some methods of approximation to make the problem uniquently solvable are proposed. Particularly P_0 angular flux information and the spherically-symmetrical geometry of an infinite medium are considered. The numerical calculation using this method related to sooner evaluated direct problem data gives promising agreement with primary densities. This approach would be the basis for further application in an elemental analysis of a medium, using an isotopic neutron source and a moving, energy-dependent neutron detector.

Streszczenie

Problem odwrotny dla stacjonarnego wielogrupowego anizotropowego transportu neutronów wykorzystany został do określania gęstości izotopów w ośrodku złożonym z wielu pierwiastków. Przestrzennie i kątowo wycałkowana postać równania transportu, zawierająca korelacje przestrzenne między strumieniami neutronów w grupach energetycznych i gęstościami składników izotopowych ośrodka, stanowi układ funkcjonałów całkowych określanych gęstości, ważonych przez strumienie wielogrupowe. Proponowane przybliżenia służą do jednoznacznego rozwiązania tego problemu, w szczególności rozważana jest izotropowa część mierzonego strumienia i nieskończony ośrodek sferycznie-symetryczny. Przeprowadzono wstępne obliczenia numeryczne tą metodą, w oparciu o wcześniej policzone dane z rozwiązania problemu prostego, co dało kilkunastoprocentową zgodność z gęstościami pierwotnymi, zadanymi w problemie prostym. Podejście takie mogłoby być podstawą przyszłego zastosowania do analizy zawartości izotopowych ośrodka przy użyciu izotopowego źródła neutronów i ruchomego, energetycznie czułego detektora.

Резюме

Обратная проблема для стационарного многогруппового переноса нейтронов была использована для определения плотности изотопов в многокомпонентной системе. Уравнение переноса, после пространственной и угловой интеграции, составляет систему интегральных функционалов плотности, содержащую пространственную корреляцию между потоками нейтронов в энергетических группах и изотопическими плотностями среды. Для однозначного решения проблемы есть использована изотропическая часть измеряемого потока в бесконечной сферическо-симметрической среде. В вычислениях этой методой с данными из ранне розрешенной прямой задачи было полученное обещающие согласие из плотностями заданными для прямой проблемы. Следующий метод может быть в будущем применён для анализа изотопического содержания среды, с использованием стационарного источника нейтронов и подвижного детектора регистрирующего энергетическую зависимость.

1. Introduction

In many applications of nuclear methods and techniques the inverse problem [Sabatier, 1978] for radiation transport in different media is of interest. The direct problem provides the measurable physical data from the known parameters of a model - in our case from known: (a) the spatial distribution of several elements in a medium, (b) microscopic properties (cross-sections) of their interaction with radiation and (c) the boundary values and source distribution - to the distribution (space, energy, angle or time-dependent) of some kinds of nuclear radiation. The inverse problem answers the question how to find (and when it is possible) the transport model parameters (the matter density, cross-sections, boundary values or sources) starting from the measured radiation field data.

The Boltzman neutron transport equation has never been explicity solved in its general form for either the direct problem or the inverse one. The main research effort for the inverse problem was made so far to develop the inverse solutions related to some approximations of the direct problem. Most of the referred authors considered the one-speed approximation with finite order of angular anisotropy in the slab geometry. However the spherical [Canfield, 1974] and arbitrary [Kanal and Davies, 1979] geometry, time dependence [Canfield, 1974] and multigroup approximation [Siewiert et al 1977, Siewiert, 1978a] have been introduced. In the one-speed case the rigorous spectral analysis considerations based on the Case theory are in progress [Kanal and Mosee, 1978, McCormick and Veeder, 1978, Siewiert, 1978c, Sengupta, 1980]. Siewiert [1978c, 1980] and others managed to confine the required measurements, appropriate to inverse one-speed problem solution, to medium boundaries only instead of all of the volume radiation sampling. Interesting results for multigroup case were obtained by Larsen [1981]. In a recent work McCormick and

Sanchez made an advanced attempt for the evaluation of anisotropic scattering coefficients.

The above results concerned the homogeneous medium case and they could give the successive terms in angular expansion of macroscopic cross-sections. One can see as two main practical applications of these results: (a) the evaluation of cross-sections from experiments with not infinitely thin samples, (b) identification of the isotopic concentrations, comparing the experimental values with characteristic angular and energy radiation transfers defined for each isotope.

We will try to consider the following inverse problem: on the basis of convenient radiation measurements and knowledge of all necessary microscopic cross-sections, to reach some predictions on space distribution of a given element in a medium. The first chapter states the problem and introduces the fundamental equation (11). The introductory arguments and some standard derivation may be easily omitted by an experienced reader or directly replaced by the volume integrated neutron transport equation (12), which application is considered from practical point of view. The proceeding discussion of equation (11) illustrates only the difficulties and some possible uncertainties of the problem. In the second chapter a further analysis in the particular case of total flux, with the infinite spherical geometry is performed, and in the third chapter a numerical example is presented, with respect to possibility of future application.

2. The statement of inverse problem.

Let us consider a medium of volume V, having boundary at Γ and containing the stationary point source $\tilde{S}$ of neutrons, with energy distribution described by G groups. To solve the direct problem we need to know the boundary values of the sought neutron multigroup angular flux $\Psi(\vec{r},\vec{\Omega})$, which is G-component column vector, as is the $\tilde{S}$ group vector.

We also need to know the macroscopic cross-sections: the total $\tilde{\Sigma}^{(t)}(\vec{r})$ which is G x G diagonal group matrix, and the group-to-group scattering cross-section $\tilde{\Sigma}^{(s)}(\vec{r}, \cos\theta_{LAB})$ (containing fission and transfers to the groups of secondary particles, if they are included) which is G x G group matrix and depends on the scattering angle cosine in the laboratory system $\cos\theta_{LAB} = \vec{\Omega}' \cdot \vec{\Omega}$.

Using this notation the system of multigroup transport equations, which is to be solved in the direct problem, can be written as:

$$\vec{\Omega}\nabla\tilde{\Psi}(\vec{r},\vec{\Omega}) + \tilde{\Sigma}^{(t)}(\vec{r})\,\tilde{\Psi}(\vec{r},\vec{\Omega}) = \int_{4\pi} d\Omega\, \tilde{\Sigma}^{(s)}(\vec{r},\vec{\Omega}'\cdot\vec{\Omega})\,\tilde{\Psi}(\vec{r},\vec{\Omega}') +$$

$$+ \tilde{S}(\vec{\Omega})\,\delta(\vec{r}) \qquad (1)$$

In the inverse problem we are loking for a transformed form of that equation, convenient for determining the densities of elements, defined as a number of the particular nuclei in the unit of volume. The appropriate flux measurements will be used as data. The natural course taken by most of the authors is to integrate the transport equation (1) over the spatial and angular variables, using some pertinent functions to derive the corresponding flux moments.

We do this term by term with equation (1), using in our case the spherical harmonics $Y_{lm}(\vec{\Omega})$ (see for example Cushing 1975 for definition, both the indexes are down-written in our paper). With the use of the Gauss theorem the first term leads us to:

$$T1 = \int d^3\vec{r} \int_{4\pi} d\vec{\Omega}\, Y^*_{lm}(\vec{\Omega})\,\vec{\Omega}\nabla\tilde{\Psi}(\vec{r},\vec{\Omega}) =$$

$$= \int ds \int_{4\pi} d\vec{\Omega}\,\vec{\Omega}\,Y^*_{lm}(\vec{\Omega})\,\tilde{\Psi}(\vec{r}_s,\vec{\Omega}) = \tilde{J}_{lm} \qquad (2)$$

As one can see, the first term depends on the boundary values of the flux only, so it is written as a constant.

Let us expand the angular flux in each group (the same we will do below for the source) in terms of spherical harmonics:

$$\tilde{\psi}(\vec{r},\vec{\Omega}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \tilde{\psi}_{lm}(\vec{r})\, Y_{lm}(\vec{\Omega}) \qquad (3)$$

where:

$$\tilde{\psi}_{lm}(\vec{r}) = \int_{4\pi} d\vec{\Omega}\, Y^{*}_{lm}(\vec{\Omega})\, \tilde{\psi}(\vec{r},\vec{\Omega}) \qquad (4)$$

Then after the integration we obtain the second term:

$$T2 = \int_{V} d^{3}\vec{r}\; \tilde{\Sigma}^{(t)}(\vec{r})\, \tilde{\psi}_{lm}(\vec{r}) \qquad (5)$$

In the third scattering term we expand the multigroup macroscopic cross-sections in Legendre polynomials of $\cos\theta_{LAB}$ and replace them by spherical functions, using the well-known addition theorem (see for example Cushing, 1975):

$$\tilde{\Sigma}^{(s)}(\vec{r},\vec{\Omega}'\cdot\vec{\Omega}) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi}\, \tilde{\Sigma}^{(s)}_{l}(\vec{r})\, P_{l}(\vec{\Omega}'\cdot\vec{\Omega}) =$$

$$= \sum_{l=0}^{\infty} \tilde{\Sigma}^{(s)}_{l}(\vec{r}) \sum_{m=-l}^{l} Y^{*}_{lm}(\vec{\Omega}')\, Y_{lm}(\vec{\Omega}) \qquad (6)$$

Now we perform integration over the angles, using the orthogonality property of spherical harmonics, and finally the $d^{3}\vec{r}$ integration. We obtain:

$$T3 = \int_{V} d^{3}\vec{r}\; \tilde{\Sigma}^{(s)}_{l}(\vec{r})\, \tilde{\psi}_{lm}(\vec{r}) \qquad (7)$$

The last, source term of equation (1) gives after integration :

$$T4 = \int_V d^3\vec{r} \int_{4\pi} d\vec{\Omega} Y_{lm}^*(\vec{\Omega}) \tilde{S}(\vec{\Omega}) \delta(\vec{r}) = \tilde{S}_{lm} \qquad (8)$$

Setting together the successive terms of integrated equation (1) we obtain the following system of equations:

$$\int_V d^3\vec{r} [\tilde{\Sigma}^{(t)}(\vec{r}) - \tilde{\Sigma}_l^{(s)}(\vec{r})] \tilde{\Psi}_{lm}(\vec{r}) = \tilde{S}_{lm} - \tilde{F}_{lm} \qquad (9)$$

Assuming that the medium consists of N elements with unknown densities $g_n(\vec{r})$, $(n=1,..,N)$ and that we know all elemental microscopic cross-sections, the total $\tilde{\sigma}_n^{(t)}$ and for the multigroup anisotropic transfer $\tilde{\sigma}_{nl}^{(s)}$, from the multigroup data files, we can write:

$$\tilde{\Sigma}^{(t)}(\vec{r}) - \tilde{\Sigma}_l^{(s)}(\vec{r}) = \sum_{n=1}^{N} g_n(\vec{r}) [\tilde{\sigma}_n^{(t)} - \tilde{\sigma}_{nl}^{(s)}] = \sum_{n=1}^{N} g_n(\vec{r}) \tilde{A}_{nl} \qquad (10)$$

This equation defines the $\tilde{A}_{nl}$ multigroup matrix. We note, that the k-th diagonal element A_{nl}^{kk} /all superscript indexes are the energy group indexes introduced in that place explicitely/ is the P_l angular coefficient of microscopic cross-section for leakage from k-th group, corresponding to n-th element. Each k-th row contains the P_l term of income to k-th group from other groups, counted as negative.

Inserting definition (10) into equation (9) we obtain the basic set of problem equations:

$$\sum_{n=1}^{N} \sum_{i=1}^{G} A_{nl}^{ik} \int_V d^3\vec{r}\, g_n(\vec{r}) \Psi_{lm}^i(\vec{r}) = S_{lm}^k - F_{lm}^k \qquad (11)$$

$(k=1,..,G)$ $(m=-l,..,l)$ $(l=1,..,G)$

In the inverse problem for the isotope densities in a medium we assume the neutron multigroup angular flux to be known from the measurements, and we treat the set of N densities as N unknown functions $g_n(\vec{r})$, for which there exists $G \times L \times (2L+1)$ relations (11). Therefore equation (11) represents our starting point for the inverse problem. Let us notice that the integral expressions in equation (11) correspond to the space correlation between the n-th elemental density function and the "i,l,m -th" multigroup angular flux.

Unfortunately, in the general case of an inhomogeneous medium consisting of several elements the problem (11) is not uniquely solvable. In principle, the algebraic values of $N \times G \times L \times (2L+1)$ integral functionals of sought density functions can be uniquely obtained from $G \times L \times (2L+1)$ algebraic equations if $N=1$, i.e. one-element medium only. Moreover, it is impossible to determine an unknown function, knowing the values of its convolutions with other functions only.

There are usually two possible ways [Sebatier, 1978] of further treatement of the problem: (a) by assuming some additional limitations on g functions, for example by using the regularisation method, where the g and their derivatives are restrained and the Ψ stability is required, and (b) by trying to obtain a mean value and a few higher moments of g instead of its exact values.

The latter way is preferred and below a further analysis of equation (11) is presented, which depends on the geometrical symmetry of particular medium and the approximation assumed in the integration. Experimental complications make us confine the data required in discussion of equation (11) to the total (angle-integrated) neutron flux measurements. We note that $\tilde{\Psi}_{00} \equiv \tilde{\Psi}$ is the group vector of total fluxes, and $\tilde{\Sigma}_0^{(s)} \equiv \tilde{\Sigma}^{(s)}$ is the angle-integrated transfer cross-section matrix. In addition we consider the case of an isotropic source $S_{00} \equiv \tilde{S}$ (the "00" spherical indexes will be suppressed below). Moreover, we take into account the quasi-infinite subcritical

medium, where the boundaries Γ are so far from a source that in all groups the boundary values term $\tilde{F}_{1m}$, containing the fluxes at the boundaries vanishes. Finaly, the simplyfied form of equation (11) will be analysed:

$$\sum_{n=1}^{N} \sum_{i=1}^{G} A_n^{ik} \int_V d^3\vec{r}\, \sigma_n(\vec{r})\, \phi^i(\vec{r}) = S^k \qquad (k=1,..,G) \qquad (12)$$

We will use the spherical symmetry of a medium, which simplifies the problem presented by equation (12) even more. In the section 3.1 we suggest a practical realisation of the method in that geometry, with discrete data points integration. In the section 3.2 the problem of mean isotope densities is discussed, including the simplest case of homogeneous medium. In section 3.3 some assumptions about the flux behaviour lead us to higher spatial moments of density functions. In the section 3.4 the comparison of the data from the medium studied with those from one particular medium, called the calibration model, is made (we mean as the calibration model a medium, where both the elemental densities from independent chemical analysis and the direct problem data from measurement or neutron transport evaluation are known).

3.1 The practical analysis of the method in spherical geometry.

Let assume that the concentration zones and the flux behaviour depend on the one radial variable r only. There is a stationary isotropic source at $r=0$ and a hole, as small as possible, in which a moving detector of neutrons with proper energy ranges is placed. In nuclear well-logging for example, it would be a borehole, and in this first approach we neglect its deforming influence over the neutron flux. We move the detector away from the source to a distance, where fluxes in all

groups vanish. We register the neutron flux $\phi^i(r_p)$ in all $i=1,..,G$ at successive $p=1,..,P$ points at distances between them equal to δr_p. All these data can be recorded and integrated by rectangular, Simpson or other numerical method. In these conditions, equation (12) can be rewritten as:

$$\sum_{n=1}^{N} \sum_{i=1}^{G} A_n^{ik} \sum_{p=1}^{P} 4\pi r_p^2 \delta r_p \phi^i(r_p)\, g_{np} = S^k \qquad (k=1,..,G) \qquad (13)$$

Here the rectangular integration is being used and g_{np} are the mean values of n-th element density $g_n(r)$ in the radial interval $[r_p,\ r_p+\delta r_p]$. Using the known multigroup constants, we can form the matrix of coefficients:

$$A_{np}^k = 4\pi r_p^2\, \delta r_p \sum_{i=1}^{G} A_n^{ik}\, \phi^i(r_p) \qquad (14)$$

After the experiment is finished, we solve the obtained system of G algebraical linear equations for N x P unknowns g_{np}:

$$\sum_{n=1}^{N} \sum_{p=1}^{P} A_{np}^k\, g_{np} = S^k \qquad (k=1,..,G) \qquad (15)$$

For the uniqueness reason a minimum of $G = N \times P$ energy groups are required in order to search for densities of N elements in P radial intervals.

3.2 The case of homogeneous medium and constant concentration approximation.

Let us introduce the mean flux-averaged concentration $\langle g_n \rangle_1$ as the volume integral weighted by some group flux:

$$\langle \varepsilon_n \rangle_i = \int_V d^3\vec{r}\, \varepsilon_n(\vec{r})\, \phi^i(\vec{r}) \,/\, \int_V d^3\vec{r}\, \phi^i(\vec{r}) \qquad (16)$$

The below relation is absolutely true in a homogeneous medium: $\langle \varepsilon_n \rangle_i = \langle \varepsilon_n \rangle_j$. This situation corresponds to space neutron flux - elemental density correlation being independent of the energy group number. We would treat this assumption as a good approximation in the case of weak inhomogeneity and narrow, close energy groups with no resonance scattering. Using this, we can take out the mean concentrations $\langle \varepsilon_n \rangle$ in equation (12) from the integral and setting down:

$$A_n^k = \sum_{i=1}^{G} A_n^{ik} \int_V d^3\vec{r}\, \phi^i(\vec{r}) \qquad (17)$$

we obtain the system of linear algebraic equations:

$$\sum_{n=1}^{N} A_n^k \langle \varepsilon_n \rangle = S^k \qquad (k=1,..,G) \qquad (18)$$

where the $G = N$ energy groups are required, i.e. the number N of elements being considered requires the knowledge of N flux energy groups.

3.3 The higher spatial moments from empirical flux interpolation formulas.

Having in mind presence of continuous spectrum of the transport operator and the troubles with the rigorous spectral analysis in the case of multigroup energy dependance and inhomogeneity of a medium [Larsen and Zweifel, 1976 and others], it seems to be impracticable to expand the multigroup flux in the series of characteristic functions of transport operator. We may try, however, to approximate the problem by

fitting the fluxes within the groups to a finite set of linear-independent functions $\{ w_p(\vec{r})(p=1,..,P)\}$:

$$\phi^i(\vec{r}) = \sum_{p=0}^{P} a_p^i \, w_p(\vec{r}) \qquad (19)$$

We do not consider at this moment the justification of a practical choice of proper $\{ w_p\}$ function basis, which may be the subject of a separate analysis - we state only here a few general remarks. It is natural to take $w_p \in L^1[0,\infty]$, which means the obvious requirement, that the total neutron number in the system should be finite. In that case some simple considerations from the theory of integrable functions [Cushing, 1975] give us the density functions placed in the adjoint space $g_n \in L^\infty[0,\infty]$, and in the corresponding norm only the supremal values of the densities are bounded.

Substituting in general case:

$$A_{np}^k = \sum_{i=1}^{G} d_n^{ik} \, a_p^i \qquad (20)$$

we get in place of equation (12) a system of linear equations :

$$\sum_{n=1}^{N} \sum_{p=0}^{P} A_{np}^k \, g_{np} = S^k \qquad (k=1,..,G) \qquad (21)$$

of G equations for N x P spatial moments of densities $g_n(r)$, defined on the basis $\{ w_p\}$ as:

$$g_{np} = \int_V d^3\vec{r} \, w_p(\vec{r}) \, g_n(\vec{r}) \qquad (22)$$

There is the relation G = N x P between the required number of groups, number of elements and number of considered spatial moments.

Unfortunately, only the polynomial interpolation of the flux, which is bad in respect to the flux shape, gives an access to very simple interpretation of the p-th moment of n-th isotope density, written here as:

$$\varepsilon_{np} = \int_0^{r_\infty} dr\ 4\pi r^{2+p}\ g_n(r) \qquad (23)$$

which are in this particular case: ε_{n0} - the total content, $\varepsilon_{n1}/\varepsilon_{n0} = \langle r\rangle_n$ - the average position r, and $(\varepsilon_{n2} - \varepsilon_{n1}^2)/\varepsilon_{n0} = \langle (r-\langle r\rangle)^2\rangle_n$ - the dispersion of position, for n-th isotope spatial distribution in the medium.

However those moments may appear also in other interpolation formulas. For points not too close to a source a more realistic fit to the shape of each group flux would be an exponential multiplied by a polynomial [Davison, 1958].
A semi-empirical formula for spherical geometry is proposed:

$$\phi^i(r) = \frac{\exp(-\alpha^i r)}{4\pi r^2} \sum_{j=0}^{J} \beta_j^i (r)^j \qquad (24)$$

where α^i are the effective multigroup decay constants and β_j^i are power coefficients. Expanding the exponential term in a polynomial series we may introduce the following coefficient for $(p+j)$ power of r:

$$A_n^k(p+j) = \frac{(-1)^p}{p!} \sum_{i=1}^{G} A_n^{ik}\ \beta_j^i\ (\alpha^i)^p \qquad (25)$$

The exponential expansion is cut at $p=P$. We again have a system of G equations for N x (J+P) moments, defined as in equation (23):

$$\sum_{p=0}^{P} \sum_{j=0}^{J} A_n^k(p+j)\ \varepsilon_{n(p+j)} = S^k \qquad (k=1,..,G) \qquad (26)$$

Since the convergence of the exponential is slow, a large number of groups $G = N \times (P+J)$ must be considered.

3.4 Comparison with the calibration model.

If we perform the measurements or calculations for a calibration model, we know for one medium both the isotope densities and the flux values. The comparison between the fluxes ϕ' from model and ϕ from the considered medium may then be related to the comparison between their elemental densities g'(model) and g(medium being considered). Following the idea of Ronen [1979] we write, using the mean value theorem, that there exists such a point $\vec{r}_n$ for which:

$$\int_V d^3\vec{r}\ \phi^i(\vec{r})\, g_n(\vec{r}) = \frac{\phi^i(\vec{r}_n)}{\phi'^i(\vec{r}_n)} \frac{g_n(\vec{r}_n)}{g_n'(\vec{r}_n)} \int_V d^3\vec{r}\ \phi'^i(\vec{r})\, g_n'(\vec{r}) \qquad (27)$$

The continuous dependence of densities on position in both media is required. The product $\phi^i(\vec{r})\, g_n(\vec{r})$ represent a local value at $\vec{r}$ of virtual reaction rate due to neutrons in the group i interacted with the element n, and the integral represent a total value of virtual reaction rate. Since that the physical meaning of the theorem includes a fact, that there exists such a common point $\vec{r}_n$ in both media, where the ratio of local virtual reaction rate to the total virtual reaction rate is the same for both media.

The integrals on the right side of equation (27) denoted as B_n^i can be later calculated from measured or evaluated direct problem data for the calibration model. Of course we do not explicitely know the exact value of the point $\vec{r}_n$, but we can obtain some approximations for the medium and modal density ratio $G_n = g_n'/g_n$, using in the following matrix of coefficients:

$$A_n^k = \sum_{i=1}^{G} A_n^{ik} B_n^i \Phi^i \qquad (28)$$

the mean and the extremal values of the model and medium fluxes ratio $\Phi^i = \sigma^i / \varphi^i$. The corresponding systems of linear algebraic equations for G_n approximation are:

$$\sum_{n=1}^{N} A_n^k G_n = B^k \qquad (k=1,..,G) \qquad (29)$$

4. The numerical tests.

The first numerical test of the method was performed for homogeneous spherical medium containing two elements: carbon and silicon, introduced with densities:

$$\varepsilon_C = 0.060 \qquad \varepsilon_{Si} = 0.040$$

in [atoms/barn.cm], i.e. in the range of 1.0-2.0 $g \cdot cm^{-3}$.

The direct problem was solved using the MORSE Monte-Carlo code [Straker et al., 1970], with 10^4 machine particles on the CYBER-73 CDC 6600 computer. Two groups of fast neutrons were considered from the CASK multigroup data base: 14.0-12.2 and 12.2-10.0 MeV. There was one group source in the first group with the neutron output of 1.0 [neutrons/s]. The MORSE output analysis routine scored the both group flux for 10 detectors in 5 cm radial intervals up to 50 cm.

For the neutron flux data obtained from MORSE in [neutrons/s·cm^2·eV] we performed the group widths integration and the space integration by the Simpson rule, and we obtained the integrated fluxes in groups $\tilde{\Phi}$:

$$\tilde{\Phi} = \begin{pmatrix} 10.98 \\ 2.72 \end{pmatrix}$$

in [neutrons·cm·s^{-1}]. The cross-section matrices were:

$$\tilde{A}_C = \begin{pmatrix} 0.84 & 0.00 \\ -0.26 & 0.80 \end{pmatrix} \qquad \tilde{A}_{Si} = \begin{pmatrix} 1.18 & 0.00 \\ -0.24 & 1.28 \end{pmatrix}$$

in [barn/atom]. Setting the above data into the coefficients definition (17), we obtained the system given by equation (18) in the form:

$$9.22\,\varepsilon_C + 12.96\,\varepsilon_{Si} = 1.00$$
$$-0.68\,\varepsilon_C + 0.85\,\varepsilon_{Si} = 0.00$$

So for the energy groups used here one has obtained the inverse problem solution for densities:

$$\varepsilon_C = 0.051 \qquad \varepsilon_{Si} = 0.041$$

in [atoms/barn·cm]. The error of the method in this example is about 15% for C and 2% for Si. Making allowance for the mean statistical error of MORSE results, about 8% in our case, the agreement seems to be promising.

In the second example we used the same two elements, but there were three spherical inhomogeneity regions in the medium, with densities:

$0 < r \leq 15$ cm :	$\varepsilon_C = 0.060$	$\varepsilon_{Si} = 0.040$
$15 < r \leq 25$ cm :	$\varepsilon_C = 0.080$	$\varepsilon_{Si} = 0.080$
$25 < r \leq 50$ cm :	$\varepsilon_C = 0.010$	$\varepsilon_{Si} = 0.020$

[atoms/barn·cm]. The space and energy integrated group fluxes were similar to the first problem:

$$\tilde{\Phi} = \begin{pmatrix} 10.35 \\ 2.74 \end{pmatrix}$$

[neutrons·cm·s^{-1}]. The approximation (18) gives in [atoms/

/barn·cm] :

$\langle g_C \rangle = 0.056$ $\langle g_{Si} \rangle = 0.043$

We must remember that these are the flux-averaged (not the volume-averaged) mean elemental densities. The knowledge of real values of the densities permits us to examine the assumption of weak dependence of the flux-averaged densities upon the energy group of the flux. We have obtained the results:

$\langle g_C \rangle_1 = 0.058$ $\langle g_{Si} \rangle_1 = 0.043$

$\langle g_C \rangle_2 = 0.053$ $\langle g_{Si} \rangle_2 = 0.044$

in [atoms/barn·cm]. However, the assumption of weak dependence on energy group would not necessarily be valid if the groups are not so close to each other and there is a physical difference between them, for example for the fast and thermal groups.

5. Conclusions.

The basic aim of this paper is to obtain some information about the isotopic composition of a material under study from changes of neutron energy spectrum observed at various distances. The space- and angular-integrated form of neutron transport equation seems to be most convenient for this kind of the inverse problem of neutron transport theory.

The presented equation (11) is quite general and exact within the frame of multigroup energy dependence formulation. This equation enables us to obtain an unique solution for one element case only, and too many specific angular-energy-space neutron field measurements are required from the experimental point of view in order to obtain the solution. Therefore, using the additional assumptions, the equation (12) is obtained. We need here the data on total (i.e. angle-integrated) flux, and in the case of spherical symmetry of an infinite

medium with an isotropic source the flux-averaged densities can be obtained. The approach of section 3.3 may give some additional information about the higher spatial moments of the densities. These assumptions are strictly correct, however, for the case of homogeneous medium thus here the best validity and utility of the method could be expected. We used such model to obtain some numerical evidence for the correctness of our analysis.

The Monte-Carlo multigroup neutron transport code was used to provide the trial direct problem data, analogous to these obtained from the proposed experiment. Then, the inverse problem subroutine was employed to compute densities of two considered elements, which were compared with the input values for Monte-Carlo. The error of the results of inverse problem evaluation for the first example of homogeneous medium has not exceeded 1%. In the second example, unhomogeneous, the weak dependance of the flux-averaged densities upon the energy group of the flux was confirmed.

There is now one visible area of application of our method, which is the neutron well-logging, with variable source-detector separation distance in a logging tool. Fortunately, in this case we essentially know the content of the rock medium and only the additional admixture of one or few minerals is prospected, which gives us hope to minimise the number of unknowns.

A c k n o w l e d g e m e n t s

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